The size of the proton

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1. Introduction

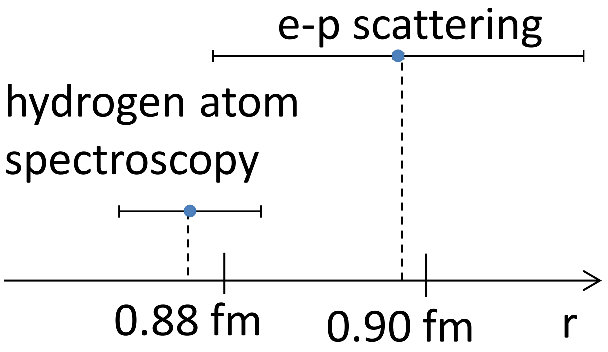
The existing data of the proton charge radius,

by electron-proton elastic scattering

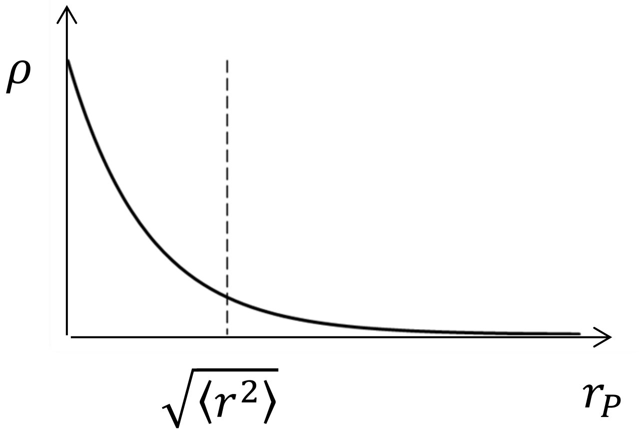
$$r_p = \sqrt{\langle r^2 \rangle} = 0.897(18)\ \text{fm}$$

by spectroscopy of hydrogen atoms

$$r_p = \sqrt{\langle r^2 \rangle} = 0.8768(69)\ \text{fm}$$



※The proton charge density distribution is not uniform. It is an exponential function. So we use the value of the root-mean-square $r_p = \sqrt{\langle r^2 \rangle}$



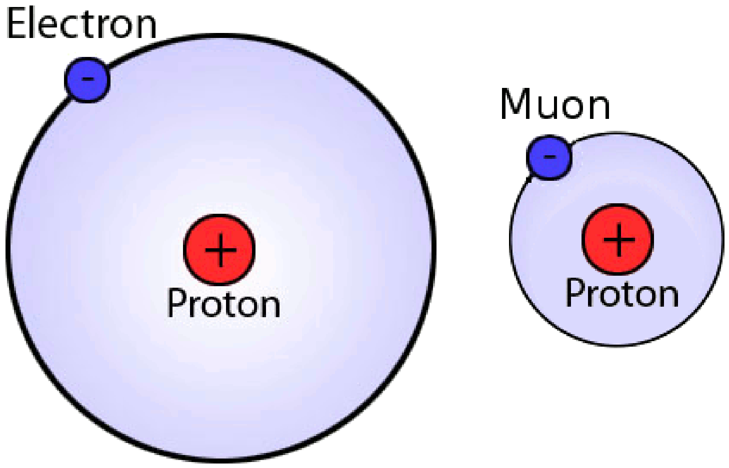
One way to improve the accuracy of the proton charge radius :

⇨ **Measure Lamb shift in muonic hydrogen**

2. Muonic atom

Muonic atom

- Electron in atom is replaced by μ^-
- Charge of μ^- is equal to charge of e^-
- Mass of μ^- is 200 times larger than mass of e^-
- From the equation of Bohr radius $r = \frac{\hbar^2 n^2}{k_0 e^2 m}$: radius of muonic hydrogen is 200 times smaller
- From the equation of binding energy $E = -\frac{m {k_0}^2 e^4}{2\hbar^2 n^2}$: binding energy of muonic hydrogen is 200 times larger



