Master’s Thesis

Flavor Asymmetry of the polarized light Sea Quarks extracted from deep-inelastic Scattering at HERMES

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Abstract

The spin content in the nucleon is an interesting subject of nuclear physics, and it has been found that the quark contribution is less than 30% in the nucleon spin in recent experiments. On the other hand from the unpolarized experiment, it was found that the unpolarized quark distributions of $u$ and $d$ are not equal: there is an asymmetry of $\bar{u}$ and $\bar{d}$.

The main subject of this thesis is the extraction of the polarized quark flavor distribution, especially the polarized flavor asymmetry extraction. The flavor asymmetry of light sea quarks is expressed as $\Delta \bar{u} - \Delta \bar{d}$, where $\Delta \bar{u}(\Delta \bar{d})$ is the polarized $\bar{u}(\bar{d})$ quark distribution. In this analysis, a new method for extraction of flavor asymmetry of light sea quarks is proposed. It is based on the Chiral Quark Soliton Model (CQSM) prediction. With the new method, the single quark distributions can be extracted as well as the flavor asymmetry.

At HERMES experiment the produced particles are measured in Deep-Inelastic Scattering as well as scattered lepton. The Ring Imaging Cherenkov Counter is operated to identify particles since 1998. The study of the detector efficiency is also discussed in this thesis. The HERMES RICH can identify pions, kaons and protons. 16 types of inclusive and semi-inclusive spin asymmetry became available. Using these asymmetries, the quark distributions are extracted and their moments with the original and the new methods are presented.
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Chapter 1

Introduction

The spin structure of the nucleon became one of the fundamental questions in physics, when results from the EMC experiment [1] showed that only a fraction of the nucleon’s spin is carried by its quarks. Before that experiment, it has been predicted, that spin of the nucleon, like its magnetic moment, is almost completely carried by the quarks. As a consequence of this discovery, a more general approach for the spin contents of a proton, taking quarks, gluons and orbital momenta into account, has to be used:

\[
S_z = \frac{1}{2} \Delta \Sigma + \Delta g + L_z, \tag{1.1}
\]

where \(\Delta \Sigma = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s}\) is the contribution of quark spins, \(\Delta g\) is that of the gluon spin and \(L\) is that of the orbital angular moment.

Further experiments [2] confirmed the EMC results and obtained: \(\Delta u \cong 0.8, \Delta d \cong -0.4\) and \(\Delta s \cong -0.1\). But the contributions of light sea quarks \((\Delta \bar{u}, \Delta \bar{d})\), gluons and the orbital angular momenta are not clear yet. In most further studies to measure the contribution of quark spins, the contributions of light sea quarks are assumed to be symmetric as \(\Delta \bar{u} = \Delta \bar{d} = \Delta s = \Delta \bar{s}\) to compensate for the lack of experimental data. The contributions of valence quarks have been extracted from the different experiments and their results are almost consistent, however, the spin structures of light sea quark are not made clear comparing the valence quarks. In addition, the asymmetry between the spin of the \(\bar{u}\) and the \(\bar{d}\) quarks have been not obtained clearly yet. In the unpolarized experiment at HERMES, a flavor asymmetry of the unpolarized sea quark densities \((\bar{u} - \bar{d} \neq 0)\) are measured already [3], and this measurement may be a hint that there might be an asymmetry for the polarized light sea quark densities. This unpolarized flavor asymmetry have been obtained in other experiment [4] [5].

