Cross Section
For Prompt Photon Production
in Proton-Proton Collisions
at $\sqrt{s} = 62.4$ GeV

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Abstract

Measurements of cross section for prompt photon production in proton-proton collisions at $\sqrt{s} = 62.4$ GeV in PHENIX are reported. The prompt photon production is expected to be useful to test the applicability of perturbative Quantum ChromoDynamics (pQCD), to determine gluon spin contribution $\Delta g$ to the proton spin through double helicity asymmetries $A_{LL}$ in polarized proton-proton collisions, and to investigate of Quark-Gluon Plasma (QGP). Once the applicability of pQCD is confirmed, the framework of pQCD can be used to predict other quantities of interest such as $A_{LL}$, cross section for other processes etc.

Proton is composed of quarks and gluons. Proton spin, 1/2, comes from the spin of quarks $\Delta q$, the spin of gluons $\Delta g$ and angular momenta of quarks and gluons. In 1980's, the quark spin contribution to the proton spin, $\Delta q$, is measured by European Muon Collaboration (EMC) at CERN, and is found to be very small. Now, measurements of $\Delta g$ and the angular momentum contribution of quarks and gluons to the proton spin are going on in various experiments.

For the measurement of prompt photon $A_{LL}$, the study of the cross section for prompt photon production is important as the first step. In this thesis work, the prompt photons are measured down to $p_T \sim 2$ GeV/$c$ for the first time. The prompt photons of low $p_T$ correspond to low $x$ gluons inside of the proton. Here $x$ is Bjorken $x$, the fraction of the parton momentum to the total proton momentum.

The prompt photon is mainly produced via quark-gluon scattering ($qg \rightarrow \gamma q$) in proton-proton collisions. The prompt photon carries information on the quark-gluon scattering. Therefore, the prompt photon production is one of the tools to extract the $\Delta g(x)$.

Quarks and gluons, which have a degree of freedom of color, are ordinarily confined in hadron as a color-singlet state. On the other hand, as the temperature or density of many-body system of hadrons increases, a normal nuclear state is expected to transform into another state of matter where quarks and gluons become color de-confined. This state is called QGP. For investigations of QGP which is produced in nucleus-nucleus collisions, the cross section for prompt photon production in proton-proton collisions is important as a reference.

PHENIX, which stands for Pioneering High Energy Nuclear Interaction eXperiment, is an experiment at Brookhaven National Laboratory (BNL) using the Relativistic Heavy Ion Collider (RHIC). My work at PHENIX is,
as the first step, to operate and optimize the modules of ERT (Electro-
Magnetic Calorimeter (EMCal)-RICH). The fire status of the ERT trigger
module and related electronics are carefully checked. The ERT trigger plays
an important role in the measurement of photon, electron, etc. As the second
step, I measured the cross section for prompt photon production in proton-
proton collisions at $\sqrt{s} = 62.4$ GeV. This $\sqrt{s}$ is achieved with the colliding
proton beam of 31.2 GeV. In this measurement, the polarization of the proton
beam is averaged out. The data are taken in 2006. The data are accumulated
with PHENIX central arm spectrometer with the ERT trigger ($\leq 0.6$ GeV).
The integrated luminosity is about 0.065 pb$^{-1}$. The PHENIX central arm
spectrometer consists of two arms which are placed almost back-to-back on
their azimuth. Each arm covers pseudo-rapidity $|\eta| < 0.35$ and 90° azimuthal
angle.

The cross section for prompt photon production as a function of $p_T$ is
measured over 4 orders of magnitude from 2 to 7 GeV/c. In particular, the
cross section in the $p_T$ region from 2 to 4 GeV/c is measured for the first
time. Here, $p_T$ is the transverse momentum. The cross section shows a
steep decrease as $p_T$ increases. The cross section is consistent with the one
measured earlier at high $p_T$ at CERN-ISR. It is found that the cross section
predicted by pQCD is lower than the measured cross section at low $p_T$ region.
The relation between the prompt photon $p_T$ and gluon $x$ is evaluated using
PYTHIA simulation. The gluon $x$ proved in this measurement is 0.025 $\sim 0.3$
corresponding to $p_T = 2 \sim 7$ GeV/c. The gluons are localized at small $x$, so
the study of gluons in this low $x$ region is important. The present result of
prompt photon production at low $p_T$ provides a basis for further theoretical
studies of QCD, gluon spin contribution measurement and the study of QGP.
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Chapter 1

Introduction

Proton is composed of quarks and gluons. The spin of proton, 1/2, comes from the spin of quarks, gluons and their angular momenta;

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta g + L_{\text{quark,gluon}}. \] (1.1)

Here \( \Delta \Sigma, \Delta g \) and \( L_{\text{quark,gluon}} \) is the contribution of the quark spin, the contribution of the gluon spin, and the contribution of the quarks and gluons angular momenta. In 1980’s, \( \Delta q \), was measured by EMC at CERN, and was found to be \( 12 \pm 9 \pm 14\% \). The remaining part of the proton spin must be carried by gluons and angular momenta of quark and gluons.

The measurement of \( \Delta g \) is being performed by PHENIX through measurement double helicity asymmetry (\( A_{LL} \)) in polarized proton-proton collisions. PHENIX is an experiment at BNL using RHIC. The RHIC is the first collider to be able to use the polarized proton beam. The \( A_{LL} \) is defined as;

\[ A_{LL} = \frac{\sigma^{++} - \sigma^{--}}{\sigma^{++} + \sigma^{+-}}, \] (1.2)

\( \sigma^{++(+-)} \) is cross section for particle production such as prompt photon, etc. in the proton-proton collision with beam helicity pattern “++ (+-)”. \( A_{LL} \) can also be calculated from theory side based on a perturbative QCD assuming the function of \( \Delta g(x) \). \( \Delta g(x) \) is the probability of finding the polarized gluons with the momentum fraction \( x \).

\[ \Delta g = \int_0^1 \Delta g(x) dx. \] (1.3)

By comparing the experimental \( A_{LL} \) with the theoretical \( A_{LL} \), the \( \Delta g(x) \) can be investigated. If the experimental \( A_{LL} \) agrees with the theoretical \( A_{LL} \), the
$\Delta g(x)$ used in the calculation can be judged to be correct. The coverage of PHENIX central arm spectrometer in pseudo-rapidity is $|\eta| < 0.35$. The probed Bjorken $x$ is estimated as follows;

$$x \sim x_T = \frac{2p_T}{\sqrt{s}},$$  \hspace{1cm} (1.4)

here $p_T$ is transverse momentum of a particle such as prompt photon, etc. So far, PHENIX has measured the $A_{LL}$ at $\sqrt{s} = 200$ GeV. In $\pi^0$ production, $\Delta g(x)$ in the region of $x = 0.02 \sim 0.3$ is studied. It is important to measure the $A_{LL}$ outside of this $x$ region as well. Therefore, the measurement of the $A_{LL}$ at different $\sqrt{s}$ such as 62.4 GeV is essential. pQCD can also predict the cross section for the prompt photon production in proton-proton collisions so that the test of pQCD prediction can be performed by comparing the measurement with the calculation.

Quarks and gluons, which have a degree of freedom of color, are ordinarily confined in hadron as a color-singlet state. As the density of many-body system is compressed by a factor of 10 compared with the ordinal density, nucleons overlap each other and are not independent particles. In such a state, quarks and gluons can freely move in the whole volume of many-body system. As the temperature of many-body system is raised with keeping the constant nucleon density, the density of many-body system increases because $\pi$'s are produced in the nucleon-nucleon interaction. Therefore, the collisions among hadrons are increased. In such state, quarks and gluons are not regarded as the confined particles in a specific hadron. These state is called QGP. After a few $\mu s$ of the Big Bang in the universe, QGP is believed to have been formed. The distance between quarks and gluons increased due to the expanding the universe and temperature of the universe decreased. So the force between quarks and gluons became stronger because of the asymptotic freedom of QCD. Then these particles are coupled with each other. By investigating QGP, the initial state of the universe can be studied.

QGP is being studied comparing nucleus-nucleus collisions and proton-proton collisions. The experimental efforts began at Bevelac at Lawrence Berkeley Laboratory with 2 GeV/A beam in 1970's. In the middle of 1980's, experiments with higher energies started at Alternating Gradient Synchrotron (AGS) in Brookhaven National Laboratory (BNL) with 14.6 GeV/A beam and at Super Proton Synchrotron (SPS) in CERN with 158 GeV/A beam. PHENIX performs the investigation of various QGP signatures using RHIC. The state of QGP can be detected by measuring the prompt photon radiation in nucleus-nucleus collisions. Photons are coupled with quarks by electromagnetic force, and the coupling constant is about 100 times weaker than the strong force. The photons are transparent in quarks matter. Therefore,
the cross section for prompt photon production in nucleus-nucleus collisions is essential quantity. The cross section for prompt photon production in proton-proton collisions is important as a reference to compare with that in nucleus-nucleus collisions.

In proton-proton collisions, the prompt photon is mainly produced via quark-gluon collision ($qg \rightarrow \gamma q$), and is also produced via quark-anti-quark annihilation ($q\bar{q} \rightarrow \gamma q$). The prompt photon is defined to be the photon from original parton-parton collisions, and not the photon from hadron decay. The diagram of the $qq \rightarrow \gamma q$ is shown in Figure 1.1. Figure 1.2 shows the

Figure 1.1: The schematic diagram of quark-gluon collision ($qg \rightarrow \gamma q$) in proton-proton collisions.

unpolarized parton distribution functions in a proton. Gluons exist much more than anti-quarks in $0 \leq x \leq 1$. Therefore, in proton-proton collisions, the $qq \rightarrow \gamma q$ process is the dominant process for prompt photon production. In particular, the gluons are localized at small $x$, so the study of gluons in this small $x$ region is important. The prompt photon carries information on the quark-gluon scattering. Therefore, the prompt photon production is one of the tools to extract the $\Delta g(x)$. The first evidence for the existence of prompt photons was obtained by R806 in proton-proton collisions. They measured the cross section as a function of $p_T$, and the measured $p_T$ range was about $4 \sim 12$ GeV/c.

As mention above, PHENIX is an experiment at BNL using RHIC and aims to study $\Delta g$ and to investigate QGP. RHIC is the first polarized proton-proton collider and high energy nucleus-nucleus collider up to $\sqrt{s} = 200$ GeV/A. My work at PHENIX is, as the first step, to operate and optimize the modules of ERT (Electro-Magnetic Calorimeter (EMCal)-RICH). The fire status of the ERT trigger module and related electronics are carefully checked. The ERT trigger plays an important role in the measurement of photon, electron, etc. As the second step, I measured the cross section for prompt photon production in proton-proton collisions at $\sqrt{s} = 62.4$ GeV. This $\sqrt{s}$ is achieved with the colliding proton beam of 31.2 GeV. In this
Figure 1.2: Unpolarized parton distribution functions (PDFs) determined by the CTEQ6.1M fit\textsuperscript{35}. Drawn as a yellow color are the PDFs by the ZEUS-JETS.

measurement, the polarized proton beam is taken the average. The data are taken in 2006. The data are accumulated with PHENIX central arm spectrometer with the ERT trigger (≤ 0.6 GeV). The integrated luminosity is about 0.065 pb\(^{-1}\), which corresponds to 894 M events. In this measurement, the polarized proton beam is taken the average. In addition, the polarization of proton beam is about 50%. The PHENIX central arm spectrometer consists of two arms which are placed almost back-to-back on their azimuth. Each of arm covers pseudo-rapidity |η| < 0.35 and 90° azimuthal angle. The used detectors for the measurement of prompt photon are EMCal and Pad Chamber, which is a kind of Multi-Wire Proportional Chamber, for the veto of charged particles.

In Chapter 2, theoretical background and prompt photon production are introduced. In Chapter 3, the RHIC facility and PHENIX detector are explained. In Chapter 4, analysis procedure for data selections, simulation studies and cross section measurement are described. Results and discussions of this research are in Chapter 5.
Chapter 2

Theoretical background and prompt photon production

2.1 Theoretical background

2.1.1 QCD Lagrangian

Quantum Chromodynamics is defined as a field theory by its Lagrange density;

\[ L_{\text{QCD}}^{\text{eff}}(\phi^i(x), \phi^i(x), A(x), c(x); g, m_f) = L_{\text{invar.}} + L_{\text{gauge}} + L_{\text{ghost}}, \quad (2.1) \]

which is a function of fields \((\phi^i(x), A(x) \text{ and } c(x))\), parameters \(g\) and \(m_f\). The \(\phi^i(x)\) is the four-component Dirac-spinors related to the field of each quark of color \(i\) and flavor \(f\). The \(A(x)\) is the gluon (Yang-Mills) fields. The \(c(x)\) is ghost fields. The \(g\) is the QCD coupling constant, and the \(m_f\) is quark mass.

The \(L_{\text{invar.}}\) is the classical density, invariant under local \(SU(N_c)\) gauge transformations, with \(N_c = 3\) (red, blue and green) for QCD. It was originally written down by Yang and Mills.
\[ \mathcal{L}_{\text{invar.}} = \sum_{f} \bar{\phi}_{f}^i [iD(A) - m_f \phi_f^i - \frac{1}{4} F^2(A)] \]
\[ = \sum_{f=1}^{n_f} \sum_{\alpha, \beta = 1}^{4} \sum_{\gamma = 1}^{N_c} \bar{\phi}_{f, \beta, \gamma} [i(\gamma)_{\beta, \alpha}^{\mu} D_{\mu, \beta, i} \phi_{f, \gamma, i} - m_f \delta_{\beta, \alpha} \delta_{\gamma, i} \phi_{f, \alpha, i}] \]
\[ - \frac{1}{4} \sum_{\mu, \nu = 0}^{3} \sum_{a = 1}^{N_c^2 - 1} F_{\mu, \nu}^a (A) F_{\alpha}^{\mu \nu} (A). \quad (2.2) \]

In the second expression, we have written out all indices explicitly, using the notations;

\[ D_{\mu, ij}(A) = \partial_{\mu} \delta_{ij} + ig \sum_{a} \frac{\lambda_{a}^{ij}}{2} A_{\mu a} , \quad (2.3) \]
\[ F_{\mu, \nu}^a (A) = \partial_{\mu} A_{\nu a} - \partial_{\nu} A_{\mu a} - g f_{abc} A_{\mu b} A_{\nu c} , \quad (2.4) \]

where the \( f_{abc} \) is the structure constants of the \( SU(3) \) algebra.

The \( \mathcal{L}_{\text{invar.}} \) cannot be quantized due to the arbitrary gauge transform of the \( \mathcal{L}_{\text{invar.}} \), and is needed to cancel non-observable physical quantities. These problems are solved by adding gauge-fixing \( \mathcal{L}_{\text{gauge}} \) and ghost densities \( \mathcal{L}_{\text{ghost}} \) to the \( \mathcal{L}_{\text{invar.}} \), as in Eq (2.1). Details of the lagrangians are described in \([29, 30, 31]\).

### 2.1.2 Asymptotic freedom

The successes of QCD in describing the strong interactions are summarized by two terms; “asymptotic freedom” and “confinement”. The asymptotic freedom\([16, 17, 18, 19]\) refers to the weakness of the short-distance interaction, while the confinement\([28]\) of quarks follows from its strength at long distance. Qualitatively, when two quarks are close together, the force is relatively weak (asymptotic freedom), but when they move further apart the force becomes much stronger (confinement). At some distance, it becomes easier to make new quarks and anti-quarks, which combine to form hadrons, than to keep pulling against the ever-increasing force. When we calculate any cross section by perturbation, it can be generally expressed as follows;

\[ \sigma = \sum_{s} c_s \alpha_s , \quad (2.5) \]

here \( c_s \) is obtained by calculating the Feynman diagram of the subjected physics process, \( \alpha_s \) is strong coupling constant. Such a method is called
perturbative Quantum ChromoDynamics (pQCD). It could be diverged to infinity if there is a loop in the Feynman diagram. It is common to use dimensional regularization method\cite{20} which manages the infinity divergence. The infinity divergence appears as the following expression;

\[ \Delta = \frac{2}{\varepsilon} - \gamma_E + \ln(4\pi) + \ln(\mu^2). \]  

(2.6)

There are methods to subtract only \(2/\varepsilon\) called minimum subtraction scheme and \(\ln(4\pi) - \gamma_E\) called modified subtraction scheme\cite{21}. In the dimensional regularization method, parameter \(\mu\) which has mass dimension is introduced to be consistent with the dimension of physics quantity. After the subtraction \(\ln(\mu^2)\) remains, therefore the \(c_i\) depends on the subtraction method and the \(\ln(\mu^2)\). However, cross sections do not depend on how to apply to \(\mu^2\). Exactly speaking, according to dimensional analysis it depends on \(t (= \ln(Q^2/\mu^2))\) with the assumption of energy scale of the cross section as \(Q\). That is to say that the change with \(\mu^2\) has to cancel out with the change of \(\alpha_s\). The following equation has to be formed;

\[ \mu^2 \frac{d\sigma(\tau, \alpha_s)}{d\mu^2} = (\mu^2 \frac{\partial}{\partial \mu^2} + \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} \frac{\partial}{\partial \alpha_s})\sigma(\tau, \alpha_s) \]

\[ = 0. \]  

(2.7)

Eq.2.7 can be transformed as follows;

\[ (-\frac{\partial}{\partial \tau} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s})\sigma(\tau, \alpha_s) = 0. \]  

(2.8)

Here the \(\beta(\alpha_s)\) is defined as follows. It can be calculated by perturbation.

\[ \beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = -\frac{\beta_0}{2\pi} \alpha^2 - \frac{\beta_1}{4\pi^2} \alpha^3 + \frac{\beta_2}{64\pi^3} \alpha^4 + .... \]  

(2.9)

\(\beta_0 = 11 - \frac{2}{3} n_f\), and the contribution of 11 comes mainly from the non-abelian diagram. The \(-\frac{2}{3} n_f\), which shows the due to weakness asymptotic freedom, comes from the fermion loop diagram. \(\beta_1 = 51 - \frac{19}{7} n_f\), \(\beta_2 = 2857 - \frac{3033}{9} n_f + \frac{337}{27} n_f^2\). The \(n_f\) is the number of flavors of quarks.

The Eq.2.7 can be written in terms of \(\alpha_s\) as follows;

\[ \alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)} [1 - \frac{2\beta_1}{\beta_0^2} \ln[\ln(\mu^2/\Lambda^2)] + \frac{4}{\beta_0^2 \ln^2(\mu^2/\Lambda^2)} ([\ln[\ln(\mu^2/\Lambda^2)] - \frac{1}{2})^2 + \frac{\beta_2 \beta_0}{8\beta_1^2} - \frac{5}{4})]. \]  

(2.10)
where, $\Lambda$, which is scale parameter in QCD, provides initial condition of Eq. 2.9. The $\alpha_s$ or the $\Lambda$ is ideally determined through the reaction process which does not depend on hadronization model and etc. This process is prompt photon production in proton-(anti-)proton collisions, $Q^2$ evolution in deep inelastic scattering, QCD correction to decay width of $Z$ and etc.\cite{22}. Figure 2.1 shows $\alpha_s$ as a function of $\mu$. The average value of $\alpha_s$ in these several reactions at $\mu = m_Z$ is $0.1183 \pm 0.0025$\cite{58}. The $m_Z$ is invariant mass of $Z$. On the other hand, according to \cite{15}, the $\alpha_s(m_Z) = 0.1176 \pm 0.0020$. When the $\mu^2$ is above $\sim 1$ GeV$^2$ the pQCD can be applied.

![Figure 2.1](image.png)

Figure 2.1: Strong coupling constant $\alpha_s$. The value of the $\alpha_s$ changes as a function of $\mu$. Solid lines indicate $\pm 1\sigma$ limits, and dot line shows the central value.

2.1.3 Lorentz invariant cross section

Parton model cross sections\cite{23, 24, 25, 26} in proton-proton collision are calculated from sub-process cross section as partonic scattering, which is calculated from tree graphs (no loops), by combining them with parton distribution function $f_{a/A}(x_a)$ and fragmentation function $D_{C/e}(z_c)$. The $f_{a/A}(x)$ is
the probability of finding a parton “a” in a hadron “A”, carrying momentum \( xp \), \( 0 \leq x \leq 1 \). The \( D_{c/z}(z_c) \) is the probability of obtaining a hadron “C”, carrying momentum \( zp \), \( 0 \leq z \leq 1 \). These functions cannot be calculated using perturbation theory and must be obtained from experimental data of various types of hard-scattering processes. The cross section for the process under consideration is built up by summing over all the possible constituent scattering, each of which is weighted by the appropriate parton distribution and fragmentation functions. The hard scattering in the parton model is described by the lowest-order sub-processes for high-\( p_T \) particle, jet, or prompt photon production, corresponding to a two-body scattering. This is shown in Figure 2.2. The corresponding expression for the invariant cross section is

\[
E_C \cdot \frac{d\sigma}{d^3p_C} (AB \to C + X) = \sum_{abcd} \int \int \int dx_a dx_b dx_c f_{a/A}(x_a) f_{b/B}(x_b) \times D_{c/z}(z_c) \times \frac{\hat{s}}{z^2 \pi} \frac{d\hat{\sigma}}{d\hat{t}} (ab \to cd) \cdot \delta(\hat{s} + \hat{t} + \hat{u}). \tag{2.11}
\]

The \( \delta \) function in Eq[2.11] is appropriate for the two-body scattering of massless partons and follows simply from two-body phase space. Furthermore, the initial and final partons have been assumed to be collinear with the corresponding initial and final hadrons. In Eq[2.11] pQCD calculates the part of

\[
\frac{d\hat{\sigma}}{d\hat{t}} (ab \to cd). \tag{2.12}
\]

### 2.1.4 Parton distribution function (PDF)

The proton structure function \( F_2(x, Q^2) \) is measured by lepton-proton deep inelastic scattering (DIS). Many DIS experiments with electron, muon or neutrino beam have measured the structure function of the proton and neutron (with deuteron target). \( F_2^p(x) \) and \( F_2^n(x) \) are well summarized in [15]. In the quark-parton model the structure function can be written as a sum of parton distribution function;

\[
F_2(x, Q^2) = x \cdot \sum_f e_f^2 (q_f(x, Q^2) + \bar{q}_f(x, Q^2)) \tag{2.13}
\]

in the leading order, and the quark distribution functions can be extracted by global analysis of all the experimental data. Also the gluon distribution
Figure 2.2: Schematic representation of a high-p$_T$ reaction factorized into parton distribution functions ($f$), parton fragmentation functions ($D$) and a hard-scattering sub-process.

$g(x)$ can be determined with DIS data via the scaling violation ($Q^2$ dependence) of the $F_2(x, Q^2)$, which appears in higher-order calculation due to gluon radiation:

$$F_2(x, Q^2) = x \sum_q e_q^2 \{q(x, Q^2)$$
$$+ \frac{\alpha_s(Q^2)}{2\pi} (C_q(x, \alpha_s) \otimes q(x, Q^2))$$
$$+ \frac{1}{N_f} C_g(x, \alpha_s) \otimes g(x, Q^2)) \}. \quad (2.14)$$
The convolution \( C_f(x, \alpha_s) \otimes f(x, Q^2) \) is defined as

\[
C_f(x, \alpha_s) \otimes f(x, Q^2) = \int_x^1 \frac{dy}{y} C\left(\frac{y}{x}, \alpha_s\right) f(y, Q^2)
\]

(2.15)

where \( f(y, Q^2) \) is the distribution density of parton \( f \) with a momentum fraction \( y \), and \( C\left(\frac{y}{x}, \alpha_s\right) \) is the probability that a quark \( q \) with a momentum fraction \( x \) is produced from the parton \( f \) with a momentum fraction \( y \).