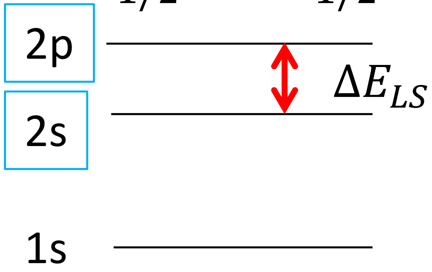
200 : 1

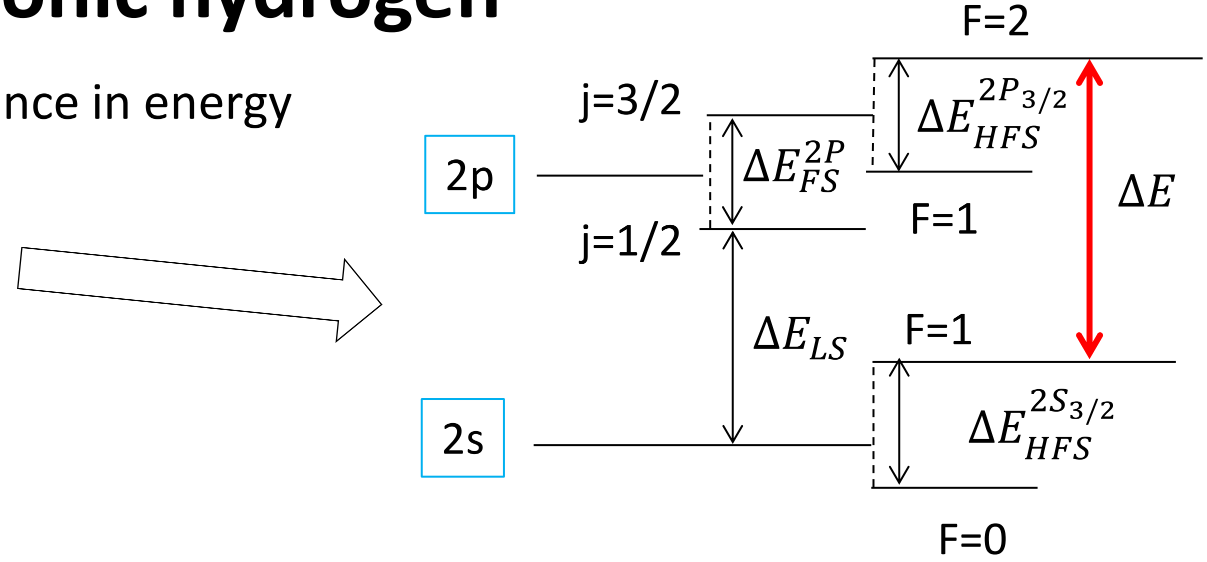
The ratio of H radius to muonic hydrogen radius

The contribution of the charge radius to Lamb shift is enhanced because of these properties

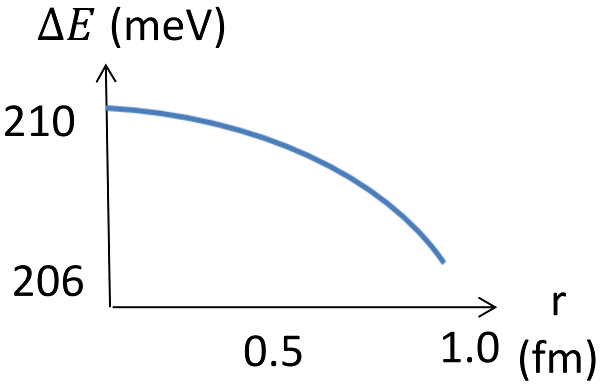
Lamb shift in muonic hydrogen

Lamb shift is a small difference in energy between $S_{1/2}$ and $P_{1/2}$





The energy ΔE is sum of :
Lamb shift ΔE_{LS},
fine structure ΔE^{2P}_{FS},
hyperfine structure $\Delta E^{2P_{3/2}}_{HFS}$, ΔE^{2S}_{HFS}



$$\Delta E = \Delta E_{LS} + \Delta E^{2P}_{FS} + \frac{3}{8}\Delta E^{2P_{3/2}}_{HFS} - \frac{1}{4}\Delta E^{2S}_{HFS}$$