This polarized flavor asymmetry of light sea quarks is theoretically predicted by the \(1/N_c\)-expansion of Quantum ChoromoDynamics (QCD) [6], where it is calculated in the large-\(N_c\) limit by the Chiral Quark Soliton Model. This model predicts a quite large polarized sea quark flavor asymmetry. From the theoretical side, the estimation of the flavor asymmetry are achieved in [7] [8]. Additionally there are the extraction of flavor asymmetry with the HERMES and SMC data in [9] [10] and it suggests the existence of the flavor asymmetry, although it is not confirmed because the statistics is not enough. It is very interesting and challenging to study of the flavor asymmetry of light sea quarks, \(\Delta \bar{u} - \Delta \bar{d}\), for solving the spin structure of nucleon.
One of the main tasks of the HERMES Experiment at DESY is the determination of the quark spin distributions with semi-inclusive measurements in deep-inelastic scattering. Semi-inclusive measurement allows us to study the spin contributions of each quark in the nucleon by tagging the struck quark with help of the identification of the leading hadron in the final state. The HERMES Experiment started in 1995 and has obtained many important basic results already. Since 1998 a Ring Imaging Cherenkov Counter (RICH) is installed, and this is one of the most important devices to tag the struck quark from the target. The HERMES RICH uses two radiators, aerogel and C$_4$F$_{10}$ gas. Incident photons make a dual ring image on the photon detector plane, which allows us to detect three types of hadrons ($\pi$, K, p) in the momentum region of $2 \sim 15$ GeV. This dual radiator RICH provides accurate identification of the detected hadrons. With this RICH data we can extract the contributions of quark spins to the total spin of the nucleon.
Chapter 2

Deep-Inelastic Scattering (DIS)

2.1 DIS Kinematics

Deep-Inelastic Scattering (DIS) is one of the most important tools to investigate the internal structure of the nucleon. The DIS process of the interaction between a nucleon \(N\) and a lepton \(l\) can be written as follows:

\[
l + N \rightarrow l' + X,
\]

where \(l'\) is the scattered lepton, \(h\) is the detected hadron and \(X\) stands for other undetected particles. The Feynman diagram of this process is shown in Fig. 2.1.

The incoming lepton which hits the nucleon interacts with the quarks by the weak or electromagnetic interaction, by exchanging a neutral vector boson \(Z^0\) or a virtual photon \(\gamma^*\). But in the energy region of HERMES the contribution of the \(Z^0\) boson can be neglected because the mass of the \(Z^0\) boson is far beyond the kinematics of HERMES. So from here on, we consider the electro-magnetic process which the exchange of one photon \(\gamma^*\) only.

![Feynman diagram](image)

Figure 2.1: The diagram of DIS. The kinematical valuables are expressed in Tab.2.2. \(X\) expresses the produced particles in the fragmentation processes.
### The inclusive DIS variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = (E, \vec{k})$</td>
<td>Four-momentum of the incoming lepton</td>
</tr>
<tr>
<td>$k' = (E', \vec{k}')$</td>
<td>Four-momentum of the scattered lepton</td>
</tr>
<tr>
<td>$s = (0, \vec{s})$, $\vec{s} = \frac{1}{m}(\frac{k}{</td>
<td>k</td>
</tr>
<tr>
<td>$s' = (0, \vec{s}')$</td>
<td>Spin four-vector of the scattered lepton</td>
</tr>
<tr>
<td>$P = (M, \vec{0})$</td>
<td>Four-momentum of the target nucleon</td>
</tr>
<tr>
<td>$S = (0, \vec{S})$, $\vec{S} = (\sin \alpha \cos \beta, \sin \alpha \sin \beta, \cos \alpha)$</td>
<td>Spin four-vector of the target nucleon</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Scattering angle of the lepton in the lab system</td>
</tr>
<tr>
<td>$\nu = \frac{P \cdot q}{M} \overset{\text{lab}}{=} E - E'$</td>
<td>Energy transfer to the target</td>
</tr>
<tr>
<td>$q = (\nu, \vec{q})$</td>
<td>Four-momentum transfer to the target</td>
</tr>
<tr>
<td>$Q^2 \overset{\text{lab}}{=} 4EE' \sin^2 \theta \frac{\theta}{2}$</td>
<td>Negative square of the four-momentum transfer</td>
</tr>
<tr>
<td>$W^2 = (P + q)^2 \overset{\text{lab}}{=} M^2 + 2M\nu - Q^2$</td>
<td>Invariant of the hadron final state</td>
</tr>
<tr>
<td>$x = \frac{Q^2}{2P \cdot q} \overset{\text{lab}}{=} \frac{Q^2}{2M \nu}$</td>
<td>Bjorken scaling variable</td>
</tr>
<tr>
<td>$y = \frac{P \cdot q}{P \cdot k} \overset{\text{lab}}{=} \frac{Q^2}{2M \nu}$</td>
<td>Fractional energy transfer of the virtual photon</td>
</tr>
</tbody>
</table>

Table 2.1: Definition of DIS kinematical values. These variables of DIS are shown in Fig.2.2 and Fig.2.3.
Figure 2.2: Definition of DIS kinematic variables. The angles $\theta$, $\phi$, $\alpha$ and $\beta$ are defined by the incoming lepton $k$, the scattered lepton $k'$ and the spin four-vector of target nucleon $S$.