\( C_f(x, \alpha_s) \) is called “splitting function”. Figure 2.3 summarizes the PDFs determined by

- the ZEUS-JETS fit\[32\], which uses inclusive DIS cross section, inclusive jet cross section and di-jet cross section in \( e^-p \) collisions by the ZEUS experiment,
- the ZEUS-S fit\[33\], which uses inclusive DIS cross section in \( e^-p \) collisions by the ZEUS experiment and fixed-target DIS cross section,
- the MRST2001 fit\[34\] and CTEQ6.1M fit\[35\], which uses inclusive DIS data, inclusive jet cross section in proton-anti-proton collisions, etc.

In Figure 2.3 the PDFs by the ZEUS-JETS fit are shown in yellow in all the plots for comparison. All the analyses have determined the PDFs with a good consistency.

The \( Q^2 \) dependence of the distribution functions is described by the DGLAP evolution equation\[36\] that has the schematic form of

\[
\frac{\tau}{\partial \tau} \left( \frac{q_i(x, t)}{g(x, t)} \right) = \frac{\alpha_s(Q^2)}{2\pi} \sum_{q_i, q_j} \int_x^1 \frac{dy}{y} \left( P_{q_i q_j}(\frac{y}{x}, \alpha_s(Q^2)) \right) \left( \begin{array}{c} \frac{q_i(y, t)}{g(y, t)} \\ P_{g q_j}(\frac{y}{x}, \alpha_s(Q^2)) \end{array} \right) \right) \right),
\]

(2.16)

where \( P_{ab}(\frac{y}{x}, \alpha_s(Q^2)) \) is the probability that a parton \( a \) is produced from a parton \( b \) via \( q \to gq, g \to gg \) splittings with a fraction \( \frac{y}{x} \) of the longitudinal momentum of the parton \( b \). \( \tau \) is defined as \( \mu^2 \).

2.1.5 Fragmentation function (FF)

Then PDFs have a close relation to the quantity which is called fragmentation functions (FFs). The probability interpretation of FFs \( D_i^H(z) \) is similar. The \( D_i^H(z) \) is the number density of the hadron \( H \) in the original parton \( i \) with the momentum fraction \( z \) of the parton \( i \). FFs contain all the long distance information of the fragmentation of a parton to a hadron. The fragmentation
Figure 2.3: Unpolarized parton distribution functions (PDFs) determined by the ZEUS-JETS fit\cite{32} (Top left), the ZEUS-S fit\cite{33} (Top right), the MRST2001 fit\cite{34} (Bottom left), and the CTEQ6.1M fit\cite{35} (Bottom right). The PDFs by the ZEUS-JETS fit are drawn as a yellow color in all the plots for comparison.
functions have been measured by ALEPH \cite{37, 38}, OPAL \cite{39, 40, 41, 42, 43}, DELPHI \cite{44} and L3 \cite{45} at CERN, HRS \cite{46}, MARKII \cite{47, 48} and TPC \cite{49} at SLAC, TASSO \cite{50, 51} at DESY, AMY \cite{52} at KEK in $e^+e^-$ collisions using the $e^+e^- \rightarrow \gamma^*$ or $Z^* \rightarrow h + X$ process.

2.2 Prompt photon production

2.2.1 Prompt photon production in pQCD

Prompt photon is produced via quark-gluon collision ($qg \rightarrow q\gamma$) subprocess, which is shown in Figure 2.4 at Leading-Order (LO), and quark anti-quark annihilation ($q\bar{q} \rightarrow g\gamma$) subprocess, which is shown in Figure 2.5 at LO. The photons from hadron decay such as $\pi^0 \rightarrow \gamma\gamma$ is not included in the definition of the prompt photon. The invariant cross section for the subprocess of the prompt photon at a LO $a + b \rightarrow \gamma + c$ is

$$\frac{d\sigma}{dt}(qg \rightarrow \gamma q) = \frac{\pi\alpha_s}{3s^2} e_q^2 \frac{\hat{u}^2 + \hat{s}^2}{\hat{u}\hat{s}}$$  \hspace{1cm} (2.18)
in the quark-gluon collision process and

$$\frac{d\hat{\sigma}}{d\hat{t}}(q\bar{q} \rightarrow \gamma g) = \frac{8\pi\alpha_s}{9\hat{s}^2}\frac{\hat{u}^2 + \hat{t}^2}{\hat{u}t}$$  \hspace{1cm} (2.19)$$

in the quark anti-quark annihilation process. $e_q$ is the charge of the interacting quark, $s, t$ and $u$ are Mandelstam variables\[54\]. To obtain the unpolarized cross section, we average/sum over the initial/final quark and gluon spins and colors. Figure 2.6 shows the subprocess fractions at $\sqrt{s} = 62.4$ and 200 GeV in proton-proton collisions.

The prompt photon is also produced by bremsstrahlung from a quark. The photon fragmentation functions can be calculated with QED as follows;

$$zD_{\gamma/q}(z, Q^2) = e_{q}^2 \frac{\alpha_s}{2\pi} \{1 + (1 - z)^2\} \ln(Q^2/\Lambda^2), \text{ and} \hspace{1cm} (2.20)$$

$$zD_{\gamma/gluon}(z, Q^2) = 0,$$  \hspace{1cm} (2.21)$$

where $z$ is the fraction of the parent parton’s momentum, $\Lambda$ is the scale parameter in QCD, and $e_{q_i}$ is the charge of the $i$th quark. The photon fragmentation function increases uniformly with the scale $Q^2$ over the whole $z$ region and evolves with the $Q^2$ as a result of $q\bar{q}$ pair production and gluon bremsstrahlung. Further details are described in \[54\].

In hadron-hadron collisions, the cross section depends on $\sqrt{s}$ and $p_T$. Table 2.1 summaries cross section for each subprocess in two-body scattering of parton-parton. Figure 2.7 shows the cross section for prompt photon production of pQCD calculation to NLO at $|\eta| < 0.35$, $\sqrt{s} = 62.4$ GeV.

For prompt photons produced at $\eta = 0$ in the colliding hadrons center of mass frame, initial state partons of

$$x \sim x_T \equiv \frac{2p_T}{\sqrt{s}}$$  \hspace{1cm} (2.22)$$

are probed when energy of proton beam is $\sqrt{s}/2$. In the case of the two colliding partons with the same $x$, the momentum of each parton has $x\sqrt{s}/2$ which should be equal to $p$. In the central rapidity ($\eta = 0$), $p \sim p_T = x_T\sqrt{s}/2$.

As shown in Figure 2.3, the gluon density is much larger than the anti-quark density in all $x$ region. Therefore, the prompt photon production in proton-proton collisions mainly comes from the $q + g \rightarrow \gamma + X$ process for all $p_T$ region. This suggests that the prompt photon production is a useful tool to extract the gluon distribution function.
Figure 2.6: The fraction of the quark-gluon collision subprocess (blue) and quark-anti-quark annihilation subprocess as a function of $p_T$ at $\sqrt{s} = 62.4$ GeV (Top) and $\sqrt{s} = 200$ GeV (Bottom) in proton-proton collisions.
Table 2.1: Subprocess cross section in parton-parton two-body scattering. Factors of $\pi\alpha_s^2/s^2$, $\pi\alpha_s/s^2$ and $\pi\alpha^2/s^2$ have been factored out of the purely strong interaction, the single photon production and the double photon production processes, respectively. The $e_q$ is the electric charge of quark. The $s$, $t$ and $u$ are the Mandelstam variables [5].

<table>
<thead>
<tr>
<th>Subprocess</th>
<th>Cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$qq' \rightarrow qq'$</td>
<td>$\frac{4}{9} \frac{s^2+u^2}{t^2}$</td>
</tr>
<tr>
<td>$qq \rightarrow qq$</td>
<td>$\frac{4}{9} \left( \frac{s^2+u^2}{t^2} + \frac{s^2+v^2}{u^2} \right) - \frac{8}{27} \frac{s^2}{tu}$</td>
</tr>
<tr>
<td>$q\bar{q} \rightarrow q'\bar{q}'$</td>
<td>$\frac{4}{9} \frac{t^2+u^2}{s^2}$</td>
</tr>
<tr>
<td>$q\bar{q} \rightarrow q\bar{q}$</td>
<td>$\frac{4}{9} \left( \frac{s^2+u^2}{t^2} + \frac{v^2+s^2}{s^2} \right) - \frac{8}{27} \frac{u^2}{s^2}$</td>
</tr>
<tr>
<td>$gg \rightarrow gg$</td>
<td>$\frac{4}{9} \left[ \frac{4}{3} \left( \frac{s}{u} + \frac{u}{s} \right) + \frac{s^2+u^2}{t^2} \right] - \frac{8}{27} \frac{t^2+u^2}{s^2}$</td>
</tr>
<tr>
<td>$gg \rightarrow q\bar{q}$</td>
<td>$\frac{1}{6} \left[ \frac{t}{u} + \frac{u}{t} \right] - \frac{3}{8} \frac{t^2+u^2}{s^2}$</td>
</tr>
<tr>
<td>$gg \rightarrow gg$</td>
<td>$\frac{9}{2} \left[ 3 - \frac{tu}{s^2} - \frac{su}{t^2} - \frac{st}{u^2} \right]$</td>
</tr>
<tr>
<td>$gg \rightarrow \gamma\gamma$</td>
<td>$\frac{2}{3} e_q^2 \left[ \frac{u}{s} + \frac{s}{u} \right]$</td>
</tr>
<tr>
<td>$g\bar{g} \rightarrow \gamma\gamma$</td>
<td>$\frac{8}{9} e_q^2 \left[ \frac{u}{s} + \frac{s}{u} \right]$</td>
</tr>
<tr>
<td>$g\bar{g} \rightarrow \gamma\gamma$</td>
<td>$\frac{2}{3} e_q^2 \left[ \frac{u}{s} + \frac{s}{u} \right]$</td>
</tr>
</tbody>
</table>

\[
\left[ \sum_{i=1}^{n} e_q^2 \right]^2 \left[ \frac{s^2+u^2}{t^2} \ln^2 \left[ -\frac{s}{u} \right] + 2 \frac{u-s}{t} \ln \left[ -\frac{s}{u} \right] \right]^2 \\
+ \left[ \frac{s^2+u^2}{t^2} \ln^2 \left[ -\frac{s}{u} \right] + 2 \frac{s-u}{t} \ln \left[ -\frac{s}{u} \right] \right] + \left[ \frac{t^2+u^2}{s^2} \ln^2 \left[ \frac{t}{u} + \pi^2 \right] + 2 \frac{t-u}{s} \ln \left[ \frac{t}{u} \right] \right]^2 \right]
\times \frac{1}{2} \left[ \frac{s^2+u^2}{t^2} \ln^2 \left[ -\frac{s}{t} \right] + 2 \frac{s-t}{u} \ln \left[ -\frac{s}{t} \right] + \frac{s^2+u^2}{t^2} \ln^2 \left[ -\frac{s}{u} \right] + 2 \frac{s-u}{t} \ln \left[ -\frac{s}{u} \right] \right]
+ \frac{t^2+u^2}{s^2} \ln^2 \left[ \frac{t}{u} + \pi^2 \right] + 2 \frac{t-u}{s} \ln \left[ \frac{t}{u} \right]
\times \frac{1}{2} \left[ \frac{s^2+u^2}{t^2} \ln \left[ -\frac{s}{t} \right] + \frac{s-t}{u} \ln \left[ -\frac{s}{t} \right] \right]^2 + \frac{s^2+u^2}{t^2} \ln \left[ -\frac{s}{u} \right] + \frac{s-u}{t} \ln \left[ -\frac{s}{u} \right]^2 + 4 \right] \]
The measurement of the high transverse momentum prompt photon production provides an excellent tool both for precision tests of pQCD and for measurements of gluon distribution functions. The high transverse momentum prompt photon production is closely related with high transverse momentum jet production. Here, transverse momentum is given as follows:

\[ p_T = p \cdot \sin \theta, \]

where the \( \theta \) is the angle from beam axis, and \( p \) is the absolute momentum of the photon or hadron. The 4-vector of a photon can be reconstructed with a better precision than the 4-vector of a jet. The prompt photon is one particle, whose position and energy can be well measured in an electromagnetic calorimeter. On the contrary, a jet consists of a number of particles spread
over a fairly wide area of phase space. Jet energy is deposited in both electromagnetic and hadronic calorimeters. In addition, there is an ambiguity when we judge which particles belong to the jet and which particles belong to the underlying event. The underlying event here means particles originating from beam collisions.

On the other hand, the rate for the prompt photon production is much smaller than that for the jet production. The lowest order prompt photon production is proportional to $\alpha_s^2$ while jet production is proportional to $\alpha_s^2$. The prompt photon measurements suffer from large background, primarily from those jets in which a large fraction of the momentum of the jet is carried by a single $\pi^0$, and one of two photons of the $\pi^0$ decay is not detected. Backgrounds can come from other sources such as $\eta \rightarrow \gamma \gamma$, $\omega \rightarrow \pi^0 \gamma$, etc. decays as well, but the bulk of the background originates from $\pi^0$'s.

The following are four different methods of the prompt photon measurement.

- $\pi^0$ tagging method: This technique involves simply measuring the positions and energies of the two photons and requiring the resultant mass to be consistent with that of the $\pi^0$ within experimental resolution. Prompt photon yields are extracted from inclusive photons by subtracting background photons. The inclusive photons also have $\pi^0$ decay photons, which is dominant background photons, and also $\eta$, $\eta'$, $\omega$ decay photons.

The $\eta$, $\eta'$ and $\omega$ decay photons are evaluated using the ratio of the cross section for these meson productions to that for $\pi^0$ production and the ratio to photons. It is expressed with the following equation.

$$ A = \sum \frac{\sigma^i}{\sigma^{\pi^0}} \cdot \frac{Br(i \rightarrow \gamma(\gamma))}{Br(\pi^0 \rightarrow \gamma\gamma)}. \tag{2.24} $$

Index $i$ corresponds to $\eta$, $\eta'$ and $\omega$ particles. The $\eta$, $\eta'$ and $\omega$ decay photons can be evaluated by multiplying the $A$ by the number of observed $\pi^0$ decay photons.

The $\pi^0$ decay photons can be categorized into two groups. One is the group of two tag photons, which means that both of two photons from $\pi^0$ decay are detected, therefore, $\pi^0$ invariant mass can be reconstructed. Another group is one tag photon, which means that one photon from $\pi^0$ decay escapes from a detector or it cannot be detected due to the bad channel of the detector. The effect of the one tag photon is estimated using a fast Monte-Carlo simulation. The simulation calculates $R$ which is the ratio of one tag photons to two tag photons. Then, the one tag photon is evaluated by multiplying the $R$ by the number of two tag photons. The prompt photon yields can be evaluated with the following equation;

$$ N_\gamma = N^{incl.} - (1 + A) \times (1 + R) \times N_{\pi^0}^\gamma, \tag{2.25} $$

21
where $N_\gamma$, $N_{\gamma}^{incl.}$ and $N_{\gamma}^{0}$ is the number of prompt photons, the inclusive photons and the two tag photons. Figure 2.8 shows the schematic drawing of photons and fine granularity detector. A typical invariant mass distribution of two photons in the $\pi^0$ mass range is also shown in the figure.

Figure 2.8: A schematic drawing of a detector which can be used for the measurement of prompt photon using the $\pi^0$ tagging method. An electromagnetic calorimeter of fine granularity to resolve the two photons from $\pi^0$ decays is used to detect prompt photons and $\pi^0$s. A typical invariant mass distribution of two photons in the $\pi^0$ mass range is shown in the bottom panel which is also shown that in the $\eta$ mass range.

Isolation method The Isolation method bases on the $\pi^0$ tagging method. It additionally requires that the photon candidate should be unaccompanied inside a cone of a radius $R = \sqrt{(\Delta \eta^2 + \Delta \phi^2)}$; typically $R = 0.5 - 1.0$) centered on the photon direction. $\eta$ is the pseudo-rapidity and $\phi$ is the azimuthal angle. “Unaccompanied” means that the amount of additional energy inside the cone is less than a certain fraction of the photon’s energy or less than some fixed scale. The application of isolation discriminates strongly the $\pi^0$ events, since a $\pi^0$ is usually accompanied by additional particles from the fragmentation function of the jet. Prompt photons from the leading order processes are unaffected, since the photon is isolated. Photons originating from bremsstrahlung processes are also strongly discriminated, because of the presence of a nearby jet. The effect of an isolation cut on the prompt photon signal can be calculated in a Next-to-Leading Order (NLO) calculation. When a cross section is measured using this method, it has to be
corrected with the efficiency of this requirement. On the other hand, when $A_{LL}$ is measured with this method, the efficiency is not needed to require because the efficiency is canceled out in the ratio.

**Conversion method** The conversion method attempts to measure the photons energy with electromagnetic calorimeter located downstream of a thin converter, but not to distinguish $\pi^0 \rightarrow \gamma \gamma$. Scintillator hodoscopes are placed immediately upstream and downstream of the converter to determine whether a conversion occurred in the converter. The Conversion method for establishing prompt photon is to measure the fraction of events which have conversion in the converter, and compare this to the expected conversion probability assuming the absence of the prompt photon. For example, the non-conversion probability $P_\gamma$ of a photon in a 1.0 radiation length converter is approximately given

$$P_\gamma = \exp^{-\frac{x}{2}} \sim 0.46,$$

(2.26)

where $x$ is the converter thickness. The non-conversion probability $P_{\pi^0}$ of both photons from $\pi^0$ decay is given by square of Eq.(2.27). By measuring the observed non-conversion probability $P_{\text{obs}}$, the fraction of all observed events which correspond to prompt photon conversions, $f_\gamma$;

$$P_{\text{obs}} = f_\gamma P_\gamma + (1 - f_\gamma) P_{\pi^0}. $$

(2.27)

To obtain the correct average value of $P_{\text{obs}}$, the contributions of $\eta$, $\eta'$ and $\omega$ should be considered. This method was applied in R108 at Intersecting Storage Ring (ISR) and to the isolated prompt photons in UA2 at SpS.

**Profiles method** Even if the two photons cannot be resolved, a measurement of the transverse and/or longitudinal profile of the electromagnetic shower may allow a discrimination between prompt photons and $\pi^0$s. Showers originating from $\pi^0$'s appear broader due to the opening angle of the two photons. This technique becomes inefficient as the $\pi^0$ energy increases, since the opening angle decreases as the the $\pi^0$ energy. The longitudinal development of prompt photon and $\pi^0$ showers will also differ as the average energy of a $\pi^0$ photon is half that of the prompt photon. The longitudinal development of an electromagnetic shower varies only logarithmically with photon energy.

**2.2.3 Experiments in the past**

**R806** The R806 presented the first convincing evidence for the existence of prompt photons. The follow-up experiments extended the measured range of prompt photon production to higher transverse momenta. These
experiments were performed at the ISR for proton-proton collisions at \(90^\circ\) in \(\sqrt{s} = 63\ \text{GeV}\) with a luminosity of \(0.9 \times 10^{-37} \text{cm}^2\) \((0.9\ \text{pb}^{-1})\). The measured \(p_T\) was between 4 and 12 \(\text{GeV}/c\).

The photon detector consisted of two identical lead-liquid-argon shower counters. A sketch of the counter geometry is given in Figure 2.9. Along the direction of the incident photon, each device was segmented in three regions of 2.5, 2.5 and 12.0 radiation lengths. The first layer consisted of azimuthal \(\phi\) strips, the second layer of \(U\) and \(V\) strips oriented at \(\pm 20^\circ\) with respect to the \(\phi\) strips, and the layer of polar-angle \(\theta\) strips. The transverse granularity in the first two layers was 2 cm, and provided position resolution of 0.5 cm and two shower separation of 5.0 cm. It was possible to detect \(\pi^0\)s with a high efficiency. It could identify the background of prompt photon candidates because of high granularity, and it could reach a good energy resolution and moderate solid angle acceptance.

Figure 2.9: Sketch of the geometry of the liquid-argon calorimeters used in R806. Dimensions are given in cm. The top drawing shows a cut in the calorimeter in the plane perpendicular to beams. The layout of the strips is shown in the middle and bottom panels.

Both calorimeters placed 1.65 m from the crossing, opposite to the ISR c.m. motion, \(+23.75^\circ\) with respect to the scattering plane. The ISR c.m.
motion means the central direction of circular motion of proton beams. This geometrical configuration is shown in Figure 2.10. The coverage of the two calorimeters in rapidity direction, and in azimuthal angle is $|\eta| < 0.46$, and $\phi \sim 22^\circ \times 2$, respectively.

![Figure 2.10: The apparatus of the R806 experiment is schematically shown. In the figure, the ISR c.m. motion means the central direction of circular motion of proton beams.](image)

Data were recorded when all the following conditions were satisfied; An energy deposition was found in a localized regions in $U$ and $V$ strips above some threshold and an energy depositions above some threshold in the $\Phi$ strips.

Figure 2.11 shows the result of prompt photon cross section measurement. The result of R108 is also shown in Figure 2.11 and it is described below. Further information about the R806 experiment is in Reference [6, 56].

**R108** The R108 experiment[57] which provided convincing evidence for prompt photon at high $p_T$ was executed at the ISR using the conversion method described before. Data were collected at $\sqrt{s} = 63$ GeV with an integrated luminosity of $7.6 \times 10^{37}$ cm$^{-2}$ (76 pb$^{-1}$).

The apparatus is shown in Figure 2.12. It consisted of two identical $12 \times 14$ arrays of lead glass blocks (each $15 \times 15$ cm$^2$ in transverse size) located at 1.1 m from interaction point. The coverage of the PbGl is $|\eta| < 0.26$ in rapidity direction, and $\phi = 90^\circ \times 2$ in azimuthal angle. The coverage of the cryostat is $|\eta| < 0.32$ in rapidity direction, and $\phi = 360^\circ$ in azimuthal angle.
Figure 2.11: The result of prompt photon cross section measurement of R806 (red) and R108 (blue).

The arrays were located outside a superconducting solenoid containing drift chambers and a scintillator hodoscope (the A counter in Figure 2.12). Two hodoscopes of 12 counters each (the B counters in Figure 2.12) were located outside the coil. The coil and cryostat corresponded to 1.0 radiation lengths of material, which served as the photon converter.

Data were recorded whenever the total signal in either lead glass array was above some threshold and when at least one A counter had a signal. In the off-line analysis, it was required that the interaction point had at least two reconstructed tracks, and that four A counters had signals. These criteria were effective in rejecting cosmic-ray events and beam-gas interactions. Clusters were defined in order to localize and isolate depositions of energy in $3 \times 3$ matrices of blocks; using this definition, $\pi^0$‘s with momenta above 3 GeV/$c$ appeared as localized signal clusters, as did other decays of $\eta$’s, $\eta'$’s, $\omega$‘s and so on. A cluster was rejected from consideration as a neutral particle candidate if a reconstructed charged particle pointed to 30 cm away from the cluster.

Conversions were defined by the presence of signals in one of the two B
counters nearest the candidate shower that had a pulse height corresponding to at least 1.5 times the value expected for a minimal ionizing particles.

The result of prompt photon cross section measurement is shown Figure 2.11 and the cross section was measured from 4.5 to 8.5 GeV/c. Further information about the R108 experiment is in Reference 57.