ΔE_{LS} depends on r_P and others are constants

$$\Delta E = 209.9779(49) - 5.2262\, {r_P}^2 + 0.0347\, {r_P}^3 \text{ meV}$$

Measure ΔE , then r_P can be evaluated

3. Experiment

the muon beam

- the accelerator at the PSI was used
- The proton was accelerated to 590 MeV by the cyclotron at PSI
- By collision with a target, pions were produced in nuclear reaction
- Pion decays to μ^-

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

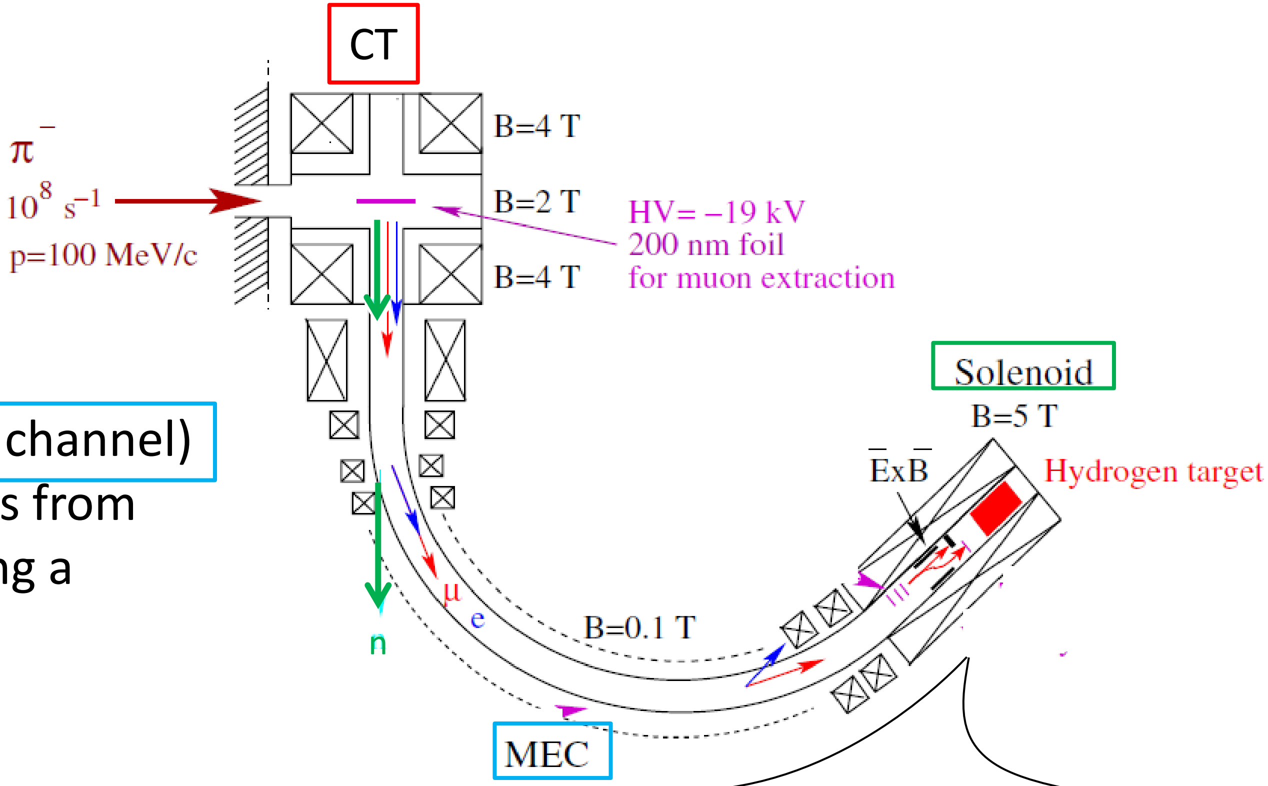
The Paul-Scherrer-Institute(PSI) in Switzerland

π^- enters the CT

- CT (Cyclotron trap)

π^-decays to μ^-

μ^- is decelerated by a thin foil



- MEC (muon extraction channel)