Figure 2.3: Definition of scattering angles in DIS. The angles which are shown here are identical with those in Fig.2.2.
We next define the kinematics of the DIS process. The kinematic variables are defined in Tab.2.1. In this thesis, the discussion is based on the laboratory (lab) system, which is the special case where an incoming lepton with four-momentum \( k = (E, \vec{k}) \) is scattered by a fixed target with four-momentum \( P = (M, 0) \). \( M \) is the rest mass of the target nucleon. All quantities are expressed in units of \( c \) and \( \hbar \) as \( c \equiv \hbar \equiv 1 \) and skip all \( c \) afterwards.

First the four-momentum transfer \( q \) between the incoming lepton and the target nucleon is defined as:

\[
q^2 = (k' - k)^2_{\text{lab}} \approx \frac{4EE'}{c^2} \sin^2 \frac{\theta}{2},
\]

and the negative square of the four-momentum transfer squared is defined:

\[
Q^2 \equiv -q^2 = -(k' - k)^2.
\]

The invariant mass \( W \) of the hadron final state is obtained by the four-momentum transfer of the exchanged photon and the target four-momentum \( P \) of the initial state:

\[
W^2 = (P + q)^2 = M^2 + 2Pq + q^2 = M^2 + 2M\nu - Q^2,
\]

where the Lorenz invariant variable \( \nu \) is defined as:

\[
\nu = \frac{P \cdot q}{M} \equiv \frac{E - E'}{E'}. \tag{2.5}
\]

In the laboratory frame \( \nu \) gives the transfer energy from the incoming lepton to the target by the virtual photon. Using this \( Q^2 \) the two dimensionless variables \( x, y \) are given by:

\[
x = \frac{Q^2}{2P \cdot q}_{\text{lab}} \equiv \frac{Q^2}{2M\nu}, \tag{2.6}
\]

\[
y = \frac{P \cdot q}{P \cdot k}_{\text{lab}} \equiv \frac{\nu}{E}. \tag{2.7}
\]

\( x \) is the variable of Bjorken scaling and \( y \) is the fractional energy transfer and these can be determined in the region of 0 to 1.

In the elastic scattering where \( W = M \):

\[
2M\nu - Q^2 = 0. \tag{2.8}
\]

On the other hand in the inelastic scattering where \( W \) is larger than \( M \) the relation:

\[
2M\nu - Q^2 > 0, \tag{2.9}
\]

is obtained. The Bjorken scaling variable \( x \) is a measure for the inelasticity of the scattering process. In the elastic process \( x \) is defined as:

\[
x = 1. \tag{2.10}
\]

In the inelastic process \( x \) takes the value from 0 to 1:

\[
0 < x < 1. \tag{2.11}
\]
2.2 Cross Section and Nucleon Structure Function

In DIS, the process occurs in a large region of the square of four-momentum transfer $Q^2$ and energy transfer $\nu$. The cross section of DIS can be written as:

\[
\frac{d^2 \sigma}{dE' d\Omega} = \frac{\alpha^2}{Mq^4} \cdot \frac{E'}{E} L^\mu \nu W_{\mu \nu}, \tag{2.12}
\]

where $\alpha$ is the fine structure constant, $L^\mu \nu$ is the lepton tensor and $W_{\mu \nu}$ is the hadron tensor. The lepton tensor can be obtained from QED calculation:

\[
L^\mu \nu (k, s; k', s') = \left[ \bar{u}(k', s') \gamma^\nu u(k, s) \right] \left[ \bar{u}(k', s') \gamma^\mu u(k, s) \right]^* \tag{2.13}
\]