2.3 Proton spin

It is now considered that proton spin, which is one half, is due to the quark spin, gluon spin and angular momentum of gluons and quarks as follows;

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta g + L_{\text{quark, gluon}}. \tag{2.28}
\]

Here \( \Delta \Sigma \), \( \Delta g \) and \( L_{\text{quark, gluon}} \) is the contribution of the quark spin to the proton spin, the contribution of the gluon spin to the proton spin, and the contribution of the angular momentum of quarks and gluons. Before 1988, it was thought that the proton spin is carried only by quarks. In 1988, EMC measured the spin dependent structure function with a higher statistics and a wider kinematic range than before. They reported that the contribution of quark spin to the proton spin is only \( 12 \pm 9 \pm 14\% \) 11 12. After EMC reported their experimental result, E142 10, E143 8, E154 9 and E155 10 at SLAC, SMC 11 at CERN and HERMES 12 13 at DESY measured \( \Delta q \) through
deep inelastic scattering (DIS) with lepton beams. The recent result\[14\] of HERMES is

$$\Delta \Sigma = 0.330 \pm 0.011(\text{theory}) \pm 0.025(\text{exp.}) \pm 0.028(\text{evol.})$$

(2.29)

at \(Q^2 \sim 5\) GeV\(^2\), which is only \(\sim 30\%\) of the proton spin. DIS with lepton beams are sensitive to the polarized quark distribution function \(\Delta q_f(x)\) because its interaction is electromagnetic interaction \(lq \rightarrow lq\). \(\Delta q_f(x)\) is defined as

$$\Delta q_f(x) = q_f^+(x) - q_f^-(x), \text{ and}$$

(2.30)

$$\Delta \Sigma = \sum_f \int_0^1 \Delta q_f(x) dx,$$

(2.31)

where \(x\) is the Bjorken scaling variable and \(q_f^{+(-)}(x)\) means the probability of finding flavor \(f\) quarks with the spin parallel (anti-parallel) to the proton spin and with the momentum fraction \(x\) of the proton momentum. DIS with lepton beams are partly sensitive to the polarized gluon distribution function \(\Delta g(x)\), which is defined in the same way of \(\Delta q_f(x)\), via the \(Q^2\) dependence only of the \(\Delta q_f(x)\). However, \(\Delta g(x)\) measured by DIS can be discussed within the uncertainty of the analysis. The longitudinally-polarized proton-proton collision is suited for the measurement of the \(\Delta g(x)\) because gluon-involved scatterings, such as \(qg \rightarrow qg, gg \rightarrow gg\) or \(qg \rightarrow \gamma g\), take place in the lowest order.

### 2.3.1 Double helicity asymmetry

Double helicity asymmetry \((A_{LL})\) is defined as

$$A_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}} = \frac{1}{P_1 P_2} \frac{N^{++} - N^{+-}}{N^{++} + N^{+-}}$$

(2.32)

where \(P_1\) and \(P_2\) are the polarization of the proton beams. \(N^{++/+} \) represents the number of events of prompt photon production when the helicity of beam1 and beam2 is \(++/+-\), respectively.

\(A_{LL}\) is measured to extracted \(\Delta g(x)\) Many systematic uncertainties are cancel out in the ratio and thus a high precision can be achieved. In a gluon-gluon s-channel scattering \((gg \rightarrow g^* \rightarrow gg)\), for example, the spins of two initial-state gluons have to be anti-parallel with each other to make
a valid spin \((S_z = 0)\) of the virtual gluon \(g^*\). Such spin conservation, or helicity conservation in other processes, makes spin-dependent asymmetry in gluon-gluon scattering cross section. This asymmetry selects one spin state (parallel or anti-parallel to the proton spin) of gluons in polarized proton, and thus the spin-dependent distribution can be extracted from polarized proton-proton collisions.

![Diagram of proton spin and beam direction](image)

Figure 2.13: A sketch of the collisions of the polarized protons. The green arrows, blue arrows denote the direction of proton beam, the direction of the proton spin, respectively. \(\sigma^{++}(+-)\) represents the cross section for prompt photon production when the helicity of beam1 and beam2 is \((++, +(-)), \) respectively.

On the other hand, \(A_{LL}\) can be predicted by pQCD, and can be written in Leading-Order (LO) for prompt photon production as follows;

\[
A_{LL} \approx \frac{\Delta g(x_1)}{g(x_1)} \cdot \left[ \frac{\sum_{f} e_f^2 [\Delta q_f(x_2) + \Delta \bar{q}_f(x_2)]}{\sum_{f} e_f^2 [q_f(x_2) + \bar{q}_f(x_2)]} \right] \cdot \hat{a}_{LL}(gq \rightarrow \gamma g) + (1 \leftrightarrow 2). 
\]  

(2.33)

The \(e_f\) is the electric charge of quarks, the \(\hat{a}_{LL}\) is the partonic asymmetry and is calculable with pQCD. For the inclusive measurement of prompt photon production, it does not involve fragmentation function because the photon does not fragment. The fragmentation function is not necessary to consider. Therefore, the prompt photon \(A_{LL}\) can be precisely predicted.

In \(\pi^0\) production at \(\sqrt{s} = 200\ \text{GeV}\), \(\Delta g(x)\) in the region of \(x = 0.02 \sim 0.3\) is studied. The corresponding \(x\) is \(p_T = 1 \sim 10\ \text{GeV/c}\). It is important to measure \(p_T\) down to 1 GeV/c in different \(\sqrt{s}\), because \(A_{LL}\) in the outside of this measured \(x\) region is measured. Therefore, the measurement of the \(A_{LL}\) at different \(\sqrt{s}\) such as 62.4 GeV is essential.
2.4 Quark gluon plasma

Quarks and gluons are confined inside hadrons as described in Section 2.1.2. In a high density and/or high temperature condition, the hadrons are expected to form another state of matter. One of other states is called quark gluon plasma (QGP). It is thought that the QGP was formed a few $\mu$s after the Big Bang in the universe. Figure 2.14 shows a phase diagram as functions of temperature and density. According to the recent lattice calculation\[60\], the transition temperature at 0 baryon density is 170 MeV. The QGP searches have been performed in nucleus-nucleus collisions at the Bevatoron at Berkeley, the AGS, RHIC at BNL, and the SPS at CERN.

![Phase diagram of hadronic matter. The transition from hadronic to quark matter is illustrated.](image)

One of the possible signal of QGP is jet quenching effect. The parton loses its energy with strong interaction when it goes through the QGP, which is predicted with the QCD calculation\[61, 62, 63, 64\]. The jet production in nucleus-nucleus collisions is reduced with strong interactions in the QGP matter, on the other hand, prompt photon production in nucleus-nucleus collisions is not reduced because photons do not interact with strong interaction. That is the jet quenching scenario. Therefore, the prompt photon production in proton-proton collisions is an important reference for QGP study.
Chapter 3

Experimental setup

3.1 Relativistic heavy ion collider (RHIC)

The Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) is a circular accelerator for collision of ions such as gold-gold collision, copper-copper collision, deuteron-gold collision and polarized proton-proton collision. The heavy ions and the polarized protons are accelerated up to $\sqrt{s} = 200$ and 500 GeV per nucleon, respectively. The RHIC has two rings called “blue-ring” and “yellow-ring”. The beam circulates clockwise in the blue-ring and anti-clockwise in the yellow-ring. The design luminosity of gold ions and the polarized protons is $2 \times 10^{26}$ and $2 \times 10^{32}$ cm$^{-2}$s$^{-1}$, respectively. Crossing intervals of bunch is 106 nsec or 31.9 cm when two rings store 120 bunches. The number of bunches that are actually filled with ions was typically 55 at the beginning of 2005 and 111 at the end of 2005 and in 2006. A set of beam injection, acceleration, store and dump is called a “fill”. One fill lasts typically 4 ~ 8 hours. The polarization direction of the filled bunches are different in each fill. As the result, all four polarization patterns $++, +-, -+, --$ take place in collisions. The polarization pattern of the blue-ring is $++--++--...$, and that of the yellow-ring is $+-+-+-+-...$. The polarization of the proton beam was 58% at $\sqrt{s} = 200$ GeV and 50% $\sqrt{s} = 62.4$ GeV in 2006 [66]. By using such bunch patterns the candidates of bunch-dependent systematic uncertainties are canceled out.

Figure 3.1 shows the air view of the RHIC and complexes around the RHIC. It is composed of the LINAC, the Booster, the Tandem Van de Graff, the Alternating Gradient Synchrotron (AGS), and the RHIC. The RHIC has six beam interaction points called “2 o’clock”, “4 o’clock” ... “12 o’clock”. The PHENIX experiment is located at the 8 o’clock. The polarized proton beam are accelerated up to 200 MeV at the Linac, up to 23.4 GeV at the
AGS, and up to 31.2 GeV or 100 GeV at the RHIC. The circumference of the RHIC is 3.83 km.

![An air view of the RHIC and complexes around the RHIC.](image)

Figure 3.1: An air view of the RHIC and complexes around the RHIC.

### 3.1.1 Beam luminosity

For measurements of physics quantity, the determination of absolute integrated luminosity is needed. The absolute luminosity \( L \) is determined with the number of events \( N_{BBC} \) triggered by BBC detector which is one of the PHENIX detector and BBC trigger cross section \( \sigma_{BBC} \):

\[
L = \frac{N_{BBC}}{\sigma_{BBC}}. \tag{3.1}
\]

The \( \sigma_{BBC} \) is obtained via Vernier scan method\[^{65}\]. In the scan, the transverse profile of the beam overlap is measured by sweeping one beam
across the other in steps while monitoring the BBC trigger rate, and beam profiles are assumed to be Gaussian. The $\sigma_{BBC}$ is calculated as follows:

$$\sigma_{BBC} = \frac{R_{\text{max}}}{L_{\text{machine}} \cdot \varepsilon_{\text{vertex}}}.$$  \hspace{1cm} (3.2)

The $R_{\text{max}}$ is the BBC trigger rate in case of maximally overlapping of beams. The $\varepsilon_{\text{vertex}}$ is the vertex selection efficiency obtained by the vertex reconstruction analysis of BBC detector.

The $L_{\text{machine}}$ is provided as follows;

$$L_{\text{machine}} = \frac{f_{\text{beam}}}{2\pi \sigma_x \sigma_y} \cdot \sum_{\text{crossings}} N_B \cdot N_Y.$$  \hspace{1cm} (3.3)

The $f_{\text{beam}}$ is a recurrence frequency for one bunch collision in the beams. It is $f_{\text{beam}} = 78$ kHz. The $N_B$ and $N_Y$ are the number of ions stored in the blue and yellow rings, respectively. The number of ions ($N_B$ and $N_Y$) are measured with two types of equipments; the Direct Current Current Transformer (DCCT) and the Wall Current Monitor (WCM). The DCCT is high-permeability copper toroidal coil and has been installed in both the blue and yellow rings near the 2 o’clock interaction point. It measures the electric current induced in itself by the beam current and can precisely determine the integral of beam current during $\sim 1$ sec with an accuracy of $\sim 0.2\%$. Due to the long integral time, it does not distinguish bunched ions from debunched ions. The debunched ion means here the between a bunched ion and a bunched ion. The WCM is large-RLC circuit placed between two adjacent installed barrels of the beam pipe and has been installed in both the blue and yellow rings near the 2 o’clock interaction points. It measures, with a sampling time of $\sim 0.25$ nsec, the voltage across the circuit that is caused by an image charge dragged by the beam current on the beam pipe. It is not as precise as the DCCT but can distinguish bunched from debunched ions because of its short sampling time. A possible contribution of debunched ions is checked with the comparison between the two measurements. The number of ions stored in one bunch are $90 \times 10^9$ (ions/bunch) at $\sqrt{s} = 62.4$ GeV in 2006[66].

The “crossings” means pairs of colliding bunches at the PHENIX interaction region. In Eq. 3.3, the $\sigma_x$ and the $\sigma_y$ are the width in $x$ and $y$ direction of collisions between the two beams. They are measured in the horizontal and vertical directions by the Vernier scan. They are represented with the widths of the blue beam and the yellow beam at the collision point ($\sigma_{x}^2 = (\sigma_{i_x}^2)^2 + (\sigma_{i_y}^2)^2$; $i =$ blue or yellow).

Thus, the absolute luminosity is obtained. At $\sqrt{s} = 62.4$ GeV in 2006, the average luminosity is $23 \times 10^{30}$ (cm$^{-2}$s$^{-1}$). The integrated luminosity derived
by the RHIC is 1.05 pb$^{-1}$. Additional information about the performance of the RHIC can be found in Reference [66–67].

### 3.2 PHENIX detector: overview

The PHENIX detector[68] consists of spectrometers and beam counters. It can be grouped into three parts; the Inner detectors, the Central arms and the Muon arms. Figure 3.2 is a schematic drawing of PHENIX detector in a side view. Figure 3.3 shows the beam cross section of the central arms.

![PHENIX detector schematic](image.png)

**Figure 3.2:** The schematic drawing of PHENIX detector in a side view. The colored subsystems with red are Beam-beam detectors called BBC.

Figure 3.4 shows the magnet field lines formed by four magnets called Central magnet inner, Central magnet outer, Muon magnet north and Muon magnet south.

**Figure 3.5** shows the PHENIX coordinate system. The $z$ axis is along the beam direction. The $x$ and $y$ axes are in horizontal and vertical directions, respectively. $+x$, $+y$ and $+z$ directions point to the West arm, the top and the North arm, respectively.

Table 3.1 summarizes the coverage of the main PHENIX detector subsystems. In this section below, each subsystems are explained. In particular, the subsystems used in this analysis are described in detail.
Table 3.1: Coverage of the main PHENIX Detector subsystems. $\Delta \eta$ is the pseudo-rapidity coverage, and $\Delta \phi$ is the azimuthal angle coverage.

<table>
<thead>
<tr>
<th>Element</th>
<th>$\Delta \eta$</th>
<th>$\Delta \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central magnet</td>
<td>$\pm 0.35$</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>Muon magnet north</td>
<td>-1.1 $\sim$ -2.2</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>Muon magnet south</td>
<td>1.1 $\sim$ 2.4</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>BBC</td>
<td>$\pm (3.1 \sim 3.9)$</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>ZDC</td>
<td>$\pm 2$ mrad</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>DC</td>
<td>$\pm 0.35$</td>
<td>$\pi/2 \times 2$</td>
</tr>
<tr>
<td>PC</td>
<td>$\pm 0.35$</td>
<td>$\pi/2 \times 2$</td>
</tr>
<tr>
<td>TEC</td>
<td>$\pm 0.35$</td>
<td>$\pi/2$</td>
</tr>
<tr>
<td>RICH</td>
<td>$\pm 0.35$</td>
<td>$\pi/2 \times 2$</td>
</tr>
<tr>
<td>ToF</td>
<td>$\pm 0.35$</td>
<td>$\pi/4 \times 2$</td>
</tr>
<tr>
<td>PbSc EMCal (West)</td>
<td>$\pm 0.35$</td>
<td>$\pi/2$</td>
</tr>
<tr>
<td>PbSc EMCal (East)</td>
<td>$\pm 0.35$</td>
<td>$\pi/4$</td>
</tr>
<tr>
<td>PbGl EMCal (East)</td>
<td>$\pm 0.35$</td>
<td>$\pi/4$</td>
</tr>
<tr>
<td>Muon tracker (South)</td>
<td>-1.15 $\sim$ -2.25</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>Muon tracker (North)</td>
<td>1.15 $\sim$ 2.44</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>Muon identifier (South)</td>
<td>-1.15 $\sim$ -2.25</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>Muon identifier (North)</td>
<td>1.15 $\sim$ 2.44</td>
<td>$2\pi$</td>
</tr>
</tbody>
</table>
The beam cross section of the central arms and etc. The colored subsystems with red, sky-blue, and blue are used in this analysis. The red is Pad chamber3 (PC3), sky-blue is lead-scintillator sampling calorimeter (PbSc), and blue is lead-glass Cerenkov radiator (PbGl). The ratio of the acceptance of the central arm to the total acceptance is 17.5%.

3.3 Inner detector

The Inner detectors have the Beam-beam counters (BBC) and the Zero-degree counters (ZDC).

3.3.1 Beam beam counter (BBC)

The BBC is two identical sets of counters placed at the north and the south sides of the collision point [70]. The distance between the collision point and the BBC is 144 cm. It covers a pseudo-rapidity of $3.0 < |\eta| < 3.9$ over the full azimuth. Each counter is composed of 64 one-inch diameter mesh-dynode PMTs (Hamamatsu R6178) equipped with 3 cm quartz as a Cherenkov radiator. Figure 3.3 shows the arrangement of the 64 BBC elements. The outer and inner diameters are 30 cm and 10 cm, respectively. The gain of each PMT is adjusted using a minimum ionizing particle peak in pulse height distribution. During data taking, drifts of timing each element are monitored.
Figure 3.4: The view of magnet fields in PHENIX detector.

and calibrated using a laser signal.

The main role for the BBC is to provide the time of beam-beam collisions for the time-of-flight measurement, to produce a signal for a trigger, to measure the collision point of beams ($z$−BBC) along the beam axis, and determine the beam luminosity. Time ($T_0$) and the $z$−BBC of a collision is then determined as

$$T_0 = \frac{T_S + T_N}{2L} - \frac{L}{c},$$  \hspace{1cm} (3.4)

$$z_{BBC} = \frac{c \cdot |T_S - T_N|}{2},$$  \hspace{1cm} (3.5)

where $L = 144$ cm is the distance between collision point and the BBC. The resolutions of $T_0$ and $z$−BBC in proton-proton collisions are $\sim 20$ ps and $2 \sim 3$ cm, respectively.
3.3.2 Zero degree calorimeter (ZDC)

The ZDC is a hadron calorimeter designed to measure the number of neutrons from collisions. It is placed at 18m north and south from the collision point. Charged particles are eliminated from the acceptance because bending magnets of RHIC are set inside of the ZDC to bend the beams in the RHIC.

The ZDC consists of alternating layers of tungsten absorbers and sampling fibers, and have 150 radiation length and 5.1 interaction length. The Shower maximum detector (SMD) is inserted between the first and second ZDC modules to determine the hit position of neutrons. It consists of 16 scintillator hodoscopes; eight in vertical and eight in horizontal. The position of an incident neutron is evaluated as the centroid of deposit energies in the hodoscopes.

3.4 Central arm spectrometers

The Central arms have a charged particle tracking system and an Electromagnetic calorimeter (EMCal). The charged particle tracking system consists of Pad chambers (PC1, PC2 and PC3), Drift chambers, Ring-imaging Cerenkov detectors (RICH), etc. Silicon vertex detector (SVD) will be installed around the collision point in 2009, and detects charged particles to trace its tracks.
Figure 3.6: **Top** A BBC array consists of 64 BBC elements. **Bottom** A single BBC consists of one-inch mesh dynode photo-multiplier tubes with a 3 cm thick quartz radiator.
The SVD aims to detect decay points of mesons which contain heavy quark flavor by reconstructing the decay tracks. I have worked for the development of the SVD, and Appendix A describes my work.

### 3.4.1 Drift chamber (DC)

The DC each covers $\frac{\pi}{2}$ in azimuthal and 180 cm in the beam pipe centered around mid-rapidity. The DCs are positioned outside the magnetic field at 2.02 m to 2.46 m radial distance from the interaction point. They consist of the 3 different direction of the wire; the same directions as the beam pipe and $\pm 6^\circ$ tiled angle with respect to the beam pipe direction.

Charged particles passing through the DC ionize the gas mixture, 50% argon and 50% ethane with $\leq 1$% alcohol. The released electrons are measured on the wires. The hit position is measured by the drift time of the electrons hitting the wires. Momentum measurements are performed by measuring the angular deflection of the track from a straight line trajectory through the interaction point with the PC. The DCs are also used to measure the azimuthal angle of the tracks. Details on the DC can be found in Reference [68].

### 3.4.2 Pad chamber (PC)

The Pad Chambers (PC) are multi-wire proportional chambers (MWPC) and form three separate layers which are called PC1, PC2 and PC3. The PC1 is placed at 2.5 m from the beam pipe and is used for determining the momentum vector together with the DC by providing the z coordinate. The PC2 is placed at 4.1 m from the beam pipe. The PC3 is placed at 5.0 m from the beam pipe and in front of the EMCal. The PC3 is used for a charged particle veto of the EMCal. The PCs consist of a single plane of anode and field wires lying in a gas volume between two cathode planes (sec 3.7). Argon and ethane by 50% each are used as the gas volume. One cathode plane is segmented into pixels and the other is solid copper, and signals from the pixels are routed outside the gas volume.

Figure 3.8 shows that three pixels form a cell at the center of the figure. One cell has an effective readout size of 8.2 mm in $z \times 8.4$ mm in $x-y$ plane, and is divided into three pixels; one center pixel and two side pixels. The size of the cell is built with the consideration of the spread size of the released electrons. Hit reconstruction of the PC requires that all three pixels in a cell must have a signal simultaneously. Nine pixels shown in Figure 3.8 as the same colors compose a pad that is read out by a single pre-amplifier and
Figure 3.7: The construction of the PC. There is the plane of anode wires between two cathode planes. One of the cathode planes is pixel plane.

discriminator. The hit position are determined with signals from the three pads because the cell in the three pads is uniquely determined.

There are 14,400 cells per m$^2$ on the PC1 and 172,800 electronics read out channels distribute over the PC1 and PC3 in both arms and the PC2 in the West arm only. Electric charges caused by a particle in the gas volume spread over one or two cells, and the hit reconstruction of the PC system requires that all three pixels in a cell must have a signal simultaneously. This requirement reduces mis-reconstructions due to electrical noises. Further information on the PC can be found in Reference [68].

In order to veto charge particles in front of the EMCal using the PC3, a degree of matching of the position between pc3dhi and emcdhi in $\phi$ direction and between pc3dz and emcdz in $z$ direction are evaluated. The variables such as pc3dhi are defined in Figure 3.9. A $\text{emcpcd}\phi (\equiv \text{pc3dhi} - \text{emcdhi})$ by a charged particle passing the PC3 and the EMCal is ideally 0. The $\text{emcpcd}\phi$ fluctuates in practice so it is distributions are distributed with the center at 0. The veto of charged particle can be performed by determining the matching place to define it. The same approach is applied in $z$-direction
3.4.3 Ring imaging cherenkov detector (RICH)

The RICH has a volume of roughly $40 \text{m}^3$ and minimum thickness of 87 cm of the pressured gas, which is CH$_2$, N$_2$ or CO$_2$ gas depending on run periods. The Cerenkov photons produced in the pressured gas are reflected on the mirror and are detected with photo multiplier tubes (PMT’s).

The average size of the Cerenkov ring is 8 cm and the average number of the Cerenkov photons produced by an electron is 11 on the plane where the PMT’s are set. The RICH provides $e/\pi$ discrimination below the $\pi$ Cerenkov threshold which is set at about 4 GeV/c. In combination with the EMCal in each arm and the TEC in one arm, the goal is to limit the false identification of hadrons as $e^+$ and $e^-$ to less than $10^{-4}$, for momenta below the Cherenkov threshold. The EMCal is capable of rejecting about 90% hadrons at momenta $\leq$ 1 GeV/c, and the TEC (present in only one arm of PHENIX) provides $dE/dx$ separation of electrons from $\pi$’s for momenta below about 1 GeV/c. Further information on the RICH can be found in Reference [68].

Figure 3.8: Nine connected pixels which are shown by the same color in the figure form a pad. The three pixels (red, blue and green) form a cell at the center of the figure.

with emcpdz ($= pc3dz - emcdz$).
Figure 3.9: Definition of variables in the Central Arm tracking.
3.4.4 Time expansion chamber (TEC)

The TEC is composed of a set of 24 large multi-wire tracking chambers arranged in four, six-chamber sectors, which reside only in the East Arm.

It measures all charged particles passing through its active area, providing dielectron direction vectors that are matched to additional track information from the DC’s and PC’s. The tracking information is used to solve the complex pattern recognition problems associated with the high particle multiplicities in heavy ion collisions. The detector system allows systematic studies of tracking efficiency and background rejection versus multiplicity in coordination with the DC. The TEC also enhances the momentum resolution of the PHENIX Central Arm at \( p_T \leq 4 \text{ GeV}/c \) by combining with the DC to provide a long lever arm for improved track-angle resolution. In addition the TEC measures ionization energy losses (dE/dx) of charged tracks which enables particle identification, particularly \( e/\pi \) separation. Further information on the TEC can be found in Reference [68].