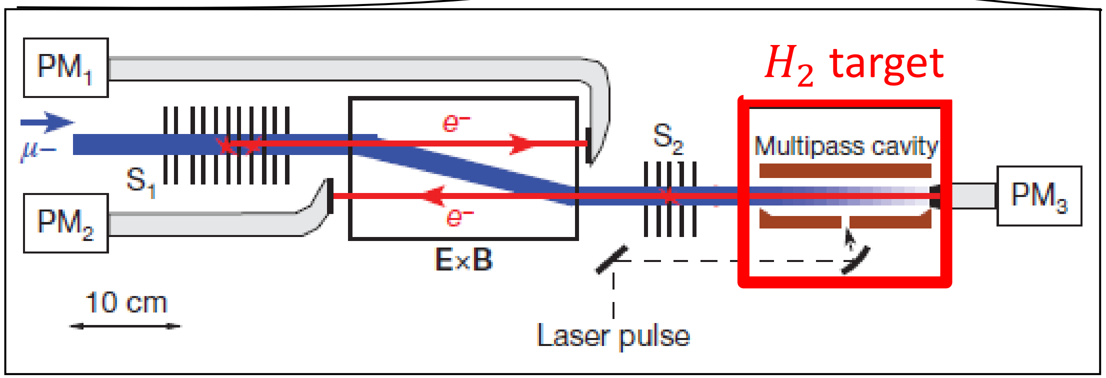
The MEC separates muons from background radiation using a magnetic field

- Solenoid

- High magnetic field of 5 T makes radial size of the beam small
- The purpose of the carbon foils S_1, S_2

① decelerate the beam

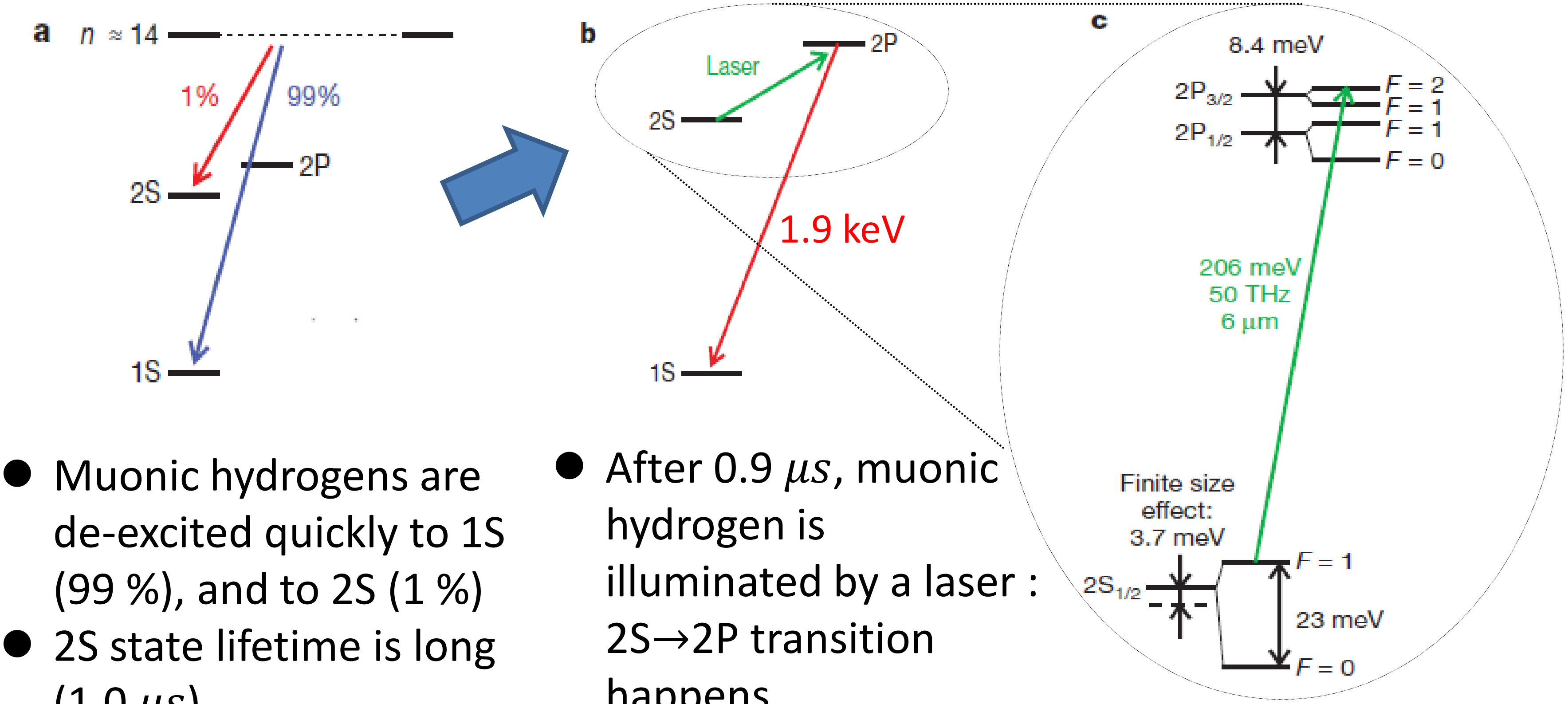
② measure the incident time by detecting secondary electron emission from the beam