and we can split into symmetric $(S)$ and anti-symmetric $(A)$ part:

\[
L^\mu \nu (k, s; k', s') = L^\mu \nu_{(S)} + iL^\mu \nu_{(A)}, \tag{2.14}
\]

where

\[
L^\mu \nu_{(S)} = 2 \left\{ k^\mu k'^\nu + k'^\mu k^\nu - g_{\mu \nu} (k \cdot k' - m^2) \right\}, \tag{2.15}
\]

\[
L^\mu \nu_{(A)} = 2m \epsilon_{\mu \nu \lambda \sigma} s^\lambda (k - k')^\sigma, \tag{2.16}
\]

where $m$ is the lepton mass, $s$ and $s'$ are covariant spin four-vector of the initial and final lepton, respectively. In Eq.2.15-Eq.2.16, the notations are used, which are shown as:

\[
\epsilon_{0123} = \pm 1, \tag{2.17}
\]

\[
g_{\mu \nu} = \begin{cases} 
1 & \mu = \nu = 0, \\
-1 & \mu = \nu = 1, \text{ or } 2 \text{ or } 3, \\
0 & \mu \neq \nu. 
\end{cases} \tag{2.18}
\]

Like the lepton tensor, the hadron tensor is separated into symmetric $(S)$ and anti-symmetric $(A)$ part:

\[
W^\mu \nu (q; P, S) = W^\mu \nu_{(S)} (q; P) + iW^\mu \nu_{(A)} (q; P, S). \tag{2.19}
\]

The hadron tensor reflects the structure of hadrons. Although the lepton tensor can be obtained exactly from the calculation, the hadron tensor cannot be calculated because hadrons are not pointlike particles. Therefore it can be parameterized by so called structure functions. In the following sections, the cross section and structure function in the unpolarized and polarized case will be discussed.
2.2.1 Unpolarized Cross Section and Structure Function

The spin-independent part of the hadron tensor \( W_{(S)}^{\mu\nu} \) can be expressed by dimensionless structure functions, \( F_1(x, Q^2) \) and \( F_2(x, Q^2) \):

\[
W_{(S)}^{\mu\nu}(q; P) = 2 \left( -g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{Q^2} F_1(x, Q^2) \right) + \left( P_\mu + \frac{P \cdot q}{Q^2} q_\mu \right) \left( P_\nu + \frac{P \cdot q}{Q^2} q_\nu \right) \frac{F_2(x, Q^2)}{P \cdot q}.
\]

(2.20)

The structure functions \( F_1(x, Q^2) \) and \( F_2(x, Q^2) \) are Lorentz-invariant and reflect the internal structure of the nucleon. With the symmetric parts of the lepton and hadron tensors, the unpolarized differential cross section in the lab system (Eq. 2.12) is:

\[
\frac{d^2\sigma}{dxdQ^2} = \frac{d\sigma}{d\Omega} \frac{1}{EE'x} \left[ F_2(x, Q^2) + \frac{2\nu}{M} F_1(x, Q^2) \tan^2 \left( \frac{\theta}{2} \right) \right].
\]

(2.21)

These two structure functions \( F_1(x, Q^2) \) and \( F_2(x, Q^2) \) are independent or have very weak dependence of the value of \( Q^2 \) for the fixed \( x \). Therefore, \( F_1(x, Q^2) \) and \( F_2(x, Q^2) \) can be rewritten in the limit of the large \( Q^2 \):

\[
F_1(x, Q^2) \rightarrow F_1(x),
\]

(2.22)

\[
F_2(x, Q^2) \rightarrow F_2(x),
\]

(2.23)

where \( x = \frac{Q^2}{2M} \) is the momentum fraction which the parton carries.

The inelastic structure functions \( F_1(x) \) and \( F_2(x) \) depend only on the \( x \) variable, which is called the Bjorken scaling.