3.4.5 Time of flight counter (ToF)

The ToF serves as a primary particle identification device for charged hadrons. It is designed to have about 100 ps timing resolution in order to achieve a clear particle separation in the high momentum region, i.e, \( \pi/K \) separation up to 2.4 GeV/c and K/proton separation up to 4.0 GeV/c.

The ToF is placed at a distance of 5.1 m from the collision point of beams, between the PC3 and the EMCal (discussed in the next section) in the East arm of PHENIX. It is designed to cover the \( |\eta| < 3.5 \) and \( \pi/6 \) in azimuth. The ToF consists of 10 panels of ToF walls. One ToF wall consists of 96 segments, each equipped with a plastic scintillator slat and photo-multiplier tubes which are read out at both ends.

3.4.6 Electromagnetic calorimeter (EMCal)

The EMCal[72] is located at a 5 m distance from the beam pipe to measure the position and energy of photons, electrons and positrons. The EMCal consists of four sectors in each of the East and West Arms, and each sector has a size of \( 2 \times 4 \text{ m}^2 \). There are two kinds of calorimeter which are a shashlik type lead-scintillator sampling calorimeter (PbSc)[73,74] and homogeneous calorimeter made of lead-glass Cerenkov radiator (PbGl). All the four sectors in the West Arms, which are called W0, W1, W2 and W3 from the bottom to top, and the two sectors of East Arms, which are called E2 and E3 from the bottom to top, are the PbSc. The other sectors which are E0 and E1
of the East Arms are the PbGl. Table 3.2 shows basic parameters of the EMCal. A super module is composed of 12 x 12 channels for the PbSc and 4 x 6 channels for the PbGl. The one channel is named as tower. A sector is composed of 18 super-modules for the PbSc and 192 super-modules for the PbGl.

<table>
<thead>
<tr>
<th></th>
<th>PbSc</th>
<th>PbGl</th>
</tr>
</thead>
<tbody>
<tr>
<td>radiation length ($X_0$) [mm]</td>
<td>21</td>
<td>29</td>
</tr>
<tr>
<td>Moliere radius ($R_M$) [mm]</td>
<td>$\sim 30$</td>
<td>37</td>
</tr>
<tr>
<td>channel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cross section [mm$^2$]</td>
<td>$55.35 \times 55.35$</td>
<td>$40 \times 40$</td>
</tr>
<tr>
<td>depth [mm]</td>
<td>375</td>
<td>400</td>
</tr>
<tr>
<td>$X_0$ [mm]</td>
<td>18</td>
<td>14</td>
</tr>
<tr>
<td>super-module</td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of channels</td>
<td>144(12x12)</td>
<td>24(4x6)</td>
</tr>
<tr>
<td>sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of super-modules</td>
<td>18(3x6)</td>
<td>192(12x16)</td>
</tr>
<tr>
<td>total system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of sectors</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>number of super-modules</td>
<td>108</td>
<td>384</td>
</tr>
<tr>
<td>number of channels</td>
<td>15552</td>
<td>9216</td>
</tr>
<tr>
<td>$\eta$ coverage</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>$\phi$ coverage</td>
<td>$\pi/2+\pi/4$</td>
<td>$\pi/4$</td>
</tr>
</tbody>
</table>

Table 3.2: Basic parameters of two kinds of EMCal. Moliere radius is defined as $R_M = 21 \text{MeV} \times X_0 / E_c$, where $E_c$ is the critical energy. 99% of the energy is inside a radius $3R_M$.

**PbSc** The PbSc is a sampling calorimeter read out by PMT. It consists of alternating layers of lead absorbers and scintillator. A tower of the PbSc has a size of $55.35 \times 55.35 \times 375$ mm$^3$ and a 18 radiation length. It consists of 66 cells of a 1.5 mm lead and a 4.1 mm scintillator. An organic scintillator p-bis[2-(5-Phenylxazoloyl)]-benzene (POPOP) and a fluorescent additive p-
Terphenyl (PT) are used as the scintillators. 36 wavelength-shifting fiber penetrates all cells, where each fiber loop-backed at the front of the tower, so that it penetrates all cells twice, and guide light signals to the PMT. Figure 3.10 shows one tower. Figure 3.12 shows the PbSc of W0 and W1.

The resolution of energy $\sigma_E$ and position $\sigma_x$ are given by

$$\frac{\sigma_E}{E}(\%) = \frac{8.1}{\sqrt{E(\text{GeV})}} \oplus 2.1 \quad \text{and} \quad (3.6)$$

$$\sigma_x(E, \theta)(\text{mm}) = \left( \frac{5.7}{\sqrt{E(\text{GeV})}} \oplus 1.55 \right) \oplus \Delta \sin \theta, \quad (3.7)$$

respectively. Here the $\theta$ is the incident angle of photons with respect to the towers surface and the $\Delta$ is the radiation length. The “$\oplus$” denotes a root of the quadratic sum, $\alpha \oplus \beta = \sqrt{\alpha^2 + \beta^2}$. The energy resolution is shown in Figure 3.13.

**PbGl** The PbGl is a homogeneous calorimeter made of lead-glass Cerenkov radiator read out by PMT. A tower of the PbGl has a size of $4 \times 4 \times 40 \text{ cm}^3$ (Figure 3.11) and a 14.4 radiation length. According to electron beam test experiments, the resolutions of energy ($\sigma_E$) and position ($\sigma_x$) are

$$\frac{\sigma_E}{E}(\%) = \frac{5.9}{\sqrt{E(\text{GeV})}} \oplus 0.8 \quad \text{and} \quad (3.8)$$

$$\sigma_x(E, \theta)(\text{mm}) = \left( \frac{8.4}{\sqrt{E(\text{GeV})}} \oplus 0.2 \right) \oplus \Delta \sin \theta, \quad (3.9)$$

respectively. Here the $\theta$ is the incident angle of photons with respect to the towers surface and the $\Delta$ is the radiation length. The energy resolution is shown Figure 3.13.

Clustering algorithm of the EMCal is described below. It is common to the PbSc and the PbGl. In the heavy-ion collisions, many photons, hadrons and leptons hit the EMCal, and they are observed as clusters. The maximum occupancy is about 15 % for the PbSc in the Au+Au collisions at $\sqrt{s} = 200 \text{ GeV}$. In such condition, the sum of the energy of all towers is easily affected by the other clusters unlike the test beam experiments. For instance, the observed ionization energy for the minimum ionizing particles increases by about 6 % in the Au+Au collisions. Instead of taking all clusters for energy measurement, a few towers above an energy threshold which is above 10 MeV for the PbSc and 14 MeV for the PbGl are taken.

First of all, neighboring towers that have an energy above the threshold are gathered to form a first-level cluster, which may be made by more than
Figure 3.10: The construction of the PbSc module.

Figure 3.11: The construction of the PbGl modules.
one electromagnetic shower. Then, the first-level cluster is split into peak-area clusters in the way that the number of the peak-area cluster is equal to the number of local maxima. The energy deposited in a tower between the local maxima is shared according to the breakdown of the deposited energy that is predicted from the energies of the local maxima and the shape of the electromagnetic shower. The energy of a PbGl cluster is summed up all towers in the cluster.

On the other hand, the energy of a PbSc cluster ($E_{core}$) is the sum of energy of “core”.

$$E_{core} = \sum_{i}^{\text{core}} E_{i}^{meas} , \quad (3.10)$$

where the $E_{i}^{meas}$ is the measured energy in $i$-th tower and the $\sum_{i}^{\text{core}}$ is defined as summing up of the towers belonging to the “core” towers. The condition
Figure 3.13: EMCal energy resolution of the PbSc (Top) and the PbGl (Bottom) measured with test beam experiments. The test beam experiments for the PbSc and PbGl have been performed at CERN and BNL. The blue line and red line show the result of fitting by linear (+) and quadratic (⊕) convolution formula, respectively.
of the “core” towers are satisfied with

$$\frac{E_i^{\text{pred}}}{E_i^{\text{meas}}} \geq 0.02 \quad \text{and}$$

$$E_{\text{all}}^{\text{meas}} = \sum_{i} E_i^{\text{meas}},$$

where the $E_{\text{all}}^{\text{meas}}$ is the sum of measured energy in all towers belonging to the cluster, $E_i^{\text{pred}}$ is the predicted energy using the shower profile in $i$-th tower. The average number of towers belonging to the “core” tower is 4 towers. The $E_{\text{core}}$ contains 91.8 % to total energy on average and weakly depends on energy, incident angle and position. The energy of a cluster is corrected for incident angle and energy non-linearity based on the studies with test beam experiments and GEANT simulations. The energy non-linearity is caused by a shower leakage, and for the PbSc, light attenuation in the wavelength-shifting fiber. The corrected form of the light attenuation is $\exp X_0 \ln(E)/\lambda$, where the $\lambda = 120 \text{ cm}$. The shower leakages are also estimated as 1 % at 10 GeV and 4 % at 100 GeV of photon. Figure 3.14 shows the correction functions for these effects.

The shape of a cluster as the spacial distribution of tower energies in a cluster is also parameterized with a value “photon probability” to distinguish an electromagnetic shower from hadrons. It represents the probability that the cluster is produced by a photon. The confidence level is computed with

$$\chi^2 = \sum_i \frac{(E_i^{\text{meas}} - E_i^{\text{pred}})^2}{\sigma_i^2}.$$  

Here the $i$ is the tower index in a cluster, the $E_i^{\text{meas}}$ is energy measured in the $i$-tower, the $E_i^{\text{pred}}$ is energy predicted to be deposited in the $i$-tower, and the $\sigma_i$ is an error on $E_i^{\text{pred}}$. Figure 3.15 shows $\chi^2$ distribution for showers induced by 2 GeV/c electrons and pions in the PbSc. For example, a cut of “photon probability $\leq 0.02$” gets rid of 2 % of photon cluster and $\sim 50$ % of hadron. The arrow in Figure 3.15 represents the $\chi^2$ cut corresponding to 90% electron efficiency.

### 3.5 Muon arms

The Muon arms have the Muon magnets, the Muon trackers (MuTr) and the Muon identifiers (MuID). Each PHENIX Muon arm has a large geometric acceptance of about on steradian and excellent momentum resolution and
Figure 3.14: EMCal energy correction. The angle dependence is corrected with $E' = E / \epsilon_{\text{angle}}$, and then the non-linearity is corrected with $E'' = E' / \epsilon_{\text{nonl}}$. The horizontal axis in the bottom two plots is the energy that has already been corrected for the angle dependence. The correction for the angle dependence of the PbSc includes the ratio of core energy to total energy, 91.8\%.
muon identification. The Muon arms provide a mean of studying vector meson production, the Drell-Yon process via the detection of $\mu$ pairs and heavy quark production. $Z$ and W production will be also studied at forward rapidities via the detector of single high-$p_T$ muons. Additional information about the Muon arms can be found in Reference \[68\]. The Muon arms are only briefly described below as the Muon arm are not used in the present data analysis.

### 3.5.1 Muon tracker (MuTr)

The specific design of the MuTr are driven by the requirements that it is able to 1) allow a clean separation of $J/\Psi$ from $\Psi'$, $\Upsilon(1S)$ from $\Upsilon(2S, 3S)$ and $\rho/\omega$ from $\Phi$, 2) provide a large enough signal to background and acceptance for vector mesons to be able to do statistically significant physics measurements, 3) have low enough occupancy to be able to reconstruct tracks efficiency in central Au-Au events and 4) still perform well in the lower occupancy but higher rate p-p and p-A physics program. The relative mass resolution is approximately given by $\sigma(M)/M = 6%/\sqrt{M}$, where $M$ is in GeV. This mass resolution enables a clear separation of the $\rho/\omega$ peak from the $\phi$, $J/\Psi$ and $\Psi'$, with an acceptable separation of $\Upsilon$ and $\Upsilon'$. This is consistent with a
special resolution of 100 μm.

The above design requirements led to a MuTr design which is comprised of three stations of cathode strip readout tracking chambers (CSC) mounted inside conical-shaped muon magnets with multiple cathode strip orientations and readout planes in each station. All of the CSC are in shape of octants built with a 3.175 mm half gap, 5 mm cathode strips and with alternate strips readout. The anode planes are alternating structure of 20 μm gold-plated Tungsten sense wires and 75 μm gold-plated Cu-Be field wires with a sense wire spacing of 10 mm. Half of the cathode planes have strips at stereo angle between 0° and ±11.25° with respect to the perpendicular strips. The chamber gas mixture is 50% Ar +30% CO₂ +20% CF₄ with a gas recirculation system. The typical operating conditions for this gas are that the HV is 1850 V with a gain of approximately 2×10⁴. The charge deposited by a minimum ionizing particle in the CSC is assumed to be 100 electrons. This results in a total cathode charge of 80 fC. This is an average and the charge is Landau distributed.

3.5.2 Muon identifier (MuID)

The irreducible \( \frac{\mu}{\pi} \) ratio due to weak decay before the nose-cones inside PHENIX is approximately \( 10^{-3} \). We set detector design criteria to 1/4 of this, namely \( 2.5 \times 10^{-4} \), for a \( \pi \) from the vertex or a hadronic descendant to be misidentified as a \( \mu \). The factor of 1/4 provides more than an order of magnitude suppression for the pair background. Thus, the irreducible background of \( \mu \)'s reaching the muon identifier (MuID), as opposed to the MuID design and algorithms used to reject the larger hadron background, will set the ultimate physics background level.

Of this required net \( \frac{\mu}{\pi} \) separation, approximately \( 10^{-2} \) is provided by the presence of steel preceding the MuID which filters out \( \pi \)s. This level is 3% as the maximum tolerable of the charged \( \pi \)'s which may subsequently be misidentified as \( \mu \)s.

In order to set the punch-through probability for pions of up to 4 GeV/c to be 3% or less, a total steel depth of 90 cm (5.4 hadronic interaction lengths) is required beyond the nose-cone and central magnet. Subtracting the thickness of the muon magnet backplate, a total depth 60 cm of the steel is required in the MuID itself. A \( \mu \) at the collision point must have a mean energy of at least 1.9 GeV to reach the MuID system. The mean minimum original energy for a \( \mu \) which penetrates completely through the MuID is 2.7 GeV. Segmentation of the absorber into multiple layers improves the measurement of the trajectory in the MuID. It is desirable to have the early absorber layers to be divided more finely to increase the acceptance for
\( \phi \) meson detection. The segmentation is chosen to be a total of four steel absorbers after the 30 cm thick muon magnet backplate of the north arm. The thick of the segmentations are 10, 10, 20 and 20 cm. The 5 gaps created by the absorbers are instrumented with the MuID panels. The MuID for the south arm is identical to that for the north arm although the muon magnet backplate is only 20 cm thick and at the same distance from the interaction point.

### 3.6 Data acquisition (DAQ) system

#### 3.6.1 DAQ system overview

PHENIX is supplied blue clocks and yellow clock by the RHIC facility. They synchronize with bunches in either the blue ring or the yellow ring. The two clocks make a common timing when two beams collide, and the two PHENIX DAQ synchronizes the blue clock practically. A block diagram of the DAQ system is shown in Figure 3.16.

![Block diagram of PHENIX DAQ system.](image-url)

The blue clock is input to the Master Timing Module (MTM). The MTM provides the distribution of clock timing to the Granule Timing Modules (GTM). Each GTM supplies the clock timing to the group of detectors such as the BBC, the PC3 West and the EMCal in the West Arm, and the scheme
enables us to operate many groups of detectors separately at a time. The GTM clock is input to the Front-end Electronics Module (FEM) of the group of detectors, and the FEM is placed in the interaction region. The FEM has data from its detector on the Analog/Digital Memory Unit (AMU/DMU) over many RHIC clocks. When a trigger decides to take an event, the data of the event stored on the AMU/DMU are read out to the Data Collection Module (DCM) after a digitalization if needed. The DCM is placed in the counting house, and between the FEM and the DCM is connected with fiber-optic links. The DCM formats the data to the PHENIX Row Date Format (PRDF) and sends the PRDF to the Sub Event Buffers (SEB) and then to the Assembly and Trigger Processors (ATP). The SEBs and the ATPs gather all data of each event and simultaneously write the event data to five or six hard disks called PHENIX Buffer Boxes.

### 3.6.2 Front-end electronic module of EMCal

The purpose of the FEM is to digitize analog signal from detectors and to buffer the data to allow for LVL1 trigger decisions. The scheme of the EMCal FEM is shown in Figure 3.17. One EMCal FEM deals with the signals from 12x12 towers. The PHENIX EMCal PMT emits a negative current pulse within a 5 ns rise time, is terminated by 93 Ω register, and is stored into the 500 pF capacitance. The stored charge in the capacitance causes a decrease of the voltage from a reference voltage of ±4 V. The difference of two voltages after and before the decrease is the target for the energy measurement. The voltage profile at the point A in Figure 3.17 depends on the current profile and has a fast rise time as the input pulse. The voltage profile of the point B is the integral of the current, and a step function of an about 100 ns rise time. The fast and slow voltage pulses are input to an ASIC chip, which is specialized for the PHENIX EMCal system, and are used as a timing and energy signals, respectively.

The energy signal passes through a Variable Gain Amplifier (VGA), which is exclusive to each PMT and whose gain can be set remotely to a range from x4 to x12 with 5-bit resolution. The VGAs compensate for the gain variation of PMTs which share a common high voltage. The dynamic range of physics signals from the EMCal is too large to be covered with a single 12-bit ADC conversion. Thus, two different levels of amplification are adopted; one signal as a “low-gain” is amplified only once by the VGA and another signal as a “high-gain” is amplified by the VGA and an x16 amplification. The low-gain and high-gain signals are sampled in AMUs on the EMCal FEM.

The timing signal is discriminated either in a leading-edge mode or a constant-fraction mode, where the choices of mode and threshold voltage are
remotely selected via Arc-Net which is commonly used for the monitoring and for the slow control of the PHENIX FEMs. The discriminator starts a voltage ramp generator. The ramp is stopped on the next edge of the RHIC clock providing a common-stop mode TAC. The voltage after a stop is held for two clock cycles and then sampled in an AMU on the EMCal FEM.

The TAC, low-gain and high-gain signals are sampled once per RHIC clock tick and each signal is held in a ring buffer of 64 AMUs exclusive to each signal. Therefore, the signals are preserved for 64 RHIC clock ticks or \(~7\) ms, which covers the latency of the PHENIX hardware trigger acceptance on an event. The signals of the event stored in an AMU unit are read out and converted in the ADC. The ADC outputs are controlled, collected and reformatted by several Xilinx FPGAs and are sent to the DCM.

### 3.6.3 Level 1 Triggers

My work at PHENIX was to optimize and operate the modules of ERT (Electro-Magnetic Calorimeter (EMCal)-RICH). The fire status of the ERT trigger module and related electronics were carefully checked.

A number of triggers have been designed to ensure a rareness of the physics events of interest, and use the full available luminosity. The triggers consist of two type for the prompt photon measurement and this analysis. One is minimum bias trigger to ensure the event of proton-proton collisions. The other is the ERT trigger designed to enhance the electron, positron, pair
of electrons, $\pi^0$ and prompt photon.

**BBCLL1** The main trigger which is called BBCLL1 for events in PHENIX depends on a coincidence between the two BBCs. In proton-proton collisions, no less than one charged particle at both the forward region ($3 < \eta < 3.9$) and backward region ($-3.9 < \eta < -3$) are required. The reconstructed $z$ collision point is required to be within $\sim \pm 50$ cm in order to assure an flat acceptance PHENIX detectors. The BBCLL1 is used as a minimum bias trigger. In addition, a trigger configuration without cut of collision point is called BBCLL1wide. It is used in the measurement of the BBC cross section. The cross section is used to normalize all cross sections measured at PHENIX.

The efficiency of the BBCLL1 for hard scattering such as prompt photon production has been evaluated using data triggered by a high-$p_T$ photon trigger, which is called ERTLL1$_{2\times2}$ and is described later, because the particles produced in the hard scattering bring large energy from initial beam energy and the energy of spectators is relatively small. The probability of the BBC fired by the spectators could be small. Therefore, the evaluation of the BBCLL1 efficiency is necessary. The evaluation is done as follows;

$$\varepsilon_{BBC} = \frac{N_{\pi^0}^{ERT \& BBCLL1}}{N_{\pi^0}^{ERT}} , \quad (3.14)$$

where the $N_{\pi^0}^{ERT}$ is the number of $\pi^0$s in events taken by the ERTLL1$_{2\times2}$, and $N_{\pi^0}^{ERT \& BBCLL1}$ is the number of $\pi^0$s in events taken by both the ERTLL1$_{2\times2}$ and BBCLL1. The data analyzed in the $\varepsilon_{BBC}$ were taken only by the ERTLL1$_{2\times2}$, and a BBCLL1 trigger bit stored in event data was checked in the off-line analysis. Figure 3.18 shows the efficiencies as function of $p_T$ of $\pi^0$ when $\pi^0$s were measured with the EMCal. The results are that the efficiencies show $p_T$ dependence and decrease from $\sim 0.42$ at the lowest $p_T$ bins to $\sim 0.25$ at near 7 GeV/c.

**ERTLL1** In order to record rare events, such a particle with high $p_T$, an additional trigger is required beyond the BBCLL1. That is EMCal-RICH Trigger (ERTLL1). The ERTLL1 has various configurations concerning the energy threshold or the number of EMCal modules in which the total detected energy is calculated. A basic trigger signal is from 2x2 EMCal modules (called 2x2 tile) summed up in ASIC chip on EMCal FEM. However, in this case, if a high energy particle splits its energy in neighboring tiles, the splitted energy could be below the threshold of configurations in a tile. Therefore, overlapping trigger tiles are set up, consisting of 2x2 neighboring basic tiles, to create a 4x4 tower trigger. There are three configurations in it, which are 4x4a, 4x4b and 4x4c. They differ only in the energy threshold and
require a 2.1 (4×4a), 2.8 (4×4b) and 1.4 GeV (4×4c) energy deposit in a 4×4 tower block made up four neighboring basic tiles. The scheme of the ERTLL1 signal summing part of the EMCal is shown in Figure 3.19 shows. For the measurement of prompt photon production in $\sqrt{s} = 62.4$ GeV, the ERTLL1.2×2 trigger was used.

The efficiency $\varepsilon_{ERT}(p_T)$ of the ERT.2×2 has been evaluated using the BBCLL1 and the ERTLL1.2×2 data;

$$\varepsilon_{ERT} = \frac{N_{\pi^0}^{ERT\&BBCLL1}}{N_{\pi^0}^{ERTLL1}}, \quad (3.15)$$

where the $N_{\pi^0}^{ERT\&BBCLL1}$ is the number of $\pi^0$'s in events taken by both the ERTLL1.2×2 and BBCLL1, and $N_{\pi^0}^{ERTLL1}$ is the number of $\pi^0$'s in events taken by the ERTLL1.2×2. Figure 3.20 shows the efficiencies as function of $p_T^{\pi^0}$. The $\varepsilon_{ERT}(p_T)$ reaches a plateau at $p_T^{\pi^0}$ of about 3 GeV/c. A plateau level for $\pi^0$ in the PbSc and the PbGl are 0.98±0.005 and 0.88±0.02, which is due to the geometry of inactive trigger tiles. For a plateau level of photons, it is known from past experiences that the ERT trigger efficiency is about 50 % at the ERT trigger thresholds and about 100 % at twice the ERT trigger thresholds. Therefore, the plateau level of photon starts above about 1.2 GeV/c on the ERT.2×2.