↑inside of solenoid

The μ^- beam stops in H_2 gas, and muonic hydrogens are formed

- The muons stop in H_2 gas, and highly excited μ-p atoms are formed



- Muonic hydrogens are de-excited quickly to 1S (99 %), and to 2S (1 %)
- 2S state lifetime is long (1.0 μs)

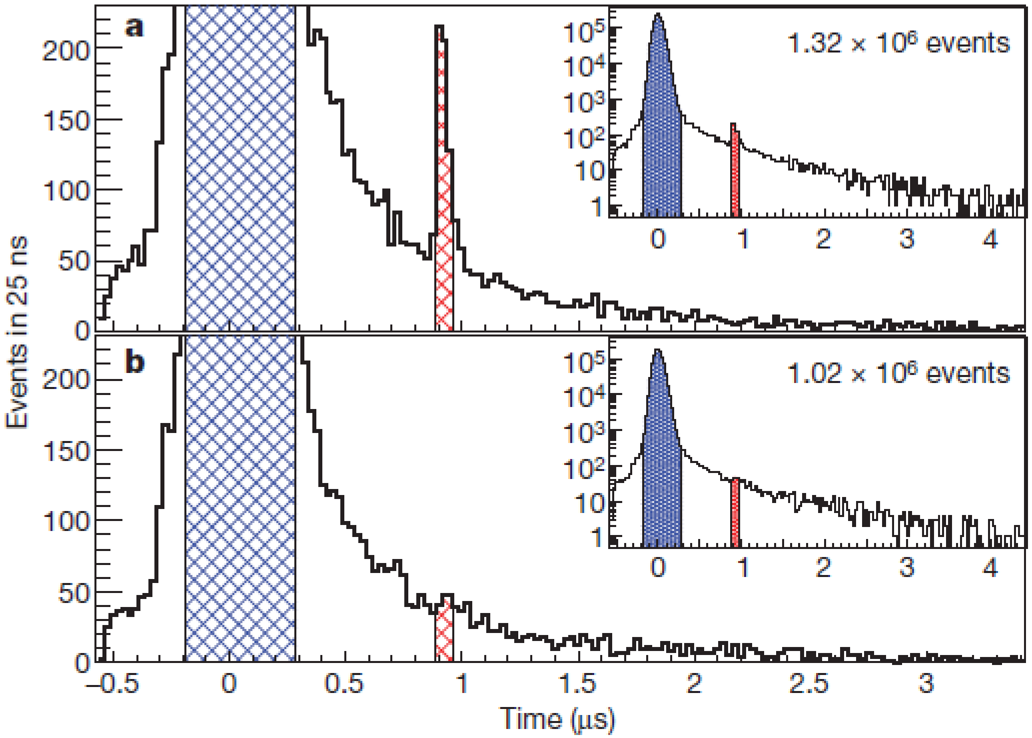
- After 0.9 μs, muonic hydrogen is illuminated by a laser : 2S→2P transition happens
- Immediately, 2P→1S de-excitation happens via emission of 1.9 keV X-rays

- LAAPDs (highly sensitive photo diodes) record the X-rays.