2.2.2 Polarized Cross Section and Structure Function

In the polarized case, the cross section includes two types of structure functions. One is the two spin-averaged structure functions \( F_1(x, Q^2) \) and \( F_2(x, Q^2) \) (containing in \( W_{(S)}^{\mu\nu} \)) like in the unpolarized case, other is the two spin-dependent structure functions \( g_1(x, Q^2) \) and \( g_2(x, Q^2) \) (containing in \( W_{(A)}^{\mu\nu} \)). \( W_{(A)}^{\mu\nu} \) is expressed by \( g_1(x, Q^2) \) and \( g_2(x, Q^2) \):

\[
W_{(A)}^{\mu\nu}(q; P, s) = \epsilon^{\mu\nu\lambda\sigma} \cdot \frac{q^\lambda}{\nu} \left[ S^\sigma g_1(x, Q^2) + \left( s^\sigma - \frac{S \cdot q}{P \cdot q} P^\sigma \right) g_2(x, Q^2) \right].
\]

(2.24)

The spin-dependent structure functions can be obtained by measuring the differences of the cross sections in the opposite target spin states because the spin-independent structure functions are canceled in the cross section differences. The differences of cross section has only the \( L_{(A)}^{\mu\nu} W_{(A)}^{\mu\nu} \) term and are obtained by \( g_1(x, Q^2) \) and \( g_2(x, Q^2) \) as follows:

\[
\sum_{s'} \left[ \frac{d^2\sigma}{d\Omega'dE'}(k, s, P; -S; k', s') - \frac{d^2\sigma}{d\Omega'dE'}(k, s, P, S; k', s') \right]
\]

\[
= \frac{\alpha^2}{2Mq^4} \cdot \frac{E'}{E} A L_{(A)}^{\mu\nu} W_{(A)}^{\mu\nu}
\]

(2.25)

\[
= \frac{8\pi\alpha^2}{Q^4} \frac{y}{E} \left( (E + E' \cos \theta) g_1(x, Q^2) - \frac{Q^2}{\nu} g_2(x, Q^2) \right).
\]

(2.26)
In the Bjorken limit, the spin-dependent structure functions $g_1(x)$ and $g_2(x)$ which are dependent only on $x$ are obtained:

\[ g_1(x, Q^2) \rightarrow g_1(x), \quad (2.27) \]
\[ g_2(x, Q^2) \rightarrow g_2(x). \quad (2.28) \]

## 2.3 The Quark Parton Model

The basis of the Quark Parton Model (QPM) is that the proton structure is obtained from the longitudinal momentum of constituents which are called ‘partons’. The charged partons are the quarks and others are the gluons. We can easily treat the deep-inelastic scattering in the Breit-Frame (Fig 2.4), where the transverse momentum and the rest mass of the partons can be neglected.

If $P$ is defined as the four-momentum vector of proton target and $\xi P$ is the part of the proton momentum which is carried by the parton the following relation is obtained in deep-inelastic scattering by this parton and lepton:

\[ (\xi P + q)^2 = m_q^2. \quad (2.29) \]

Neglecting the mass of the parton $m_q$, $\xi$ is given by:

\[ \xi = \frac{Q^2}{2 M \nu} \cdot \frac{2}{1 + \sqrt{1 + Q^2/\nu^2}} = \frac{2x}{1 + \sqrt{q + 4(Mx)^2/Q^2}}. \quad (2.30) \]

In case of $Q^2 \gg M^2$, Eq. (2.30) simplifies:

\[ \xi \approx \frac{x}{1 + (Mx)^2/Q^2} \approx x. \quad (2.31) \]

In the Breit-Frame, $\xi$ (Nachtmann valuable) which expresses the ratio of the longitudinal momentum of parton to the total momentum of proton is identical with the Bjorken $x$ in the large-$Q^2$ limit. In this frame, we notice that the photon which has the four-momentum $q$ in the lab-frame interacts with the parton which has the momentum fraction $x = Q^2/2M \nu$.

![Figure 2.4: DIS Process in Breit-Frame. $\xi P$ expresses the momentum which is carried by the parton.](image-url)
From the above consideration, we can define the structure of nucleon using the parton momenta. The nucleon contains two types of quarks, the valence and the sea quarks. The valence quarks decide the quantum number of the nucleon and the sea quarks are pairs of quarks and anti-quarks which are produced virtually from the gluons by the strong interaction.