I optimized and operated the ERT trigger circuit modules. The fire status of the ERT trigger module (see Figure 3.21) and the rejection factor
Figure 3.19: The scheme of the EMCal signal summing.
with a ERT on-line monitor were carefully checked. The rejection factor is the number of BBCLL1 triggered events divided by the number of ERT triggered events. This value is ordinary 5 in the ERT_2×2 in 2006. If the rejection factor is larger or smaller than the ordinal value, problems like a hot trigger module appear in ERT_2×2 trigger circuit module or BBCLL1. The hot ERT modules were stopped if hot ERT trigger modules appear. Trigger threshold, in other words, discriminator level is a constant, but the pedestal of discriminator level changes. Therefore, hot ERT trigger module appears. Thus, I investigated and solved all the problems of the ERT. I also coordinated experimental shift crew to monitor and optimize the ERT. This enabled the trigger for small $p_T$ prompt photon.
Figure 3.21: An example of the fire status of the ERT trigger module. These histograms show the number of triggers fired by each trigger tile (horizontal axis) for each trigger type (4x4A, 4x4B, 4x4C, 2x2, and RICH). Green tower means this trigger tile is masked, and red tower means this trigger tile is hot (noisy) in current run.
Chapter 4

Data analysis

4.1 Outline

Analysis of prompt photon cross section as a function of $p_T$ is described in this section. The data taken in 2006 were used. After the run selections which are described in sections 4.2, 4.3, and 4.4, the integrated luminosity of usable data is 0.065 pb$^{-1}$. The number of total events is $894 \times 10^6$ with the trigger condition of ERTL1,2x2&BBCLL1. The $\pi^0$ tagging method which is described in Section 4.5 is used to extract prompt photons from inclusive photons. Figure 4.1 shows the flow of the analysis with $\pi^0$ tagging method. The inclusive photons are described in Section 4.5.1. They were detected with the EMCal, and charged particles were eliminated with the PC3. The background photon is described in Section 4.5.2 and 4.5.3. Then the prompt photon yields are corrected with correction factors such as acceptance etc. It is described in Section 4.7.

The $p_T$ range of prompt photons in the present analysis is from 2 to 7 GeV/c. In this $p_T$ range, photons from a $\pi^0$ decay do not go into the same tower of the EMCal. In other words, each EMCal detects the photons separately. The fine-grained transverse segmentation of the tower is, therefore, important. For large $\pi^0$ momenta, the distribution of opening angle between the two photons in the laboratory is sharply peaked at the minimum value;

$$\theta_{\text{min}} \sim \frac{2m_{\pi^0}}{E_{\pi^0}}. \quad (4.1)$$

Figure 4.2 shows the schematic drawing of photons from a $\pi^0$ decay. Thus, if $d_{\text{min}}$ is the minimum separation distance between the $\gamma$’s at the tower, and $l$ is the distance from the interaction point to the EMCal, a relation is given...
Figure 4.1: The flow of analysis with $\pi^0$ tagging method.
as,

\[ \frac{d_{\text{min}}}{l} = \frac{2m_{\pi^0}}{E_{\pi^0}}. \] (4.2)

Now \( \theta_{\text{c.m.}} = 90^\circ \), \( p_T^{\pi^0} \sim 14 \text{ GeV}/c \), and \( l = 5 \text{ m} \), \( d_{\text{min}} \) is approximately 19 cm for PHENIX. For example, the PbSc tower size is 5.535 cm. Therefore, the photons from \( \pi^0 \) decay can be measured separately. Figure 4.3 shows the region on the EMCal in which 95% of the decay photon is contained at \( p_T^{\pi^0} = 4 \) and 6 GeV/c. The diameter of the region on the EMCal is \( \sim 1 \) and 0.75 m, which correspond to the 95% probability, at \( p_T^{\pi^0} = 4 \) and 6 GeV/c, respectively. The details are explained in Section 4.3. More details on the \( \pi^0 \) kinematics are described in Appendix B.

![Figure 4.2: A schematic drawing of photons from a \( \pi^0 \) decay.](image)

The prompt photon yield \( (N_\gamma^{\text{corr}}) \) is normalized with the integrated luminosity etc. to determine Lorentz invariant cross section. The cross section is

\[ E \cdot \frac{d^3\sigma}{dp^3} = E \cdot \frac{d^3\sigma}{dp_x dp_y dp_z}. \] (4.3)
Figure 4.3: A schematic drawing of the decay photons from $\pi^0$. Rad circle on the EMCal shows the area where photons are contained with 95% probability at $p_T^{\pi^0} = 4$ and 6 GeV/c.

It can be written as follows;

$$E \cdot \frac{d^3\sigma}{dp_x dp_y dp_z} = E \cdot \frac{d^2\sigma}{dp_T dp_T d\phi dp_z}$$

$$= E \cdot \frac{d^2\sigma}{2\pi p_T dp_T dp_z}$$

$$= \frac{d^2\sigma}{2\pi p_T dp_T dy}$$

$$\sim \frac{d^2\sigma}{2\pi p_T dp_T d\eta}$$

$$= \frac{d^2 N}{2\pi L p_T dp_T d\eta}$$

$$= \frac{\Delta N_{\text{corr}}}{2\pi L p_T \Delta p_T \Delta y},$$

(4.4)

where the rapidity $y$ and pseudo-rapidity $\eta$ are given as

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$$

$$\sim \frac{1}{2} \ln \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$= \eta.$$

(4.7)
The coverage of rapidity $\Delta \eta$ is included in the $\Delta N_{\text{corr}}$ in this thesis. The following relation was used to transform from Eq. 4.4 to Eq. 4.5:

$$\frac{dy}{dp_z} = \frac{1}{E}. \quad (4.8)$$

Various checks on systematic uncertainties are described in Section 4.6 and 4.8.

### 4.2 Run selection

The PHENIX DAQ ordinary takes data continuously for a half to one hour, and such a continuous data taking period is called “run”. All quality assurances are done run-by-run.

Obvious problems such as high voltage trips of the PC3 and the EMCal which cause an increase of dead area are monitored. Variations of gain of the EMCal modules were traced using a test pulse of laser or LED. Detector QA detects further problems; Noisy EMCal modules were detected as a large $\pi^0$ combinatorial background. Runs with bad scaler counts were discarded because scaler counts are used in the luminosity evaluation.

### 4.3 Quality assurance and calibration of EM-Cal

#### 4.3.1 Bad-tower map

The EMCal consists of 24768 individual towers. Some number of the towers can be either dead or hot. The dead here means the towers which do not register energy deposit, and the hot means the towers which are electronically noisy due to the faults of the EMCal front end electronics, photomultiplier tube (PMT), or the EMCal-RICH (ERT) trigger threshold configuration.

Such towers give false signals or wrongly reconstructed cluster energy. The cluster means that photons, hadrons and leptons hit the EMCal, and they are observed as clusters. Dead or hot-towers are detected by checking the number of hits per tower compared to its mean value in a sector. All the edge towers are not well calibrated because electromagnetic shower could be lost into the space next to the tower. Therefore, a layer of tower at the edge is included in the bad-tower map. Uncalibrated towers in tower-by-tower energy calibration which is described later were also labeled bad. The average electromagnetic shower in the EMCal is roughly 2 tower wide.
Therefore, towers in a 3x3 tower group a bad tower are also included in the
bad-tower map. Figure 4.4 shows the bad-tower map.

4.3.2 Tower by tower energy calibration

The tower-by-tower energy calibration was performed with $\pi^0$ mass peak
position, which is 135 MeV/c^2 (15) using a part of data taken in 2006. The
correction of the calibration is represented with one parameter $\varepsilon_i$ for each
tower (index $i$), with which a calibrated tower energy is given by $E^\text{calib.}_i =
\varepsilon_i E^\text{original}_i$.

The $\pi^0$s were reconstructed with photon pairs in which one photon hits
the tower $i$ with an energy of $\geq 0.8$ GeV and another photon hits other tower
with an energy of $\geq 0.1$ GeV. The photon probability of $\geq 0.02$ was required
to all the photons, and only those with $p_T^\gamma \geq 1.0$ GeV/c were used. The $\pi^0$
mass peak position was obtained by fitting a “Gaussian + 2nd / 3rd order
polynomial” function to it, and the $\varepsilon_i$ was set to the ratio of the Particle
Data Group value to the measured peak position. Here we assumed that the
peak position depended only on the entry of the tower $i$, but in reality it
depends also on the energies of towers around the tower $i$ and the energy of
the pair photon.

Because the $\varepsilon_i$ was not accurate due to the above assumption, the pro-
cedure was iterated on all the towers seven times. After each iteration, a
photon energy was calibrated with $\varepsilon$ which is obtained in the previous itera-
tion as $E_{\text{new}} = \sum_j \varepsilon_j e_j$. Here the $j$ was the index of towers that composed a
photon cluster, and $e_j$ is the energy deposited in $j$th tower. Figure 4.5 shows
the distribution of the $\varepsilon_i$. Figure 4.6 and Figure 4.7 show the position and
width of $\pi^0$ mass peak before and after the correction.

4.3.3 Run-to-run energy calibration

Run-to-run energy calibration is performed by plotting $\pi^0$ mass peak position
on run-to-run basis. Figure 4.8 shows the run dependence of the $\pi^0$ mass peak
position. The $\pi^0$ mass peak increased by 2 MeV towards as the run number
increases. In runs of 205558, 205559 and 205560 W2 and W3 sectors showed
about 6% lower $\pi^0$ mass peak than the neighboring runs. They have been
excluded from the analysis.

4.3.4 Energy non-linearity calibration

Energy non-linearity, which is due to the fiber attenuation of the PbSc, energy
leakage at the residual, and non-liner a growth of electromagnetic shower in
Figure 4.4: Bad-tower map of The EMCal in 8 different sectors. The colored towers are the bad-tower and are removed in the analysis.
low energy, has been corrected for as a function of “ecore” which was defined in Section 3.4.6. The correction factors for the PbGl and the PbSc sectors have been estimated individually by adjusting $\pi^0$ mass peak positions with photon pairs with symmetric energy (energy asymmetry: $\frac{E_1-E_2}{E_1+E_2} \leq 0.2$). The reason of using $\pi^0$ instead of $e^-$ is that electrons have energy uncertainty when they reach the EMCal. Under symmetric energy condition, the effect of the non-linearity on two photons is almost the same. The correction parameters as follows which were evaluated by analysis of 2005 data are applied. This is because the $\pi^0$ statistics were low in the data of 2006.

\[
E^{\text{PbSc}}_{\text{corrected}} = \frac{E_{\text{org}}}{0.003 + (1 - 0.01/E_{\text{org}})} \quad (4.9)
\]

\[
E^{\text{PbGl}}_{\text{corrected}} = \frac{E_{\text{org}}}{0.021 + (1 - 0.02/E_{\text{org}})} \quad (4.10)
\]

After all the corrections above have been applied to the data, the energy scale and resolution of photons were confirmed by checking the position and width of $\pi^0$ mass peaks. The red lines are the result of real data, and black lines are the result of the fast Monte-Carlo (MC) simulation. The MC simulation is described details in Section 4.5. Figure 4.9 shows the position and width of the $\pi^0$ mass peaks as a function of $p\pi^0$. The red lines agree with the black lines. A 1.5 % systematic uncertainty as an energy scale uncertainty. Therefore, Eq. 4.9 and Eq. 4.10 are applied in this analysis.
Figure 4.6: Figures show the position of $\pi^0$ mass peak as a function of $p_T$ in each sector before (Black) and after (Red) the correction for the tower-by-tower gain variation. Top left is W0 and top right is W3. Bottom left is E0 and bottom right is E3.

Figure 4.7: Figures show the width of $\pi^0$ mass peak as a function of $p_T$ in each sector before (Black) and after (Red) the correction for the tower-by-tower gain variation. Top left is W0 and top right is W3. Bottom left is E0 and bottom right is E3.
Figure 4.8: Run-to-run $\pi^0$ peak. Top left is W0 and top right is E0. The horizontal axis is the run-numbers, the vertical axis is $\pi^0$ mass peak position (GeV/$c^2$).
Figure 4.9: $\pi^0$ peak positions in top panels and widths in bottom panels. Left panels are for the PbSc, and right panels are for the PbGl.
4.3.5 ERT live area

Nine super modules (W0-SM05, W3-SM04, E0-SM07, E0-SM10, E1-SM09, E1-SM10, E1-SM15, E1-SM18, E1-SM30) were masked in the data taking period. Therefore, these additional masks were also applied in the analysis. It is 5.2% of the total acceptance.

4.4 Event and particle selection

4.4.1 Event selection

The following conditions are required for event selection. (1) Events are triggered with a coincidence between ERTLL1_2x2 and BBCLL1 as a trigger requirement. (2) Events have at least one photon with $p_T \geq 2 \text{ GeV}/c$ because the probability of prompt photon production relatively becomes large in large $p_T$ region. $p_T$ is defined in Eq.\ref{2.23}. In this selection, a photon which is produced in forward region with a relatively large $|p|$ or in $\theta \sim 90^\circ$ with a relatively small $|p|$ is eliminated. (3) The off-line BBC z-vertex cut ($|z| \leq 30$ cm) was applied.

4.4.2 Photon selection in each event

Photons are selected from clusters of the EMCal. The cluster of EMCal is the shower which is produced by electrons, photons or hadrons.

Firstly, the cluster is applied to the bad-tower map of the EMCal. Then, the cluster is applied to following cuts to extract the cluster made by photon.

- photon probability cut $\geq 0.02$.
- excluding clusters which matched tracks with $|\text{emcpcd}| \leq 0.024$ rad \&\& $|\text{emcpcdz}| \leq 9.3$ cm (PC3 – EMCal matching cut).
- requiring the ERTLL1_2x2 bit.

The description of the photon probability is in Section 3.4.6. The cluster, which passes through these cuts, is named as Target photon. The PC3 is used to veto charged particle; and the PC3 - EMCal matching parameter emcpcd$\phi$, emcpcd$z$ was used to reduce the contribution of charged particles. Figure\ref{4.10} shows the matching parameter distribution of the PC3 – EMCal. The PC3 – EMCal matched clusters are defined as $|\text{emcpcd}$ $\phi| < 0.024$ rad and $|\text{emcpcdz}| < 9.3$ cm.

Against the target photon, another group of cluster named as $\pi^0$ partner photon was made to reconstruct $\pi^0$ invariant mass. The $\pi^0$ partner photon
Figure 4.10: The left figure is $\text{emcpc3d} \phi$ distribution, and the right figure is $\text{emcpc3dz}$ distribution.

is the cluster, which passes through the bad-tower map of the EMCal and Minimum energy cut $E \geq 0.15$ GeV. The 1) photon probability cut, 2) PC3 and 3) ERTL1\_2x2 bit were not used. The reasons are that 1) photons from $\pi^0$ can be identified with $\pi^0$ invariant mass, 2) $\pi^0$ partner photon could convert into an electron/ a positron by the photon conversion, 3) it is not necessarily to require the ERTL1\_2x2 bit. Figure 4.11 shows schematic drawing of clusters in one events.

4.5 $\pi^0$ Tagging Method

Prompt photons ($N_\gamma$) are extracted with Eq. (4.11) as a function of $p_T$.

$$N_\gamma(p_T) = N^{incl}(p_T) - (1 + A) \times \{1 + R(p_T)\} \times N^0_\gamma(p_T) \quad (4.11)$$

Here,

- $N^{incl.}$: The number of inclusive photons. It is corrected with the effect of charged particles, neutral hadrons and secondary origin photons.

- $N^0_\gamma$: The number of two photons from $\pi^0$. These photons are detected with the EMCal. A $\pi^0$ is identified with its invariant mass which is reconstructed from the momenta of two photons.
Figure 4.11: The schematic drawing of clusters in one event.

- **R**: The ratio of one tag photons to two tag photons. The one tag photon means that one photon from $\pi^0$ is detected and another is not detected due to the intrinsic limitations such as the bad-towers, the geometry acceptance of the EMCal, etc.

- **A**: The ratio of $\eta$, $\omega$ and $\eta'$ production cross section to $\pi^0$ production cross section and the ratio branching ratios. It is used to evaluate background photons from hadrons other than $\pi^0$. $A = \sum \frac{\sigma_{i\gamma}}{\sigma_{\pi^0\gamma}} \frac{Br(i\rightarrow\gamma\gamma)}{Br(\pi^0\rightarrow\gamma\gamma)}$. The index $i$ corresponds to $\eta$, $\eta'$ and $\omega$ particles.

The details of these terms are explained below.

### 4.5.1 Inclusive photon yields ($N^{incl.}$)

The inclusive photons are selected from clusters using the photon selection procedure\(^{[4.4.2]}\). Corrections for the contribution of charged particles, neutral hadron such as (anti-) neutrons and secondary photons are needed. The charged particles are migrated in the inclusive photons due to inefficiency of the PC3 pads i.e. incomplete veto. There is a probability for the antineutrons to pass the photon selection because we cannot surely identify the particles. These contributions are corrected for:

$$N^{incl.} = (N_1 - \frac{N_1 - N_2}{\varepsilon_1}) \times \varepsilon_2. \quad (4.12)$$
Here the $\varepsilon_1$ is the ratio of live PC3 pads area to live EMCal area, the $\varepsilon_2$ is the ratio of (inclusive photons)/(neutral particles + inclusive photons). $N_1$, $N_2$ is the number of target photons without applying the PC3, the number of target photons, respectively.

The secondary photons are produced in the process in which particles produced in proton-proton collisions hit the subsystem detector or magnet of the Central Arms. The contribution of a secondary photon is described below, and is negligible.

**The contribution of charged particles**

Figure 4.12 shows the live PC3 pads (red) and the live EMCal area (red with black point). The black area means that EMCal is alive but PC3 pad is dead. The $\varepsilon_1$ is evaluated by counting the red pads and red and black pads, which are about 90%. The uncertainty of this evaluation is about 4%. Figure 4.13 shows the ratio of the number of clusters with PC3 hits to the number of cluster without that. It is about 15% ~ 20%. The PC3 hits are due to charged particles and electron/positron from photon conversion. The photon conversion is about 11% described in Section 4.7.5.

Remaining part 4% ~ 9% is the contribution of charged particles (hadrons). The uncertainty of the charged particle contribution is

$$\text{(0.04} \sim 0.09) \times 0.04 = 0.0016 \sim 0.0036.$$ (4.13)

Therefore, it is negligible.

**The contribution of anti-neutrons**

The main contribution of neutral hadron contamination is anti-neutron. The contributions of other hadrons such as neutron and $K_L^0$ are small. The neutral hadron contamination is evaluated by considering anti-proton production, with taking advantage of iso-spin symmetry. The anti-neutrons are produced by strong interaction, and the iso-spin symmetry is conserved.

The contribution of anti-neutron in the PbSc was evaluated using the previous analysis which was based on the condition of proton-proton collisions at $\sqrt{s} = 200$ GeV.

- spectra of (anti-) neutron which is estimated from (anti-) proton spectra by considering the iso-spin symmetry.

- The EMCal response function from PHENIX Integrated Simulation Application (PISA).
Figure 4.12: The red color with black points shows the region where both EMCal and PC3 pads are alive. The black color shows the region where EMCal is alive but PC3 pads are dead. The top panel is the West Arm from the bottom, W0, W1, W2 and W3. The bottom panel is the East Arm from the bottom, E0, E1, E2 and E3.
Figure 4.13: The ratio of the number of clusters with the PC3 hits to the number of clusters without the PC3 hits. This result shows charged particles are $15 \sim 20\%$.

- A fine tuning of the response function with charged hadron cluster.

Figure 4.14 shows the calculation results of each step. The contribution of anti-neutrons to inclusive photons is mainly in low $p_T$ region. It is assigned to be $2\%$ at $p_T = 2\text{ GeV/c}$, and below $1\%$ above $3\text{ GeV/c}$.

**The contribution of secondary photons**

The contribution of secondary photons is evaluated with a simulation using PYTHIA + PISA. Figure 4.15 shows the secondary photon detected by the EMCal in $r-z$ plane. In the simulation, the $\theta_e$ in Figure 4.15 is used to distinguish the secondary photons from EMCal cluster. Figure 4.16 shows the energy distribution of EMCal clusters and the secondary photons. The ratio of the secondary photons to the EMCal cluster is less than $1\%$ above $p_T = 2\text{ GeV/c}$.

**Multiplicity of inclusive photons**

The mean multiplicity of the inclusive photons is evaluated. Figure 4.17 shows the multiplicity distribution. The mean multiplicity is about 1.3. If
Figure 4.14: The simulation results of anti-neutron spectra (red) and the anti-neutron spectra measured by the EMCal. The black line gives inclusive photons spectra. The contribution of the anti-neutrons to the photons (blue curves divided by black curves) is shown in the bottom figure.
Figure 4.15: The origins of the EMCal clusters in $r - z$ plane. The definition of $\theta_r$ is shown in the figure. Red points are regarded as the secondary photon because of large $\theta_r$.

Figure 4.16: Energy distributions of all the clusters and the cluster of secondary photons.
the acceptance of the EMCal (17.5%) to the total acceptance is taken into account, the mean multiplicity becomes $\sim 7.4$. Therefore, in this event selection and applied cuts, about 7 inclusive photons are produced in an event.

![Graph showing multiplicity distribution](image)

Figure 4.17: The multiplicity distribution in the both arms (Black), West arm (Red), and East arm (Blue).

### 4.5.2 Evaluation of $\pi^0$ decay photons

**Two tag photons ($N^\pi^0_\gamma$)**

The two tag photons are evaluated with the invariant mass of $\pi^0$, which are reconstructed from the momenta of $\pi^0$ decay photons. The mass is 135 MeV/$c^2$. A $\pi^0$ decay into two photons by the mean lifetime $8.4 \times 10^{-17}$ s after it was produced. When energy and momentum of the two tag photons are $E_1, E_2$ and $p_1, p_2$, respectively, the mass is calculated as

$$m_{\pi^0} = \sqrt{2(E_1 \cdot E_1 - p_1 \cdot p_2)}.$$ (4.14)

The calculation of the Eq. 4.14 is done for all the combinations between a photon called “target photon” and all other photons called “$\pi^0$ partner photon” in each event. The target photon and $\pi^0$ partner photon are defined...
in Section 4.4.2. The event here means one proton-proton collision event. This calculation is performed at the each $p_T$ of the target photon. The result of the calculation should give 135 MeV/$c^2$ if the combined two photons are from $\pi^0$ decay, and thus the two tag photons are identified.

Figure 4.18 shows the invariant mass distributions of the target photons at $2.0 < p_T < 2.25$ GeV/$c$. Figure 4.19 shows the invariant mass distributions with the target photons at various $p_T$ range. Then a combination of a Gaussian and a third order polynomial function was fitted in the $\pi^0$ mass region from 0.105 to 0.165 MeV/$c^2$. This fit was done with the histograms at low $p_T$ of the target photons as shown in Figure 4.20. The combinatorial background under the $\pi^0$ mass peak was subtracted by this way. On the other hand, in high target photon $p_T$ ranges, a sideband subtraction method was used due to the low statistics. The sidebands used are the mass range from 0.045 to 0.105 GeV/$c^2$ and from 0.165 to 0.225 GeV/$c^2$. The constant background was evaluated this way and was subtracted from the yields in the $\pi^0$ mass region.