The number of X-rays indicates the existence of 2S→2P transition

4. Result

- Incident time of the muon beam is 0 μs in the histogram
- The large peak around 0 μs is due to prompt event (transition to 1S directly)
- The second peak around 0.9 μs is due to delayed event (2P→1S transition after illuminating by the laser)
- When the second peak appears, the laser has a correct energy of Lamb shift



X-ray time spectra at the different laser frequency
top: 2P→2S transition is observed
bottom: the transition is not observed

This graph shows the ratio of delayed event to prompt event as a function of the laser frequency.

peak frequency

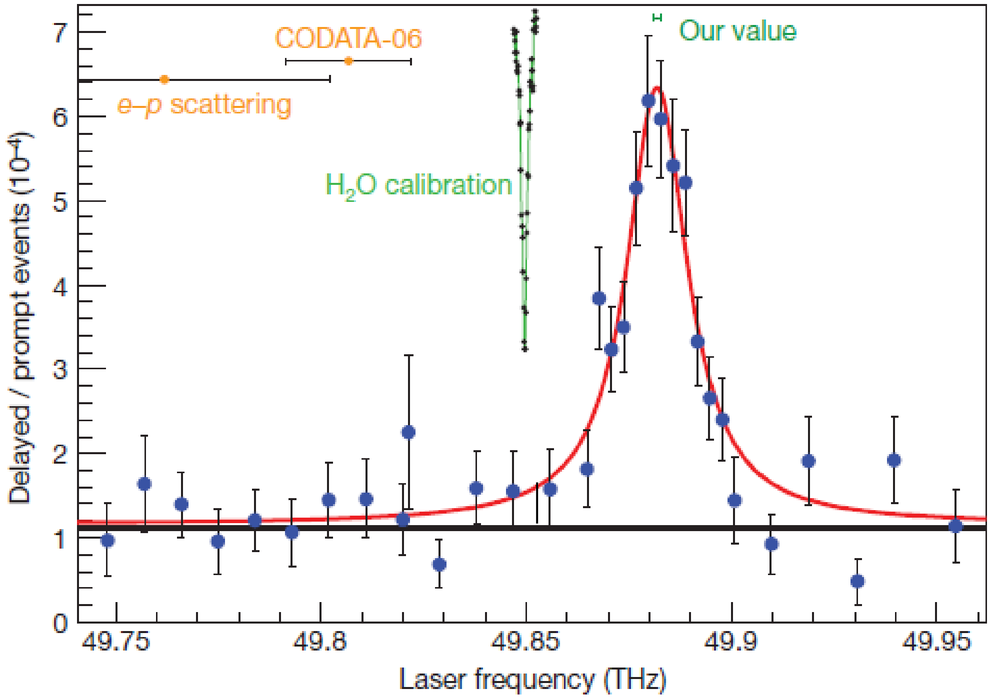
$$49{,}881.88(76)\ \text{GHz}$$

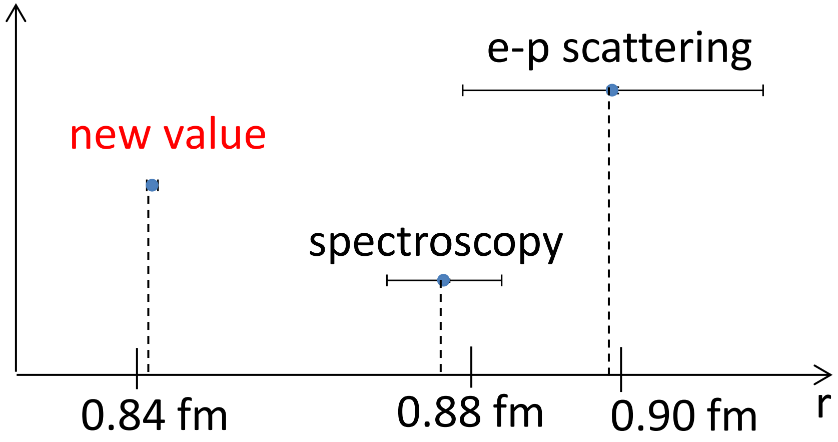
which means