The structure function $F_2(x)$ can be expressed by quark momentum distributions in the QPM as:

$$F_2(x) = x \cdot \sum_f e_f^2 (q_v(x) + q_s(x)) f,$$

in which $f$ denotes the quark flavor, $q_v(x)$ is the number density of the valence quarks, $q_s(x)$ is that of the sea quarks and $e_f$ is the corresponding electric charge. The number density of a quark with flavor $f$ which has the momentum fraction $x$ is called Parton Density Function (PDF).

### 2.3.1 The unpolarized PDF

The unpolarized PDF of quark $q$ with flavor $f$ is described as follows:

$$q_f(x) = (q_v(x) + q_s(x)) f,$$

$$q_{s,f}(x) = \bar{q}_f(x).$$

The $q(x)$ consists of the density of the valence quarks $q_v(x)$ and the sea quarks $q_s(x)$. $q_v(x)$ and the anti-quark density $\bar{q}(x)$ are identical. The PDF reflects the distribution of quark momentum in the nucleon.

Assuming that the heavy quarks can be neglected and using the isospin symmetry between the proton and neutron, we obtain the relations:

$$u^p(x) = d^n(x) \equiv u(x),$$

$$d^p(x) = u^n(x) \equiv d(x),$$

$$s^p(x) = s^n(x) \equiv s(x).$$

The valence quarks are defined as:

$$u_v(x) = u(x) - \bar{u}(x),$$

$$d_v(x) = d(x) - \bar{d}(x),$$

and we can reproduce the quantum number of the proton summing up all parton densities:

$$\int_0^1 [u(x) - \bar{u}x] dx = 2,$$

$$\int_0^1 [d(x) - \bar{d}(x)] dx = 1,$$

$$\int_0^1 [s(x) - \bar{s}(x)] dx = 0.$$
2.3.2 The polarized PDF

In the polarized QPM, the quark spin distribution $\Delta q_f(x)$ is defined as follows:

$$\Delta q_f(x) = q_f^+(x) - q_f^-(x),$$

(2.43)

where $q_f^+(x)$ is the distribution of quarks with spins in the same directions as the spin of the nucleon, $q_f^-(x)$ is that with the opposite direction. When the spin direction of the virtual photon and the quarks are longitudinally parallel or anti-parallel, we can define the polarized structure functions:

$$g_1(x) = \frac{1}{2} \sum_f e_f^2 (q_f^+(x) - q_f^-(x))$$

$$= \frac{1}{2} \sum_f e_f^2 \Delta q_f(x),$$

(2.44)

$$g_2(x) = 0.$$ 

(2.45)

In the simple QPM, $g_2(x)$ is approximated to zero. The value of $g_1(x)$ is most sensitive to the quark spin distribution. This is explained in Fig.2.5. The incoming polarized lepton emits a polarized virtual photon. This photon can be absorbed only by a quark with the opposite spin direction because the final state of a quark must have spin 1/2. In the top picture of Fig.2.5 we can measure the cross section for beam and target polarization anti-parallel and in the bottom picture we can measure that for polarization parallel. The top process is sensitive to $q_f^+(x)$ and the bottom is sensitive to $q_f^-(x)$. Since $g_1(x)$ is the difference of $q_f^+(x)$ and $q_f^-(x)$, it gives the information of the quark spin.

![Diagram polarized DIS](image)

Figure 2.5: Diagram polarized DIS. The top figure shows the process which is sensitive to quarks polarized parallel to the nucleon’s spin, and the bottom figure is the process for quarks with the anti-parallel spin to the nucleon.
2.3.3 Measurement of \( g_1(x) \)

To measure \( g_1(x) \) in the DIS experiments, we define the asymmetry of the polarized cross sections in the cases of the direction of the target polarization longitudinal \((\alpha = 0^\circ, 180^\circ \text{ in Fig.2.3})\) and transversal \((\alpha = 90^\circ, 270^\circ \text{ in Fig.2.3})\) to that of the incoming lepton:

\[
A_\parallel = \frac{\sigma(180^\circ) - \sigma(0^\circ)}{\sigma(180^\circ) + \sigma(0^\circ)},
\]

\[
A_\perp = \frac{\sigma(270^\circ) - \sigma(90^\circ)}{\sigma(270^\circ) + \sigma(90^\circ)}.
\]