![Figure 4.18: An example of two photon’s invariant mass distribution (2.0 < $p_T$ < 2.25 GeV/$c$).](image)

Figure 4.21 shows the relation between $p_T^{\pi^0}$ and $p_T^\gamma$, and the relation between the $p_T^{\pi^0}$ and opening-angle $\theta$ defined in Figure 4.2. $p_T^{\pi^0}$ is the transverse momentum of photon from $\pi^0$ decay. The minimum opening angle in the region $6.5 < p_T^{\pi^0} < 7.5$ is $\approx 0.05$ rad. The distance between the two
Figure 4.19: The mass distribution with a target photon and partner photons at each target photon’s $p_T$. The horizontal axis is mass (GeV/$c^2$) and the vertical axis is yield.
Figure 4.20: The combination of a Gaussian and a third order polynomial function is fitted to the $\pi^0$ mass region. The horizontal axis is mass (GeV/$c^2$) and vertical axis is yield. The fit was done for the spectra with $p_T$ of the target photon smaller than 3.75 GeV/$c$. At the target photon $p_T > 4$ GeV/$c$, a side band method is applied.
photons for $\theta \sim 0.05$ on the EMCal is 25 cm ($0.05 \times 5$ m). Therefore, the photons from $\pi^0$ decays up to $p_T \pi^0 \sim 7.5$ GeV/c are detected separately.

**Multiplicity of $\pi^0$s**

The mean multiplicity of the $\pi^0$s is evaluated. In particular, $\pi^0$ of $p_T ^\pi 0 > 4$ GeV/c are selected, because photons of $p_T > 2$ GeV/c are analyzed in this analysis. Figure 4.22 shows the multiplicity distribution. The mean multiplicity is about 0.01. If the acceptance of the EMCal (17.5%) to the total acceptance is taken into account, the mean multiplicity becomes $\sim$. Therefore, in this event selection and applied cuts, about 0.05 $\pi^0$ are produced in an event.

**Evaluation of the ratio of the one tag photon to two tag photon ($R$)**

The $R$ is evaluated with a fast Monte-Carlo simulation which is called as “MC simulation” here. It is also used to correct for detector acceptance and energy smearing described in Section 4.7.2. The MC simulation is not like the GEANT simulation. The MC simulation is single particle simulation and is a simulation in which the detector response is parametrized according to the performance such as energy resolution, position resolution, the bad towers, etc. during the data taking. The parameters of the MC simulation are adjusted in order to reproduce the experiment.

- The distribution of collision points which is a Gaussian distribution of 80 cm width is used. It is based on the measured distribution of collision points. A cut at $z$: ±30 cm is applied.
- The same bad-towers map described in Section 4.3.1 and the dead ERT areas described in Section 4.3.5 are used.
- The energy scale, energy non-linearity described in Section 4.3.4 and the energy and position resolution of the EMCal described in Section 3.4.6 are used.
- The cuts used in the event selection which are described in Section 4.4 and the minimum energy cut for the $\pi^0$ partner photons described in Section 4.3.2 are used.

The MC simulation calculates the $R$ in the following steps. It simulates one tag photons and two tag photons.
Figure 4.21: **Top** The relation between $p_{T\pi^0}$ and $p_{T\gamma}^{\pi^0}$. **Bottom** The relation between the $p_{T\gamma}^{\pi^0}$ and opening-angle $\theta$. 
Figure 4.22: The multiplicity distribution in the both arms (Black), West arm (Red), and East arm (Blue). Red and blue overlap each other at multiplicity 0.
1. In $|\eta| \leq 0.5$ and $0 \leq \phi \leq 2\pi$, $\pi^0$s are produced with a flat distribution. The $\pi^0$ production at $p_T^{\pi^0} = 2$ and 3 GeV/c in $\eta$ direction is estimated also with PYTHIA simulation. This simulation bases a QCD Monte-Carlo simulation. It is shown in Figure 4.23. It is almost a constant in the $|\eta| < 0.5$ and decreases by about 7% at $|\eta| = 1$ comparing with $\eta = 0$. In $p_T$ distribution, the $\pi^0$s are produced with the slope of the cross section at $\sqrt{s} = 62.4$ GeV in order to obtain the weight which is the fraction of $\pi^0$ yields at each $p_T$. The $\pi^0$ cross section[80] at $\sqrt{s} = 62.4$ GeV is obtained by fitting a function to the Lorentz invariant cross section of $\pi^0$. The cross section was already measured, and the slope is well represented with a formula;

$$
\frac{d\sigma}{dp_T} = p_T \cdot (F(p_T) \cdot \frac{p^2}{1 + p_T/p^3})^4 + (1 - F(p_T)) \cdot \frac{p^5}{p^6_T}. \quad (4.15)
$$

The expression of left-hand side in Eq.4.15 is obtained by transforming variables of the cross section from $dx$, $dy$ and $dz$ into $\eta$, $\phi$ and $p_T$, and integrating over $\eta$ and $\phi$. Figure 4.24 shows the result of the fit. Here,

$$
F(p_T) = \frac{1}{1 + \exp((p_T - p_0)/p_1)}. \quad (4.16)
$$

Eq.4.16 smoothly connects the cross section from low $p_T$ region to high $p_T$ region. The cross section in low $p_T$ region and high $p_T$ region are reproduced

$$
\frac{p^2}{1 + p_T/p^3} \quad (4.17)
$$

and

$$
\frac{p^5}{p^6_T}, \quad (4.18)
$$

respectively.

2. Then, the produced $\pi^0$s isotropically decay into two photons in its rest frame.

3. If at least one of the two photons from the $\pi^0$ decay has $p_T > 2$ GeV/c, one tag photon or two tag photons are filled taking account of the weight and the adjusted parameters above. The ratio $R$ of one tag photons to two tag photons is calculated in this way. Figure 4.25 shows the distribution of the $R$. 

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In order to check the reproducibility of the MC simulation, kinematics of $\pi^0$ such as opening-angle $\theta$ between two photons are evaluated using the experimental and numerical calculations. The numerical calculations for the kinematics of $\pi^0$ is described in Section 4.3. The $\theta$ is also defined in Figure 4.2. Note that, in the evaluation of the experimental data, it is evaluated event by event with applying $\pi^0$ mass window cut $0.105 < M_{\gamma\gamma} < 0.165$ (GeV/c$^2$). Therefore, the accidental combinatorial backgrounds in the region of the $\pi^0$ mass window are included in the experimental data. On the other hand, in the MC simulation, the combinatorial backgrounds do not exist because the MC simulation is single particle simulation of $\pi^0$.

Firstly, the relation of between the $\theta$ and $p_T^{\pi^0}$ is evaluated. Figure 4.26 and Figure 4.27 show the comparison of the experimental data with the numerical calculation described in Section 4.3, and comparison of the MC simulation with the numerical calculation, respectively. Comparing with the figures, the distribution shape in the MC simulation agrees with that in the experimental data. The upper limit of the $\theta$ is determined by the minimum energy cut of the EMCal. In this analysis, it is $< 0.15$ GeV. When the minimum energy cut becomes smaller, the $\theta$ will be distributed over a larger $\theta$.

Figure 4.28 shows the relative yields of the opening-angle $\theta$ at $p_T^{\pi^0} \sim 4$ and 6 GeV/c. The sharped peak corresponds to minimum opening-angle.
Figure 4.24: \( \pi^0 \) cross section at \( \sqrt{s} = 62.4 \) GeV with the fit function.

Figure 4.25: The distribution of the ratio \( R \) as a function of \( p_T \).
Figure 4.26: Two-dimensional histogram of $p_T^{\pi^0}$ and $\theta$. It is evaluated using the experimental data. The blue curve represents the minimum opening-angle from the numerical calculation (Eq. B.15).

Figure 4.27: Two-dimensional histogram of $p_T^{\pi^0}$ and opening-angle $\theta$. It is evaluated using the MC simulation. The blue curve represents the minimum opening-angle from the numerical calculation (Eq. B.15).
The position of the minimum opening-angle and slope in the MC simulation agree with that in the experimental data.

Figure 4.29 shows the comparison of the MC simulation with the numerical calculation defined in Eq. B.21. According to Eq. 3.7 and Eq. 3.9, the position resolution of the EMCal at photon energy $\sim 2$ GeV is $\sim 5$ mm. The discrepancy between the MC simulation and the numerical calculation at the minimum opening-angle is $\sim 10^{-5}$. On the EMCal, the discrepancy distance is obtained by multiplying the distance 5 m, which is from the beam collision point to the EMCal. It is $\sim 10^{-3}$ m, and agrees with the Eq. 3.7 and Eq. 3.9.

Next, the angle between the movement direction of $\pi^0$ and the decay photon is evaluated using the experimental data, and numerical calculation. The position and slope of the angle in the MC simulation reproduce well comparing with those of the angle in the experimental data. Figure 4.30 shows the relative yields of this angle at each $p_T^\pi^0$.

Figure 4.31 shows the comparison of the MC simulation with the numerical calculation defined in Eq. B.13. The disagreement between the MC simulation and the numerical calculation at small angle is caused by the limitation of position resolution of the EMCal. The angle of the peak position is about half value at the minimum opening-angle of Figure 4.28.

### 4.5.3 Evaluation of decay photon from $\eta$, $\eta'$ and $\omega$

Background photons come also from neutral hadron decays such as $\eta \to \gamma \gamma$, $\eta' \to \gamma \gamma$ and $\omega \to \gamma \pi^0$. These contributions can be evaluated by utilizing the ratio of cross section for $\eta$, $\eta'$ and $\omega$ production to $\pi^0$ production and their decay branching ratios. $\pi^0$, $\eta$, $\eta'$ and $\omega$ productions, and the branching ratios were already known well by experiments in the past. One can precisely evaluate these background photons $N_{BG}$:

$$N_{BG} = A \times (1 + R) \times N_{\pi^0}$$

$$A = \sum_i \frac{\sigma_i}{\sigma_{\pi^0}} \times \frac{Br(i \to \gamma(\gamma))}{Br(\pi^0 \to \gamma \gamma)}$$

where $R$ is the ratio of one tag photon to two tag photons, index $i$ corresponds to $\eta$, $\eta'$ or $\omega$. $\sigma_i$ means $\eta$, $\eta'$ or $\omega$ production cross section, and $Br_i$ is the branching ratio of $i \to \gamma(\gamma)$. According to the description below, $A = 0.23 \pm 0.05$.

The production cross section ratio The ratio of cross section for $\eta$, $\eta'$ and $\omega$ production to $\pi^0$ production was measured in several energy regions and $p_T$s. Figure 4.32 shows an example of $\eta/\pi^0$. It indicates that the ratio
Figure 4.28: The distributions of relative yields of the opening-angle $\theta$ at $p_T^{\mu^+} \sim 4$ and 6 GeV/c. The red line and black line are obtained from the experimental data and the MC simulation, respectively. The number of yields in each bin are normalized with the total yield.
Figure 4.29: The comparison of the MC simulation (black) with the numerical calculation (Blue). The plots of the MC simulation are used the data in the region of \(3.8 < p_T^\pi^0 < 4.2 \text{ (GeV/c)}\)

\[\eta/\pi^0\] is constant and is \(0.45 \pm 0.1\). It is independent of the energy of the reaction and photon’s \(p_T\) above \(1.5 \text{ GeV/c}\). The \(\eta'\) and \(\omega^0\) production ratios are also constant, and are summarized in Table 4.1. The uncertainties of the ratios are assigned as the systematic uncertainties in the \(A\). It refers to Reference [81].

**Branching ratio of neutral mesons to photon** The branching ratio of \(\eta \rightarrow \gamma \gamma\), \(\eta' \rightarrow \gamma \gamma\) are also summarized in Table 4.1. It refers to Reference [15].

### 4.5.4 Statistical uncertainty

The propagation of statistical uncertainty of prompt photon \((\delta N_\gamma)\) was calculated considering the correlation between inclusive photons \((N^{incl.})\) and two tag photons \((N_{\gamma}^{\pi^0})\). \(N_\gamma\) is written as

\[
N_\gamma = N^{incl.} - (1 + A) \times (1 + R) \times N_{\gamma}^{\pi^0} = N^{incl.} - B \cdot N_{\gamma}^{\pi^0},
\]  

(4.21)
Figure 4.30: Relative yields of the angle between the movement direction of $\pi^0$ and the decay photon at $p_T^{\pi^0} \sim 4$ and 6 GeV/c. The red line and black line are obtained from the experimental data and the MC simulation, respectively. The number of yields in each bin are normalized with the total yield.
Figure 4.31: The comparison of the MC simulation (black) with the numerical calculation (Blue). The plots of the MC simulation are used the data in the region of $3.8 < p_T < 4.2$ (GeV/c)

Table 4.1: Cross section ratio and branching ratio of neutral mesons.

<table>
<thead>
<tr>
<th>Particle/$\pi^0$</th>
<th>Ratio of cross section for production</th>
<th>Branching ratio</th>
<th>(Ratio of cross section) $\times$ (branching ratio)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta/\pi^0$</td>
<td>$0.45 \pm 0.1$</td>
<td>$\frac{Br(\eta\rightarrow 2\gamma)}{Br(\pi^0\rightarrow 2\gamma)} = \frac{39.4}{98.8}$</td>
<td>$0.18 \pm 0.04$</td>
</tr>
<tr>
<td>$\eta'/\pi^0$</td>
<td>$0.25 \pm 0.08$</td>
<td>$\frac{Br(\eta'/\rightarrow 2\gamma)}{Br(\pi^0\rightarrow 2\gamma)} = \frac{2.1}{98.8}$</td>
<td>$0.0053 \pm 0.0017$</td>
</tr>
<tr>
<td>$\omega^0/\pi^0$</td>
<td>$1.0 \pm 0.3$</td>
<td>$\frac{Br(\omega^0\rightarrow \gamma)}{Br(\pi^0\rightarrow 2\gamma)} \times 0.5 = \frac{8.9}{98.8} \times 0.5$</td>
<td>$0.045 \pm 0.014$</td>
</tr>
<tr>
<td>$A$</td>
<td>-</td>
<td>-</td>
<td>$0.23 \pm 0.05$</td>
</tr>
</tbody>
</table>

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Figure 4.32: The ratio of cross section for $\eta$ production to $\pi^0$ production. It is independent of the energy of the reaction and on photon $p_T$ above 1.5 GeV/c.
where the $B$ is $(1 + A) \times (1 + R)$. The statistical uncertainty is evaluated considering the correlation between $N_\gamma$ and $N_\gamma^{*0}$.

$$\delta N_\gamma = \sqrt{\left(\frac{\partial N_\gamma}{\partial N^{\text{incl.}}}\right)^2 \delta N^{\text{incl.}}^2 + \left(\frac{\partial N_\gamma}{\partial N_\gamma^{*0}}\right)^2 \delta N_\gamma^{*0}^2 + 2 \left(\frac{\partial N_\gamma}{\partial N^{\text{incl.}}}\right) \left(\frac{\partial N_\gamma}{\partial N_\gamma^{*0}}\right) \text{cov}(N^{\text{incl.}}, N_\gamma^{*0})}$$

$$= \sqrt{\left(\delta N^{\text{incl.}}\right)^2 + B^2 \left(\delta N_\gamma^{*0}\right)^2 + 2(-B) \left(\text{cov}(N_\gamma, N_\gamma^{*0}) + \text{cov}(N_\gamma^{*0}, N_\gamma^{*0})\right)}$$

$$\sim \sqrt{\left(\delta N^{\text{incl.}}\right)^2 + B^2 \left(\delta N_\gamma^{*0}\right)^2 - 2B \delta N_\gamma^{*0}}$$

$$= \sqrt{\left(\delta N_\gamma^{*0}\right)^2 + B(B - 2)(\delta N_\gamma^{*0})^2}$$

$$= \sqrt{\left(\delta N_\gamma^{*0}\right)^2 + (B - 1)^2(\delta N_\gamma^{*0})^2}, \quad (4.22)$$

where $N_\gamma^{*0} = N^{\text{incl.}} - N_\gamma^{*0}$, $\text{cov}(N^{\text{incl.}}, N_\gamma^{*0})$, etc. are covariance between $N^{\text{incl.}}$ and $N_\gamma^{*0}$, etc.

### 4.5.5 Prompt photon yields $(N_\gamma)$

Prompt photons as a function of $p_T$ were extracted with Eq. 4.11. It is shown (red) in Figure 4.33, which also shows the yields of the inclusive photon (black, cross), the two tag photons (green, open circle) and background photons (blue asterisk). These numbers are normalized with the $p_T$ width in the histogram. Figure 4.34 is the each photon group divided by the inclusive photons.

### 4.6 Systematic uncertainties $(I)$

The propagation of systematic uncertainties is calculated as follows;

$$\delta N_\gamma = \delta N^{\text{incl.}} - \delta B \times N_\gamma^{*0} - B \times \delta N_\gamma^{*0}. \quad (4.23)$$

Here $B = (1 + R) \times (1 + A)$ and

$$\frac{\delta N_\gamma}{N_\gamma} = \frac{\delta N^{\text{incl.}}}{N_\gamma} - \frac{\delta B N_\gamma^{*0}}{N_\gamma} - \frac{B \delta N_\gamma^{*0}}{N_\gamma}$$

$$= \frac{\delta N^{\text{incl.}}}{N_\gamma} \frac{N^{\text{incl.}}}{N_\gamma} - \frac{\delta B N_\gamma^{*0}}{N_\gamma} \frac{B N_\gamma^{*0}}{N_\gamma} - \frac{B \delta N_\gamma^{*0}}{N_\gamma} \frac{B N_\gamma^{*0}}{N_\gamma}$$

$$= \frac{\delta N^{\text{incl.}}}{N_\gamma} W - \frac{\delta B N_\gamma^{*0}}{B N_\gamma^{*0}} (W - 1) - \frac{B \delta N_\gamma^{*0}}{B N_\gamma^{*0}} (W - 1), \quad (4.24)$$

where $1/W = N_\gamma/N^{\text{incl.}}$. 

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Figure 4.33: Distributions of prompt photon (red point) with statistical uncertainties are shown. $N^{incl}$, $N^{π^0}$ and background photon are also shown as black cross, green open circle, and blue asterisk, respectively.

Figure 4.34: The distribution of each photon yields divided by the inclusive photons.
The systematic uncertainties can be grouped into three categories. One is uncertainty of \( N_\gamma \), second one is uncertainty of \( N_\gamma^{0°} \), and third one is uncertainty of \( N^{incl.} \). The second one and third one are proportional to \( \frac{1}{\sqrt{W}} - 1 \) and \( \frac{1}{\sqrt{W}} \) from Eq. 4.24 respectively. In this section, the systematic uncertainties of \( N^{incl.} \) and \( N_\gamma^{0°} \) are described. The systematic uncertainties of the \( N_\gamma \) are described in Section 4.8.

The uncertainties of \( N^{incl.} \) includes the uncertainty of anti-neutron contribution (B1), secondary photons (B2) and charged particle contribution. The charged particle contribution is negligible (Section 4.5.1). The uncertainties on \( N_\gamma^{0°} \) are as follows;

- The uncertainty of the \( A \) (C1) is \( \Delta A/A = 0.05/(1+0.23) = 4 \% \).
- The uncertainty of \( \pi^0 \) Dalitz decay contribution and conversion loss of \( \pi^0 \) partner photon (C2) is 1 %.
- \( \pi^0 \) extraction uncertainty (C3) is described in Section 4.6.1.
- The uncertainty of the \( R \) due to the function slope of \( \pi^0 \) cross section (C4) is described in Section 4.6.2.
- The uncertainty of the \( R \) due to the minimum energy cut of the EMCal (C5) is described in Section 4.6.3.
- The uncertainty of the \( R \) due to the acceptance (C6) is described in Section 4.6.4.
- The uncertainty of the \( R \) due to \( \pi^0 \) production at the \( \eta \) direction in the MC simulation (C7) is described in Section 4.6.5.
- The uncertainty of the EMCal energy resolution is negligible.

### 4.6.1 \( \pi^0 \) extraction uncertainty

\( \pi^0 \) extraction uncertainty (C3) was evaluated from the data used in this analysis. This uncertainty was assigned using the difference the number of \( N_\gamma^{0°} \)s between two different fit functions and different \( \pi^0 \) mass peak window range.

For the evaluation with two different fit functions, (Gaussian + second order polynomial) and (Gaussian + third order polynomial) were adopted. The combinatorial background is evaluated with these polynomial functions. Figure 4.44 shows the \( \frac{N_{\pi^0, pol.3} - N_{\pi^0, pol.2}}{N_{\gamma, pol.3}} \). For the evaluation of the different \( \pi^0 \)
mass peak window regions, two regions which are [0.105, 0.165 GeV/$c^2$] and [0.095, 0.175 GeV/$c^2$] are used. The uncertainty due to the different π⁰ mass peak window regions is about 1%. This 1 % uncertainty is added to the uncertainty of the evaluation.

The ratio of two tag photons to combinatorial background becomes small as $p_T$ goes to high $p_T$. Uncertainties above 4.25 GeV/$c$ are assumed to be the same as uncertainty of 3.75 GeV/$c$ because both $N_{\pi^0}$s evaluated by the function and by the side band are almost the same. It can be seen in Figure 4.36 and therefore this extrapolation can be applied.

### 4.6.2 The uncertainty of the $R$ due to the function slope of $\pi^0$ cross section (C4)

In the MC simulation, the function slope of the $\pi^0$ cross section is used to reproduce the one tag photons and two tag photons taking into account the probability for $\pi^0$ production at each $p_T^{\pi^0}$ to evaluate the contribution of one tag photons in the experiment. The function slope is determined by fitting to the $\pi^0$ cross section. Therefore, it is important to evaluate the contribution of uncertainties of parameters in the function slope to the $R$.

Parameters are firstly determined in terms of evaluating the uncertainties of parameters in the function slope. It is adequate to determine the parameters changing $\chi^2$ of the fit by ±1. Figure 4.37 shows the function slope changing $\chi^2$ by ±1 on the $\pi^0$ cross section. It is a small change on the figure.

Secondary, in the MC simulation, the determined parameters above are set the parameters of the function slope in the $\pi^0$ cross section. Then, the $R$ is recalculated. Figure 4.38 shows the ratio of (default $R$ - the $R$, which are evaluated above) to (the default $R + 1$). Here the “default $R$” means that it is used in measuring the central value of prompt photon yields. Form Figure 4.38 this uncertainty is negligible.

### 4.6.3 The uncertainty of the $R$ due to the minimum energy cut of the EMCal (C5)

The uncertainty due to minimum energy cut (C5) was evaluated using $\pi^0$‘s from the data and the MC simulation, and was assigned to be the difference between $(1 + R^{150\text{MeV}}) \times N_{\pi^0}^{150\text{MeV}}$ and $(1 + R^{500\text{MeV}}) \times N_{\pi^0}^{500\text{MeV}}$. The two values should be ideally the same. Therefore, the differences were interpreted to be the uncertainty of minimum energy cut.

Figure 4.39 shows differences of the $(1 + R^{500\text{MeV}}) \times N_{\pi^0}^{500\text{MeV}}$ to the $(1 + R^{150\text{MeV}}) \times N_{\pi^0}^{150\text{MeV}}$. 1 % uncertainty at low $p_T$ was assigned for this
Figure 4.35: The distribution of $\frac{N_{\pi, \text{pol} 3}^0 - N_{\pi, \text{pol} 2}^0}{N_{\pi, \text{pol} 3}^0}$. 

Figure 4.36: The distribution of $\frac{N_{\pi, \text{fit}}^0 - N_{\pi, \text{side}}^0}{N_{\pi, \text{fit}}^0}$. 

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Figure 4.37: Red and blue show the function slopes changing $\chi^2$ by $+1$ and $-1$ on the $\pi^0$ cross section, respectively. They almost overlap.

Figure 4.38: The distribution of $\Delta(1 + R)/(1 + R)$. Red and blue represents $\Delta(1 + R)/(1 + R)$ in the function slope changing $\chi^2$ by $+1$ and $-1$, respectively.
The uncertainty of acceptance (C6) was evaluated with $\pi^0$s from the data and the MC simulation. Firstly, two types of data were created based on new bad tower maps for the target photons, which masked the 6 towers and 12 towers around the PbSc edge. However, the bad tower maps are not applied to the $\pi^0$ partner photons. The $R$ must be reproduced based on the new types bad tower maps.