$$\Delta E = 206.2949(32)\ \text{meV}$$

From the relation between Lamb shift and charge radius r_P

$$r_P = 0.84184(67)\ \text{fm}$$





The new value is smaller compared to the previous data, $0.8768(69)$ fm, $0.897(18)$ fm

The new value is more accurate than the previous data.
There is a large discrepancy between the data.

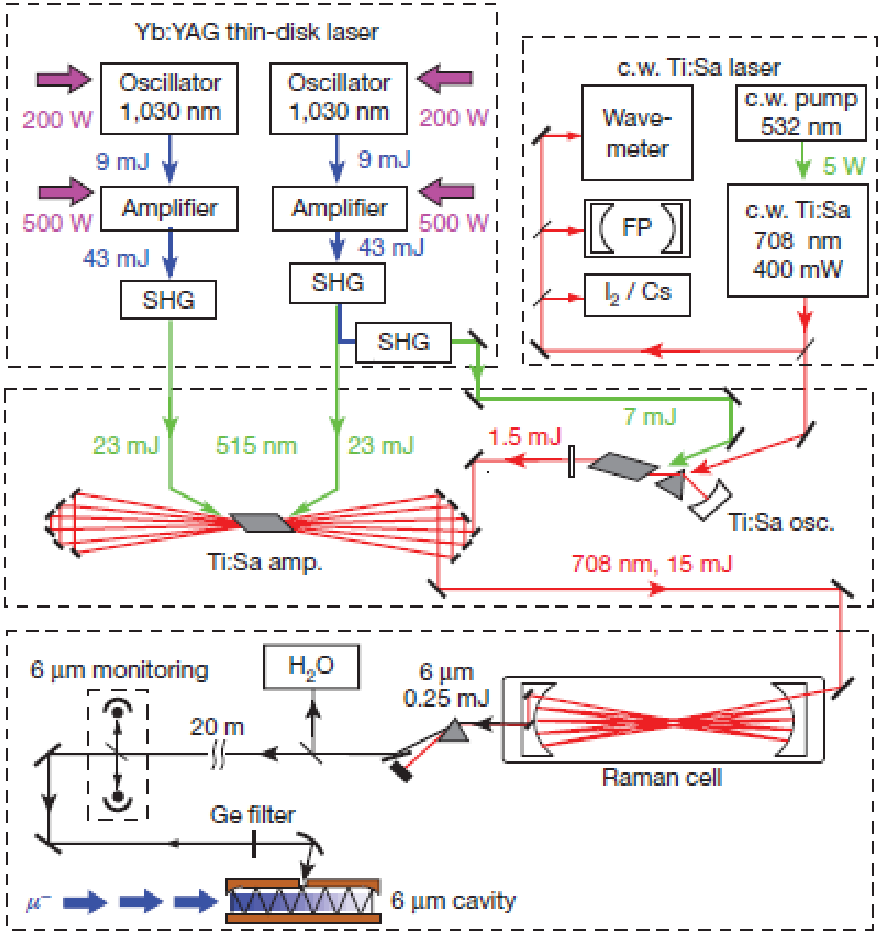
5. Summary

- In a muonic atom, an electron in an atom is replaced by μ^-.
- It is possible to determine the proton charge radius accurately by measuring Lamb shift (2S-2P transition).
- The laser illuminates muonic hydrogen. If the laser has the energy of Lamb shift, the transition is induced.
- The new value of the proton charge radius, 0.84184 fm, is smaller than the previous data.
- This discrepancy could be due to new physics.