By comparing $(1 + R) \times N_{\pi^0}$ with the default bad-tower map to the one with two new bad tower maps, the difference was estimated. Each two values should be ideally the same. Both $(1 + R) \times N_{\pi^0}$s were normalized by the used the EMCal area. Figure 4.39 shows the difference between $(1 + R) \times N_{\pi^0}$ with the default bad tower map and with the new types bad tower maps. 1% uncertainty was assigned to this.

Figure 4.39: The distribution of the difference between $(1 + R_{150\text{MeV}}) \times N_{\pi^0}^{150\text{MeV}}$ and $(1 + R_{500\text{MeV}}) \times N_{\pi^0}^{500\text{MeV}}$.

**4.6.4 The uncertainty of the $R$ due to the acceptance (C6)**

The uncertainty of acceptance (C6) was evaluated with $\pi^0$s from the data and the MC simulation. Firstly, two types of data were created based on new bad tower maps for the target photons, which masked the 6 towers and 12 towers around the PbSc edge. However, the bad tower maps are not applied to the $\pi^0$ partner photons. The $R$ must be reproduced based on the new types bad tower maps.

By comparing $(1 + R) \times N_{\pi^0}$ with the default bad-tower map to the one with two new bad tower maps, the difference was estimated. Each two values should be ideally the same. Both $(1 + R) \times N_{\pi^0}$s were normalized by the used the EMCal area. Figure 4.40 shows the difference between $(1 + R) \times N_{\pi^0}$ with the default bad tower map and with the new types bad tower maps. 1% uncertainty was assigned to this.
Figure 4.40: Distribution of $1 - \frac{(1+R_i) \times N_{i,0}}{(1+R_0) \times N_{0,0}}$. Index 0, index $i$ is the default and the veto of 6 towers (red) and 12 towers (blue) around the PbSc edge, respectively.
4.6.5 The uncertainty of the $R$ due to $\pi^0$ production in the $\eta$ region (C7)

In the MC simulation, $\pi^0$ are produced with a flat distribution in the region of $|\eta| < 0.5$. In order to check the possible photons from $\pi^0$ outside $|\eta| < 0.5$, the region is changed to $|\eta| < 1.0$. Figure 4.41 shows region of different $\eta$’s.

![Figure 4.41: The schematic view of the region of $|\eta| < 0.35$, 0.5 and 1.](image)

Figure 4.42 shows the result of the evaluation. In comparison of the $R$ in $|\eta| < 0.5$ with that in $|\eta| < 1.0$, they are difference by $\sim 0.5\%$ at $p_T \sim 2$ GeV/c. In other words, the number of one tag photons in $|\eta| < 1.0$ are only changed by 0.5% comparing to those in $|\eta| < 0.5$. The number of photons observed in each $\eta$ is summarized in Table 4.2. In the EMCal acceptance, which is $|\eta| < 0.35$, the one tag photons coming from the outside of $|\eta| < 0.35$ are the same in $|\eta| < 0.5$ and $|\eta| < 1.0$ at $p_T > 2.5$ GeV/c. If the central value of $R$ is changed by 0.5%, yields of prompt photon are changed by a few %. Therefore, this uncertainty is small and is included in a systematic uncertainty.
Figure 4.42: Rotate clockwise from top left figure, the distribution of the one tag photons and two tag photons in $|\eta| < 0.5$, the distribution of the one tag photons and two tag photons in $|\eta| < 1.0$, the $R$ distribution in $|\eta| < 0.5$ and $|\eta| < 1.0$, and the ratio of the one tag photons in $|\eta| < 0.5$ to that in $|\eta| < 1.0$ and the ratio of the two tag photons in $|\eta| < 0.5$ to that in $|\eta| < 1.0$. 

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Figure 4.43: The schematic drawing of the direction of $\pi^0$ production in $|\eta| < 0.5$ and 1.

Table 4.2: The number of photons observed at $|\eta| < 0.5$ and $|\eta| < 1.0$ in the MC simulation. It is normalized with the number of events in $|\eta| < 0.5$.

<table>
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<td></td>
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<td>93</td>
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<td>$</td>
<td>\eta</td>
<td>&lt; 1.0$</td>
<td>one tag photon</td>
<td>147</td>
</tr>
<tr>
<td></td>
<td>two tag photon</td>
<td>93</td>
<td>45</td>
<td>23</td>
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4.6.6 The ratio of $N_\gamma$ to $N^{incl}$.

The ratio of $N_\gamma$ to $N^{incl}$, which we denote $W$, was evaluated because the systematic uncertainties described above are proportional to $1/W - 1$ or $1/W$ when we consider the uncertainties of cross section. Figure 4.44 shows the ratio of $N_\gamma$ to $N^{incl}$ and the fit function of $1/W = p0 + p1 \times p_T$ on the $W$. Red lines and blue bars represent statistical uncertainties and systematic uncertainties, respectively. The systematic uncertainties evaluated above were summarized at Table 4.3.

\[ \frac{N_\gamma}{N^{incl}} \]

Figure 4.44: The ratio of $N_\gamma$ to $N^{incl}$. Red line and blue bars represent statistical uncertainty and systematic uncertainty, respectively. The black line shows a linear fit.
Table 4.3: Systematic uncertainty (I). Components of systematic uncertainty in the group of B and C are proportional to the $1/W$ and the $1/W^2$, respectively. The correction for the $p_T$ bin shift were done (Section 4.7.1). “B” and “C” represent the contribution of systematic uncertainties to the cross section.

<table>
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<th>$p_T$ (GeV/$c$)</th>
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<th>B (%)</th>
<th>1/W−1</th>
<th>C1 (%)</th>
<th>C2 (%)</th>
<th>C3 (%)</th>
<th>C5 (%)</th>
<th>C6 (%)</th>
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<td>1/1</td>
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<td>1</td>
<td>0</td>
<td>0</td>
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4.7 Correction factors

\( N_{\gamma} \) was corrected with factors as follows;

\[
N_{\gamma}^{\text{corr}} = N_{\gamma} \cdot \varepsilon_{\text{acc\&smear}} \cdot \varepsilon_{\text{conversion}} \cdot \varepsilon_{\text{high}p_T} \cdot \varepsilon_{\text{BBC} \text{bias}},
\]  

(4.25)

where \( \varepsilon_{\text{acc\&smear}}, \varepsilon_{\text{conversion}}, \varepsilon_{\text{high}p_T} \) and \( \varepsilon_{\text{BBC} \text{bias}} \) are correction factors. These correction factors and the bin shift correction are described in the following subsections.

4.7.1 Bin shift correction

Bin shift correction is a correction for the shift of central value of \( p_T \) by binning because the yield of the prompt photon decreases with \( p_T \). Eq. (4.26) is fitted to the prompt photon yields in order to estimate the corrected \( p_T^{\text{corr}} \).

\[
f(p_T) = \frac{p_0}{(p_T)^{p_1}}
\]

(4.26)

This processes are done in iterative way until converging to one value.

\[
f(p_T^{\text{corr}}) = \int_{p_T^1}^{p_T^2} \left( \frac{f(p_T)}{p_T^{p_1} - p_T^{p_1}} \right) dp_T.
\]

(4.27)

Here \( p_0 \) and \( p_1 \) are 3E+6 and \(-6.2\), respectively.

4.7.2 Acceptance and smearing correction

Acceptance and smearing correction was calculated using the MC simulation. Smearing means that the slope of the prompt photon cross section is smeared due to the decrease of prompt photon yields as \( p_T \) and also with energy resolution of the EMCal. The function of \( p_0 \times \frac{p_T}{p_T} \) was firstly fitted to the slope of the prompt photon cross section. Then, according to the prompt photon slope, prompt photons were generated with the MC simulation to determine whether the prompt photon is accepted or not. The first \( \varepsilon_i^{\text{acc\&smear}} \) was obtained this way. Then this \( \varepsilon_i^{\text{acc\&smear}} \) was used as input again to calculate the prompt photon yields. These procedures are performed in iterative way.

Figure 4.45 shows the \( \varepsilon^{\text{acc\&smear}} \), and \( p_0, p_1 \) were 0.07, 7.9, respectively. The fluctuation of the \( \varepsilon^{\text{acc\&smear}} \) due to the EMCal energy resolution was small.
4.7.3 High $p_T$ trigger correction

Photon detection with high-$p_T$ trigger was not 100% because some of the high-$p_T$ trigger tiles did not work due to the masked trigger tiles of EMCal during the run. One high-$p_T$ trigger tile was made of 12x12 towers. The masked tiles cannot trigger photons. Therefore, the photon efficiency was reduced by the trigger mask. This effect was estimated at the same time as the estimation of the $\varepsilon_{accsmear}$.

4.7.4 BBCLL1 correction

The BBCLL1 correction is described in Section 3.6.3.

4.7.5 Photon conversion correction

The prompt photon could be converted to electron-positron pairs in material before reaching the PC3. This was evaluated using two groups of two tag photons data. In one group, the PC3 veto is applied to the target photon ($N_{\gamma \text{with PC3}}^{\pi^0}$). In another group, the PC3 veto is not applied to the target photon ($N_{\gamma \text{without PC3}}^{\pi^0}$). The PC3 veto and an energy asymmetry ($\leq 0.3$) cut are applied to pair photon in the both groups. The reason of applying the energy asymmetry cut is to reduce the combinatorial background. It enables the estimation of the two tag photons precisely. Figure 4.46 shows $N_{\gamma \text{without PC3}}^{\pi^0} - N_{\gamma \text{with PC3}}^{\pi^0}$. The average value was 11. %.

The conversion uncertainty was assigned using the difference between the measured value and the calculated value from the total materials between the beam-pipe and the PC3. The probability of the prompt photon conversion before the PC3 was calculated to be 10.3%. Therefore, approximately 1% ($=11.1\% - 10.3\%$) was assigned as this uncertainty.

4.8 Systematic uncertainties (II)

The systematic uncertainties on the $N_{\text{incl}}$ and the $N_{\gamma}^{\pi^0}$ are already described in Section 4.6. In this section, the systematic uncertainties on the $N_{\gamma}$ are described. These uncertainties are as follows;

- Photon energy comes from the ADC channel of the EMCal, and the conversion coefficient has an uncertainty. The uncertainty is energy scale uncertainty (A1), and it is 10 %.

- Luminosity uncertainty (A2) is 11 %.
Figure 4.45: The distribution of acceptance and smearing correction.

Figure 4.46: Distribution of photon conversion correction.
• Photon conversion uncertainty (A3) is described in Section 4.7.5.

• The uncertainty of BBC bias efficiency (A4) is described in Section 3.6.3.

The systematic uncertainties which globally change the cross section, and the systematic uncertainties which are different \( p_T \) bin by \( p_T \) bin are separately indicated in Figure 4.47. The blue curve is dependent on \( p_T \) with following reasons. The systematic uncertainties of \( N^{incl} \) and \( N^{\pi^0} \) were multiplied by \( 1/W \) and \( 1/W - 1 \), respectively. Therefore, these uncertainties which globally change the cross section show the \( p_T \) dependence.

In Figure 4.47, the systematic uncertainties (Blue) which globally change the cross section decrease as \( p_T \) increases, and the systematic uncertainties (Red) which are different \( p_T \) bin by \( p_T \) bin are the smallest at \( p_T \sim 4 \text{ GeV/c.} \)

Figure 4.47: The systematic uncertainties (Blue) which globally change the cross section and the systematic uncertainties (Red) which are different \( p_T \) bin by \( p_T \) bin. The systematic uncertainties of \( N^{incl} \) and \( N^{\pi^0} \) were multiplied by \( 1/W \) and \( 1/W - 1 \), respectively. Therefore, the systematic uncertainties which globally change the cross section show the \( p_T \) dependence.
Table 4.4: The table of systematic error (2). “A” and “Total” represent the contribution of systematic uncertainties in $N_\gamma$ and in the cross section, respectively.

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<th>$p_T$ (GeV/c)</th>
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<th>A3 (%)</th>
<th>A4 (%)</th>
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<th>B (%)</th>
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Table 4.4 summarizes the systematic uncertainties evaluated in Section 4.6 and 4.8.
Chapter 5

Analysis results

5.1 The Lorentz invariant cross section for prompt photon production in proton-proton collisions

The Lorentz invariant cross section for prompt photon production in proton-proton collisions at $\sqrt{s} = 62.4$ GeV is shown in Figure 5.1 as a function of $p_T$ with statistical uncertainties. Figure 5.2 shows the cross section with systematic uncertainties. The numerical variables are summarized in Table 5.1. The analyzed $p_T$ range covered from 2 to 7 GeV/c. The four orders of magnitude on the cross section were obtained as a result. The cross section in Figure 5.1 shows a smooth curve as a function of $p_T$, except for the point at $p_T = 4.25$ GeV/c.

The total systematic uncertainties of the cross section depends on $p_T$. The dominant component of the systematic uncertainty at low $p_T$ came from extracting $\pi^0$ yields which contain the combinatorial background.

The slope of the cross section at lower $p_T$ was steeper than the one in higher $p_T$. This shape is typical for cross section in particle collisions at high energy.

The relations between the prompt photon $p_T$ and the $x_{\text{gluon}}$ or $Q^2$ were evaluated using the PYTHIA simulation. The $x_{\text{gluon}}$ is the fraction of gluon momentum to the proton momentum. Gluons which participated in a hard scattering was filled in the histograms. Figure 5.3 and Figure 5.4 show the the distributions of $x_{\text{gluon}}$ and $Q^2$ in events that included a prompt photon from 2 to 10 GeV/c. The region of $x_{\text{gluon}}$ which corresponds to the measured prompt photon $p_T$ is found to be about 0.025 to about 0.3 if half maximum at $2 \sim 3$ GeV/c and $7 \sim 8$ GeV/c are regarded as the edges. The range of
Figure 5.1: The prompt photon Lorentz invariant cross section as a function of $p_T$ with statistical uncertainties.
Figure 5.2: The prompt photon Lorentz invariant cross section as a function of $p_T$ with statistical uncertainties (red) and systematic uncertainties (blue).
Table 5.1: Data of the cross section with statistical uncertainties and systematic uncertainties.

<table>
<thead>
<tr>
<th>$p_T$ (GeV/c)</th>
<th>$E \cdot \frac{d\sigma}{dp_T}$ (pb GeV$^{-2}$c$^3$)</th>
<th>statistical uncertainty</th>
<th>systematic uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.12</td>
<td>136388</td>
<td>7889.1</td>
<td>105701</td>
</tr>
<tr>
<td>2.37</td>
<td>55445.9</td>
<td>4724.0</td>
<td>34099.2</td>
</tr>
<tr>
<td>2.62</td>
<td>33709.8</td>
<td>2980.8</td>
<td>18203.3</td>
</tr>
<tr>
<td>2.87</td>
<td>16190.6</td>
<td>1877.7</td>
<td>7382.9</td>
</tr>
<tr>
<td>3.12</td>
<td>5393.6</td>
<td>1263.1</td>
<td>2249.1</td>
</tr>
<tr>
<td>3.37</td>
<td>5509.6</td>
<td>868.6</td>
<td>2104.7</td>
</tr>
<tr>
<td>3.62</td>
<td>2948.7</td>
<td>614.5</td>
<td>1037.9</td>
</tr>
<tr>
<td>3.87</td>
<td>2752.6</td>
<td>424.5</td>
<td>902.9</td>
</tr>
<tr>
<td>4.23</td>
<td>69.8</td>
<td>181.5</td>
<td>21.1</td>
</tr>
<tr>
<td>4.94</td>
<td>117.4</td>
<td>55.8</td>
<td>31.8</td>
</tr>
<tr>
<td>5.95</td>
<td>26.6</td>
<td>20.8</td>
<td>7.3</td>
</tr>
<tr>
<td>6.96</td>
<td>19.9</td>
<td>14.4</td>
<td>7.5</td>
</tr>
</tbody>
</table>
$x_{\text{gluon}}$ almost agrees with the estimation with Eq. 2.22:

$$x \sim x_T \equiv \frac{2p_T}{\sqrt{s}}.$$  \hspace{1cm} (5.1)

It indicates that relatively small $x_{\text{gluon}}$ was accessed. It should be noted that the $x_{\text{gluon}}$ distribution at $p_T = 6 \sim 8 \text{ GeV}/c$ of prompt photon has a double peak. It corresponds to the collisions which took place by a large $x_{\text{quark}}$ and a small $x_{\text{gluon}}$ or vice versa.

The obtained cross section was compared with the results from other collider experiments at high energy and pQCD calculation in Section 5.2, in 5.4, respectively. The shape of the cross section is discussed using the “$x_T$ scaling” in Section 5.3. The ratio of the obtained cross section to $\pi^0$ cross section was discussed in Section 5.5.

5.2 Comparison with other experiments

The Lorentz cross section for prompt photon production from proton-proton collisions at $\eta=0$ and $\sqrt{s} = 63 \text{ GeV}$ was measured by the R806 and R108 using ISR collider at CERN in 1980. The data of the experiments were obtained from published papers, pre-prints or HEP data base. PHENIX also measured the cross section at $\sqrt{s} = 200 \text{ GeV}$ in 2003 and 2005. Figure 5.5 shows the comparison of the cross sections of R806, R108 and PHENIX at $\sqrt{s} = 200 \text{ GeV}$. The central values and slope of the R806 and R108 cross section agree with the present cross section within uncertainties. The present measurement provides the cross section at $p_T = 2 \sim 4 \text{ GeV}/c$ for the first time. In the R806 and R108, it was not possible to measure the cross section in the low $p_T$ region.

5.3 $x_T$ scaling

The proton structure function is weakly dependent of $Q^2$ and is mainly depends on the scaling variable $x$. Bjorken\cite{82} suggested that it is due to interaction of partons which are point-like objects. The $x_T$ is defined as:

$$x_T \equiv \frac{2p_T}{\sqrt{s}}.$$  \hspace{1cm} (5.2)

In this section, the examination of $x_T$ scaling of present result together with PHENIX result of $\sqrt{s} = 200 \text{ GeV}$ is presented.
Figure 5.3: The distributions of $x_{\text{gluon}}$ in events that included a prompt photon production with $p_T$ from 2 to 10 GeV/$c$. It has been evaluated with PYTHIA simulation.

Figure 5.4: The distributions of $Q^2$ in events that included a prompt photon production with $p_T$ from 2 to 10 GeV/$c$. It has been evaluated with PYTHIA simulation.
Figure 5.5: Comparison of the obtained cross section with the one of R806, R108 and PHENIX at $\sqrt{s} = 200$ GeV.
From Eq. 2.11, Eq. 2.18 and Eq. 2.19 at the pseudo-rapidity $\eta$ of 0, the basic formalism of the hadron production in hadron collisions is written as:

$$\hat{s} = -2\hat{u} = -2\hat{t}, \text{ and}$$

$$E \frac{d\sigma}{d^3p} \sim \frac{F(x_T)}{\sqrt{s}}; \quad (5.3)$$

where $n$ is constant named as power index and $F(x_T)$ does not depend on $\sqrt{s}$. The naive parton model has $n = 4$ in the leading order and $n = 4 + \alpha$ in the next-to-leading order. In the approximation, there are two assumptions; 1) the $Q^2$ scaling of the PDF and FF, 2) the $\alpha_s$ is independent of $Q^2$. Since the assumptions are violated as discussed in Section 2.1, the $n$ is not a constant and is a function of $x_T$ and $\sqrt{s}$. As $\sqrt{s}$ increases, power index is expected to decrease because the $\alpha_s$ decreases and the $Q^2$ scale breaking becomes smaller.

The examination of the $x_T$ scaling of the PHENIX result of $p_T = 200$ GeV together with other experiment was done in [83]. A good agreement was obtained with $n = 5.0$. The test of the $x_T$ scaling of present result together with PHENIX result of $\sqrt{s} = 200$ GeV is shown in Figure 5.6. The figure shows good agreements with each other at $n = 5.52$ in the measured $x_T$ range. From this result of $x_T$ scaling, point like interaction is supported.

5.4 Comparison with pQCD calculation

The cross section was compared to pQCD calculation in next-to-leading order (NLO) using the CTEQ6M parton distribution function. Figure 5.7 shows a comparison of the present data with the NLO-pQCD calculation and $\chi^2$ distribution between the measured cross section and the predicted cross section with pQCD. The three lines represent the calculation with different scales. Three curves are calculated with $\mu = \frac{1}{2}p_T$, $\mu = p_T$ and $\mu = 2p_T$. The prediction of the NLO-pQCD calculation at low $p_T$ region was lower than experimental values. It suggests a need to take into account further higher order terms in the calculation. On the other hand, at high $p_T$ region, both values agree with each other within uncertainties of measurement and calculation.

5.5 Comparison with $\pi^0$ cross section

The cross section was compared with the cross section [80] of $\pi^0$. Figure 5.8 shows the ratio of the cross section to $\pi^0$ cross section. It should be noted
Figure 5.6: \((\sqrt{s})^{5.52} \times E^3 \frac{d\sigma}{d^3p}\) at \(\sqrt{s} = 62.4\) and 200 GeV. The horizontal axis is \(x_T\). The error bar is only statistical.
Figure 5.7: **Top** Comparison of the cross section with the NLO-pQCD calculation using the CTEQ6M parton distribution function. The top line, middle one and bottom one is calculated with $\mu = \frac{1}{2}p_T$, $\mu = p_T$ and $\mu = 2p_T$, respectively. **Bottom** $\chi^2$ of each data point calculated from the experimental cross section and the predicted cross section from pQCD.
that the horizontal axis is $p_T$ of one the two photon from $\pi^0$ decay in case of $\pi^0$ cross section. It is the level of a few percent. This basically reflects the fact the electromagnetic coupling constant $\alpha$ is involved in the prompt photon production.

![Figure 5.8: The ratio of the prompt photon cross section to the $\pi^0$ cross section.](image)

Figure 5.8: The ratio of the prompt photon cross section to the $\pi^0$ cross section.
Chapter 6

Conclusion and outlook

6.1 Conclusion

This thesis reports the cross section for prompt photon production from proton-proton collisions at $\sqrt{s} = 62.4$ GeV in PHENIX. The data were taken in 2006. The subsystems used for the prompt photon measurement were the PC3 and EMCal as detector, and ERT2x2 trigger as photon trigger. The coverage of these detectors is $|\eta| < 0.35$ and $\Delta\phi = \pi$. The $\pi^0$ tagging method was used to extract prompt photons:

$$N_\gamma = N_{\text{incl.}} - (1 + A) \times (1 + R) \times N_{\pi^0},$$

(6.1)

where $N_\gamma$, $N_{\text{incl.}}$ and $N_{\pi^0}$ is the prompt photons, the inclusive photons and the two tags. Two tag means that two photons from $\pi^0$ decay were detected. By identifying the background which is the photon from $\pi^0$ decay, it was succeeded to extract the prompt photons in spite of its low event rate. The integrated luminosity of the data I analyzed was 0.065 pb$^{-1}$.

The conclusions are;

- The $p_T$ range of the obtained cross section was $2 \sim 7$ GeV/c, and the cross section was obtained over 4 orders of magnitude. The cross section at the $p_T$ from 2 to 4 GeV/c was measured for the first time in this experiment. The main source of the systematic uncertainties came from the evaluation of a missing $\pi^0$ decay photon.

- The $x_{\text{gluon}}$ which corresponds to the measured prompt photon $p_T$ was evaluated with PYTHIA simulation. The $x_{\text{gluon}}$ mean the fraction of gluon momentum to the proton momentum. The $x_{\text{gluon}}$ distribution was from about 0.025 to about 0.3. The $x_{\text{gluon}}$ which corresponds to the measured prompt photon $p_T$ was relatively small. The small $x_{\text{gluon}}$ can be accessed with prompt photon measurement.
• The cross section was compared with the cross section measured earlier by R806, R108 at $\sqrt{s} = 63$ GeV and by PHENIX at $\sqrt{s} = 200$ GeV. The R806 experiment and R108 experiment were performed with proton-proton collision at $\sqrt{s} = 63$ GeV at CERN using ISR collider in 1980’s. The cross section agreed with that of R806 and R108 within the experimental uncertainties. The cross section of about $2 < p_T < 4$ GeV/c was measured for the first time in the present experiment. We could measure the cross section at low $p_T$ region compared to others experiment. If we apply this technique of measurement of low $p_T$ prompt photon to the collision at $\sqrt{s} = 200$ or 500 GeV, we can access even smaller $x_{gluon}$.

• $x_T$ scaling was tested by a comparison of prompt photon cross section at $\sqrt{s} = 200$ GeV. Good agreements were obtained with $n = 5.25$. The success of scaling suggests point-like interaction.

• The cross section was compared to pQCD calculation of next-to-leading-order using CTEQ6M parton distribution function. The pQCD calculation can predict the Lorentz invariant cross section. The prediction of the NLO-pQCD calculation at low $p_T$ region was lower than the experimental values. It suggests a need to take into account higher order terms in the calculation. On the other hand, at high $p_T$ region the both values agree within uncertainties of measurement and calculation.

• The cross section was compared with the cross section of $\pi^0$ production.

To summarize, this thesis presents the first measurement of the cross section for prompt photon production at $p_T = 2 \sim 4$ GeV/c region in high energy proton-proton collisions. A method was developed to extract the prompt photons from inclusive photons yield using $\pi^0$ tagging method. This result is an important step for double spin asymmetry measurement in polarized proton-proton collisions, and also serves as a good reference for quark-gluon plasma study in heavy ion collisions.

6.2 Outlook

In 2009, the first run of polarized proton-proton collisions at $\sqrt{s} = 500$ GeV starts. If we apply the analysis technique of low $p_T$ prompt photon to the collision at $\sqrt{s} = 500$ GeV, we can access even smaller $x_{gluon}$.

The Lorentz invariant cross section for prompt photon production at $\sqrt{s} = 62.4$ GeV is described with the pQCD calculation, though more theoretical studies are needed for the low $p_T$ region. The pQCD can be tested
and improved using the present data. On the other hand, if prompt photons
of high $p_T$ are measured at $\sqrt{s} = 62.4$ GeV with high statistics, larger $x_{gluons}$
will be studied in details.
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Appendix A

Fluence measurement for radiation damage study of silicon stripixel sensor

A silicon vertex detector (VTX) is planned to be installed in the PHENIX Central Arm in 2009 and will be used for 10 years. In this Appendix A, I describe my work in 2006 – 2008 for the silicon stripixel sensor.

The barrel of VTX is composed of four layers (see Figure A.1). The inner two layers are silicon pixel sensors and the outer two layers are silicon stripixel sensors. These are single-sided two-dimensional read-out sensors (see Figure A.2). The silicon stripixel sensors will be placed at 10cm and 14cm away from the beam line. The expected integrated luminosity irradiating the silicon stripixel sensors for 10 years is approximately \(4 \times 10^{12} \text{ fb}^{-1}\), and thus, the total fluence is estimated to be \(3 \times 10^{12} \text{ cm}^{-2}\). Such strong radiation would cause significant damage to the silicon stripixel sensor resulting to an increase in the leakage current. The leakage current of more than 15 nA/strip will saturate the front-end electronics. Therefore, the leakage current of the silicon stripixel sensor has been studied.

We irradiated the stripixel sensor with a 16 MeV proton beam, which is equivalent to \(3 \times 10^{12} \text{ cm}^{-2}\), using the tandem accelerator at the University of Tsukuba. It is important to accurately measure the fluence to study radiation damage. The fluence that irradiated the silicon stripixel sensor was measured by two methods. One is by use of a silicon diode, whose irradiation damage characteristics are well known. The other is by the activation method using a copper (Cu) foil which was irradiated simultaneously with the silicon stripixel sensor. The measurements of the fluence during this
Figure A.1: The schematic of the barrel of the VTX. The radius of the two-layer from outside is 10 cm and 14 cm, respectively and the length of the outer is 38.2 cm. It covers $\leq 1.2$ in $\eta$ direction and almost $2\pi$ in $\phi$ direction.

Radiation damage test will be described in this Appendix. The experimental setup is shown in Figure A.3. The energy of the proton beam from the accelerator was 20 MeV. The beam size was much smaller than the size of the silicon stripixel sensor. Therefore, the proton beam was spread by a Coulomb multiple scattering through a 5 mm-thick aluminum plate placed upstream of the beam. The beam energy is degraded to about 16 MeV in this process. Then, the spread proton beam irradiated the silicon stripixel sensor to measure the radiation damage using setup A, in which the silicon stripixel sensor was mounted with a collimator to define the irradiated area.

The fluence was first measured using the Cu foil A which was placed behind the silicon stripixel sensor, and was thus irradiated with the same amount of fluence as the silicon stripixel sensor. When the Cu foil was irradiated with the 16 MeV proton beam, $^{63}\text{Zn}$ is produced in the Cu foil (eq. [A.1]).

$$p + ^{63}\text{Cu} \rightarrow n + ^{63}\text{Zn} \quad (A.1)$$

Then the $^{63}\text{Zn}$ decays to $^{63}\text{Cu}^*$ with $\beta^+$-decay followed by $\gamma$-ray emissions with energies of 662, 962 and 1412 keV. Then the fluence was obtained from the activity of the Cu foil by counting $\gamma$-rays using a germanium (Ge) detector.
through the following relations.

\[ \Phi = \frac{N_\gamma \lambda T_r}{\epsilon Br \sigma N_{Cu} d\Omega} \times F \] \hspace{1cm} (A.2)

\[ A = (1 - \exp^{-\lambda T_r}) \{ \exp^{-\lambda t_1} - \exp^{-\lambda (t_1 + t_2)} \} \] \hspace{1cm} (A.3)

Here, \( N_\gamma \) is the number of \( \gamma \)-rays emitted from the produced \( ^{63}\text{Zn} \), \( \lambda \) is the decay constant of \( ^{63}\text{Zn} \), \( T_r \) is irradiation time, \( \epsilon \) is \( \gamma \)-ray detection efficiency, \( Br \) is the branching ratio of the relevant \( \gamma \)-ray, \( N_{Cu} \) is the number of \( ^{63}\text{Cu} \) atoms, \( d\Omega \) is the solid angle between the activated Cu foil and the Ge detector, \( t_1 \) is start time for measurement of \( \gamma \)-rays, \( t_2 \) is measurement time and \( F \) is a conversion factor for 16 MeV proton to 1 MeV neutron. The energy calibration and the detection efficiency measurement of the Ge detector were carried out using \( ^{60}\text{Co} \) and \( ^{137}\text{Cs} \) standard sources. The measured fluence with statistical uncertainty is shown in Figure A.4. The fluence values obtained from three \( \gamma \)-ray lines were averaged by weighting their uncertainty. It is \( 2.4 \times 10^{12} \text{ cm}^{-2} \).
Figure A.3: Schematic of the irradiation setups. Setups A and B are setup for the measurement of the spatial intensity distribution and the radiation damage of the silicon stripixel sensor and the fluence, respectively.

The fluence was measured by an independent method using the silicon diode to use the proportionality relation between the fluence and the leakage current of the silicon diode placed by the silicon stripixel sensor. The coefficient is known from our previous experiment [92]. Knowledge of the spatial intensity distribution of the beam is necessary to transfer the fluence value at the silicon diode to the one at the position of the silicon stripixel sensor.

The spatial distribution and the fluence were measured with a foil activation method using setup B. The Cu foil B was placed at the same position where the silicon stripixel sensor had been located by revolving the turntable on which all the setups were mounted. The spatial intensity distribution of the beam is obtained through the $^{63}$Zn distribution by measuring $\beta^+$-rays and X-rays emitted in the $\beta^+$-decay of $^{63}$Zn with an imaging plate (IP).

The IP includes fluorescent material. The $\beta^+$-rays and X-rays from $^{63}$Zn generate hole-electron pairs in the IP. Then the pairs are trapped by the fluorescent material. The IP is then stimulated by the Ne-He laser to release the trapped electrons and holes. When the released electrons and holes are annihilated, ultraviolet (UV) photons are generated.
From the intensity of the UV photons, the spatial distribution of the spread beam was obtained, as shown in Figure A.5. The ratio of the intensity averaged over the silicon stripixel sensor to that at the beam center is 93 %. The ratio of the intensity at the silicon diode to that at the beam center is 73 %. It agrees with our estimation which is based on the Coulomb multiple scattering calculation. The fluence that irradiated the silicon stripixel sensor obtained using the silicon diode is $2.5 \times 10^{12}$ cm$^{-2}$. The fluence values obtained by the two methods agree with each other and are close to our goal fluence value, which is $3 \times 10^{12}$ cm$^{-2}$.

The leakage current of the silicon stripixel sensor is close to 15 nA/strip at 0 degree operation temperature.
Figure A.4: Fluence obtained from three $\gamma$-rays counts. Red solid bars show statistical uncertainties.

Figure A.5: Spatial intensity distribution of the proton beam.
Appendix B
Kinematics of $\pi^0$ decay

This appendix describes kinematics of $\pi^0$ decay.

Now a $\pi^0$ travels in one direction with the velocity of $\beta$. The direction is defined as $z$ axis, and the $\pi^0$ decays at the origin of the Cartesian coordinate system. Figure B.1 shows the rest frame of $\pi^0$ (Left) and laboratory frame (Right) at that point. In the figures, $\gamma_1$ and $\gamma_2$ are the two $\gamma$s from $\pi^0$, and $\theta$

![Diagram](image)

Figure B.1: Left: The coordinate of rest frame system $\pi^0$. Right: The coordinate of laboratory frame. The arrow of blue and red show the travel direction of $\pi^0$ and $\gamma$'s, respectively.

is the angle between the direction of $\gamma_1$ and the $z$ direction in the rest frame.
\( \gamma_1 \) and \( \gamma_2 \) are the two \( \gamma \)'s, and \( \theta_1 \) is the angle between the \( z \) direction and the direction of \( \gamma_1 \). \( \theta_{12} \) is the opening-angle between \( \gamma_1 \) and \( \gamma_2 \) in the rest frame. \( \phi \) is the angle between the projected vector of the \( \gamma \) in \( x-y \) plane and \( x \) direction. In the rest frame, the four momenta of \( \gamma_1 \) is written as follows;

\[
P_1 = \begin{pmatrix}
\frac{m_0}{2} & \frac{m_0 \sin \theta \cos \phi}{2} & \frac{m_0 \sin \theta \sin \phi}{2} & \frac{m_0 \cos \theta}{2} \\
\frac{m_0 \sin \theta \cos \phi}{2} & \frac{m_0 \sin \theta \sin \phi}{2} & \frac{m_0 \cos \theta}{2} & 0 \\
\frac{m_0 \sin \theta \sin \phi}{2} & \frac{m_0 \cos \theta}{2} & 0 & 0 \\
\frac{m_0 \cos \theta}{2} & 0 & 0 & 0 
\end{pmatrix}
\]

(B.1)

In the same way, the four momenta of \( \gamma_2 \) is written as follows;

\[
P_2 = \begin{pmatrix}
\frac{m_0}{2} & -\frac{m_0 \sin \theta \cos \phi}{2} & -\frac{m_0 \sin \theta \sin \phi}{2} & -\frac{m_0 \cos \theta}{2} \\
-\frac{m_0 \sin \theta \cos \phi}{2} & \frac{m_0 \sin \theta \sin \phi}{2} & \frac{m_0 \cos \theta}{2} & 0 \\
-\frac{m_0 \sin \theta \sin \phi}{2} & -\frac{m_0 \cos \theta}{2} & 0 & 0 \\
-\frac{m_0 \cos \theta}{2} & 0 & 0 & 0 
\end{pmatrix}
\]

(B.2)

The four momenta in the laboratory frame system can be obtained by the Lorentz transformation of the four momenta in the rest frame as follows;

\[
P_1' = \begin{pmatrix}
\gamma & 0 & 0 & \gamma \beta \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\gamma \beta & 0 & 0 & \gamma 
\end{pmatrix} \cdot P_1 = \begin{pmatrix}
\gamma \frac{m_0}{2} + \gamma \beta m_0 \cos \theta \\
\frac{m_0 \sin \theta \cos \phi}{2} & \frac{m_0 \sin \theta \sin \phi}{2} & \frac{m_0 \cos \theta}{2} \\
\gamma \beta m_0 \sin \theta \sin \phi & \gamma \beta m_0 \cos \theta \\
\frac{m_0 \cos \theta}{2} & \frac{m_0 \sin \theta \cos \phi}{2} & \frac{m_0 \sin \theta \sin \phi}{2} & \frac{m_0 \cos \theta}{2} 
\end{pmatrix}
\]

(B.3)

and

\[
P_2' = \begin{pmatrix}
\gamma & 0 & 0 & \gamma \beta \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\gamma \beta & 0 & 0 & \gamma 
\end{pmatrix} \cdot P_2 = \begin{pmatrix}
\gamma \beta m_0 \cos \theta \\
\frac{m_0 \sin \theta \cos \phi}{2} & \frac{m_0 \sin \theta \sin \phi}{2} & \frac{m_0 \cos \theta}{2} \\
\gamma \beta m_0 \sin \theta \sin \phi & \gamma \beta m_0 \cos \theta \\
\frac{m_0 \cos \theta}{2} & \frac{m_0 \sin \theta \cos \phi}{2} & \frac{m_0 \sin \theta \sin \phi}{2} & \frac{m_0 \cos \theta}{2} 
\end{pmatrix}
\]

(B.4)

where \( \gamma = \frac{1}{\sqrt{1-\beta^2}} \).

**B.1 The distribution of \( E_1' \)**

\( E_1' \) is defined as the energy of a \( \gamma \) from \( \pi^0 \) at laboratory frame as

\[
E_1' = \frac{1}{2} m_\pi \gamma (1 - \beta \cos \theta)
\]

from the Eq. \( \text{B.1} \).
The energy distribution \( \frac{dN}{dE_i} \) is
\[
\frac{dN}{dE_i} = \frac{dN}{d(\cos \theta)} \cdot \frac{1}{\gamma / \beta (\frac{1}{2}m_{\pi^0})} = \text{const.} \times \frac{2}{p_{\pi^0}},
\] (B.6)
where the \( p_{\pi^0} \) is \( \pi^0 \) momentum in the laboratory frame. At the same \( p_{\pi^0} \), the \( \frac{dN}{dE_i} \) is constant.

**B.2 The distribution of the \( \theta'_1 \)**

The solid angle distribution of \( \gamma \)s in the rest frame is calculated as follows;
\[
\frac{\Delta N}{\Delta \Omega} = \frac{d^2N}{\sin \theta d\theta d\phi} = C.
\] (B.7)

Here the two \( \gamma \)-rays are emitted isotropically. \( N \) is the number of \( \pi^0 \)s, \( \Omega \) is the solid angle, and \( C \) is constant.

\( \frac{dN_1}{d\theta'_1} \) is important in order to study the \( \theta'_1 \) distribution of the number of \( \gamma \)s (\( N_1 \)). \( \frac{dN_1}{d\theta'_1} \) can be written as follows;
\[
\frac{dN_1}{d\theta'_1} = \frac{1}{2} \frac{dN}{d\theta} d\theta d\theta'_1.
\] (B.8)

The relation between \( \theta \) and \( \theta'_1 \) is as follows;
\[
\tan \theta = \frac{\sqrt{p_x^2 + p_y^2}}{p_z} \frac{\sin \theta'}{\gamma \cos \theta'_1 - \gamma \beta}.
\] (B.9)
\[
= \frac{\sin \theta'}{\gamma \cos \theta'_1 - \gamma \beta}.
\] (B.10)

The denominator of Eq. [B.10] is obtained from Eq. [B.3] and is solved for \( \sin \theta; \)
\[
\sin \theta = \frac{\sin \theta'}{\sqrt{(\gamma \cos \theta'_1 - \gamma \beta)^2 + \sin^2 \theta'_1}}.
\] (B.11)

The Eq. [B.10] is differentiated by \( \theta'_1 \) as follows;
\[
\frac{d\theta}{d\theta'_1} = \frac{\gamma - \gamma \beta \cos \theta_1}{(\gamma \cos \theta'_1 - \gamma \beta)^2 + \sin^2 \theta'_1}.
\] (B.12)
Therefore, the Eq. \[B.8\] can be written as follows;

\[
\frac{dN_1}{d\theta_1'} = \frac{1}{2} \frac{dN}{d\theta} \frac{d\theta}{d\theta_1'} = \pi C \frac{\gamma \sin \theta_1' (1 - \beta \cos \theta_1')}{((\gamma \cos \theta_1' - \gamma \beta)^2 + \sin^2 \theta_1')^{3/2}}. \tag{B.13}
\]

Figure \[B.2\] shows the results of numerical calculation of Eq. \[B.13\] with \(C = 1/4\pi\). The \(\frac{dN_1}{d\theta_1'}\) increases at smaller \(\theta_1'\) as \(\beta\) increases. It means that the \(\gamma\)s are likely to travel forward direction of the \(\pi^0\).

Next, Eq. \[B.13\] is integrated in the region of \(0 < \theta_1 < \pi\) to study the \(\theta_1\) in which \(\gamma\)s are contained with a probability. It is shown in Figure \[B.3\] at \(p_{\pi^0} = 4\) and \(6 (\text{GeV}/c)\).
Figure B.3: The distribution of the relative sum of γs at $p_\pi^0 = 4$ (Top) and 6 (GeV/c) (Bottom). The angle which contains γs with 95% probability is 0.12 (rad) at $p_\pi^0 = 4$ (GeV/c) and 0.08 (rad) at $p_\pi^0 = 6$ (GeV/c).
B.3 The distribution of the $\theta_{12}$

$\frac{dN}{d\theta_{12}}$ is calculated to study the distribution of the opening-angle $\theta_{12}$. The following description is for the limited $\theta$, which is $0 \leq \theta_a \leq \pi/2$, in order to simplify the consideration. The following equation is obtained by using the relation $P_1 \times P_2 = P'_1 \times P'_2$, which is a Lorentz invariant.

\[
\cos\theta'_{12} = 1 - \frac{2(1 - \beta^2)}{1 - \beta^2 \cos^2 \theta_a}.
\]  \hspace{1cm} (B.14)

The region of the $\theta'_{12}$ is

\[-1 \leq \cos\theta'_{12} \leq 2\beta^2 - 1. \]  \hspace{1cm} (B.15)

Results of numerical calculation of Eq. B.14 with different $\beta$s are shown in Figure B.4. It means that the opening-angle becomes small when two gammas are emitted in vertical direction to the direction of movement of $\pi^0$.

![Figure B.4](image-url)

Figure B.4: Results of numerical calculation of Eq. B.14 with different $\beta$s. It means that the opening-angle becomes small when two gammas are emitted in vertical direction to the direction of movement of $\pi^0$. 

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On the other hand, the $\theta'_{12}$ has the following relation using $\gamma'_1$ energy ($E'_1$) and $\gamma'_2$ energy ($E'_2$).

\[
m_{\pi^0}^2 = (P'_1 + P'_2)^2 = 2P'_1 \cdot P'_2 = 2E'_1E'_2(1 - \cos\theta'_{12}) = 4E'_1E'_2\sin^2\left(\frac{\theta'_{12}}{2}\right). \tag{B.16}
\]

Eq. [B.14] is solved for $\tan\theta_a$ as follows;

\[
\tan\theta_a = \sqrt{\frac{1 + \cos\theta'_{12}}{\gamma^2(1 - \cos\theta'_{12}) - 2}}, \tag{B.17}
\]

and is differentiated by $\theta_a$ as follows;

\[
\frac{d\theta'_{12}}{d\theta_a} = \frac{\beta \cos\theta_a}{\gamma(\beta^2 \cos^2\theta_a - 1)}. \tag{B.18}
\]

Now we start to calculate $\frac{dN(\theta_a)}{d\theta'_{12}}$.

\[
\left| \frac{dN(\theta_a)}{d\theta'_{12}} \right| = \left| \frac{dN(\theta_a)}{d\theta_a} \frac{1}{\frac{d\theta'_{12}}{d\theta_a}} \right| = \left| \frac{1}{2} \frac{dN}{d\theta} \frac{1}{\frac{d\theta'_{12}}{d\theta_a}} \right| = \left| \pi C \tan\theta_a \frac{\gamma(\beta^2 \cos^2\theta_a - 1)}{2\beta} \right| = \pi C \sqrt{\frac{1 + \cos\theta'_{12}}{\gamma^2(1 - \cos\theta'_{12}) - 2}} \cdot \frac{1}{\gamma\beta(1 - \cos\theta'_{12})}. \tag{B.19}
\]

The Eq. [B.19] is valid, except for the $\beta \neq 0$. Furthermore the Eq. [B.19] can be considered for $\pi/2 \leq \theta \leq \pi$ in the same way as the $\theta_a$. As a result, the following Equation can be realized.

\[
\left| \frac{dN(\theta)}{d\theta'_{12}} \right| = 2 \left| \frac{dN(\theta_a)}{d\theta'_{12}} \right|. \tag{B.20}
\]

The $\frac{dN}{d\theta'_{12}}$ is written as follows;

\[
\left| \frac{dN(\theta)}{d\theta'_{12}} \right| = 2\pi C \sqrt{\frac{1 + \cos\theta'_{12}}{\gamma^2(1 - \cos\theta'_{12}) - 2}} \cdot \frac{1}{\gamma\beta(1 - \cos\theta'_{12})}. \tag{B.21}
\]
The Eq. B.21 is valid, except for $\beta \neq 0$. At the $\beta = 0$, it is $\delta$ function at $\theta'_{12} = \pi$. Its evidence is seen at $\beta = 0.074$. The $\frac{dN(\theta)}{d\theta'_{12}}$ rapidly increases around $\cos\theta'_{12} = 2\beta^2 - 1$ within $-1 \leq \cos\theta'_{12} \leq 2\beta^2 - 1$, and it does not exist in $2\beta^2 - 1 \leq \cos\theta'_{12} \leq 1$. Figure B.5 shows the result of numerical calculation of Eq. B.21 with $C = 1/4\pi$. In Figure B.5, the sharp peaks called “Jacobian peak” are seen at each $p_{x0}$. The opening-angle $\theta_{12}$ corresponding to Jacobian peak is minimum opening-angle $\theta_{12}^{\text{mini}}$. Figure B.5 shows the relation between the opening-angle $\theta_{12}^{\text{mini}}$ and $p_{x0}$.

Figure B.5: Results of numerical calculation of Eq. B.21 with $C = 1/4\pi$ in different $\beta$s. Sharp peaks called “Jacobian peak” are seen at each $\beta$.

Next, Eq. B.19 is integrated in the region of $0 < \theta_{12} < \pi$ to study the $\theta_{12}$ in which $\gamma$s are contained with a probability. It is shown in Figure B.7 at $p_{x0} = 4$ and $6$ (GeV/c).
Figure B.6: The relation between $\theta^{\text{mini}}$ and $p_{\pi^0}$. 
Figure B.7: The distribution of the relative sum of $\gamma$s at $p_T^\gamma = 4$ (Top) and 6 (GeV/c) (Bottom). The angle which contains $\gamma$s with 95% probability is 0.2 (rad) at $p_T^\gamma = 4$ (GeV/c) and 0.15 (rad) at $p_T^\gamma = 6$ (GeV/c).
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