Appendix

Laser system

- When muon beam incidents the solenoid, the laser system receives a signal
- After 0.9 μs laser launches
- The laser is amplified by this system
- The frequency of the laser is tunable



Appendix

Equation of $\Delta \boldsymbol{E}$

$$\Delta E_{LS} = 206.0573(45) - 5.2262 {r_P}^2 + 0.0347 {r_P}^3 \text{ meV}$$

$$\Delta E_{FS}^{2P} = 8.352082 \text{ meV}$$

$$\Delta E_{HFS}^{2P_{3/2}} = 3.392588 \text{ meV}$$

$$\Delta E_{HFS}^{2S} = 22.8148(78) \text{ meV}$$

ΔE is calculated from these terms

$$\Delta E = \Delta E_{LS} + \Delta E_{FS}^{2P} + \frac{3}{8} \Delta E_{HFS}^{2P_{3/2}} - \frac{1}{4} \Delta E_{HFS}^{2S}$$

$$\Delta E = 209.9779(49) - 5.2262\, {r_P}^2 + 0.0347\, {r_P}^3 \text{ meV}$$

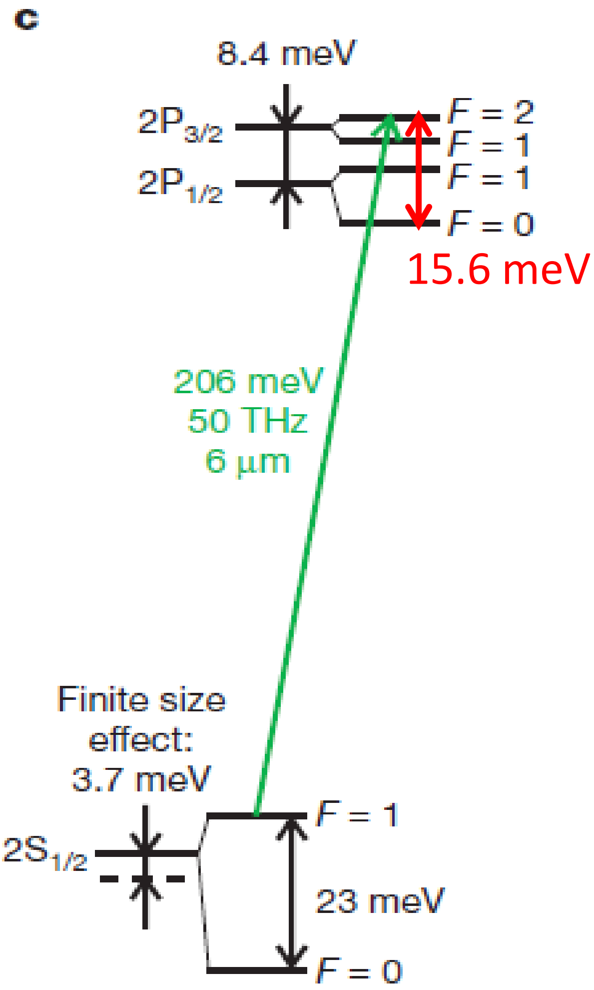
Appendix

Transition

Laser wavelength : 6.00 μm → 6.03 μm
Laser energy : 205.6 meV → 206.7 meV

$\boxed{2S_{1/2}\ (F=1) \rightarrow 2P_{3/2}\ (F=2) : 206\ \text{meV}}$
$2S_{1/2}\ (F=1) \rightarrow 2P_{3/2}\ (F=1) : 203\ \text{meV}$
$2S_{1/2}\ (F=1) \rightarrow 2P_{1/2}\ (F=1) : 198\ \text{meV}$
$2S_{1/2}\ (F=1) \rightarrow 2P_{1/2}\ (F=0) : 191\ \text{meV}$

$2S_{1/2}\ (F=0) \rightarrow 2P_{3/2}\ (F=2) : 229\ \text{meV}$
$2S_{1/2}\ (F=0) \rightarrow 2P_{3/2}\ (F=1) : 226\ \text{meV}$
$2S_{1/2}\ (F=0) \rightarrow 2P_{1/2}\ (F=1) : 221\ \text{meV}$
$2S_{1/2}\ (F=0) \rightarrow 2P_{1/2}\ (F=0) : 214\ \text{meV}$



Other transitions don't happen by this laser

F : total angular momentum