This measured cross section asymmetry is related to the asymmetry in absorption of the virtual photon which was emitted by the incident lepton:

\[
A_\parallel = D(A_1 + \eta A_2),
\]

\[
A_\perp = d(A_2 - \xi A_1),
\]

where \( D, \eta, d, \xi \) are given by:

\[
D = \frac{y(2 - y)}{y^2 + 2(1 - y)(1 + R(x, Q^2))}, \quad \eta = \frac{2\gamma(1 - y)}{2 - y},
\]

\[
d = D\sqrt{\frac{2\epsilon}{1 + \epsilon}}, \quad \xi = \frac{1 + \epsilon}{2\epsilon},
\]

\[
\epsilon = \frac{1 - y}{1 - y + \frac{Q^2}{x}}, \quad \gamma = \frac{Q}{\nu} = \frac{2Mx}{Q}.
\]

\( R(x, Q^2) \) is the ratio of longitudinal \((\sigma_T)\) to transverse \((\sigma_L)\) absorption cross section \(^1\) of the virtual photon:

\[
R(x, Q^2) = \frac{\sigma_L}{\sigma_T} = (1 + \gamma^2) \frac{F_2(x, Q^2)}{2x F_1(x, Q^2)} - 1.
\]

In the Bjorken limit, \( R(x, Q^2) \) becomes zero. The polarized structure functions \( g_1 \) and \( g_2 \) can be deduced from \( A_1 \) and \( A_2 \) by:

\[
A_1 = \frac{g_1 - \gamma^2 g_2}{F_1},
\]

\[
A_2 = \frac{\gamma(g_1 + g_2)}{F_1}.
\]

\( A_2 \) is limited by \( R(x, Q^2) \) because it contains the interference term \( \sigma_I \) between \( \sigma_T \) and \( \sigma_L \) which has to obey the triangular relation \( \sigma_I \leq \sqrt{\sigma_L \sigma_T} \) [11], leading to:

\[
|A_2| \leq \sqrt{R(x, Q^2)}.
\]

\(^1\)For photons “polarization” is usually used for the \( \leftrightarrow E \)-field direction, not for the spin direction. In this sense, transverse photon polarization means longitudinal spin direction.
From the results of recent experiment [12], it is clear that $A_2$ is close to zero. If $g_2$ and $A_2$ can be ignored, $g_1(x)$ is obtained by these simple relations:

$$A_1 \approx \frac{A_\parallel}{D}, \quad (2.54)$$

$$g_1 \approx A_1 \cdot F_1. \quad (2.55)$$

In Bjorken limit, $F_1(x)$ and $g_1(x)$ are described by:

$$F_1(x) \approx \frac{F_2(x)}{2x}, \quad (2.56)$$

$$g_1(x) \approx \frac{A_\parallel F_2(x)}{D \cdot 2x}. \quad (2.57)$$

Eq.2.56 is called the Callan-Gross relation. The recent measurement of $g_1(x)$ is shown in Fig.2.6.

![Graph showing recent results of the spin dependent structure function on the proton $g_1^p(x)$. The solid circle shows the final result for $g_1$ from 1997 data at HERMES, which are shown at $Q^2=2$ GeV$^2$ (the left figure) and $Q^2=10$ GeV$^2$ (the right figure). The results from SLAC-E143 ($Q^2=2$ GeV$^2$) and SMC ($Q^2=10$ GeV$^2$) are shown for comparison.](image)

**Figure 2.6:** The recent results of the spin dependent structure function on the proton $g_1^p(x)$. The solid circle shows the final result for $g_1$ from 1997 data at HERMES, which are shown at $Q^2=2$ GeV$^2$ (the left figure) and $Q^2=10$ GeV$^2$ (the right figure). The results from SLAC-E143 ($Q^2=2$ GeV$^2$) and SMC ($Q^2=10$ GeV$^2$) are shown for comparison.

### 2.4 Sum Rules

#### 2.4.1 Gottfried Sum Rule

The Gottfried Sum Rule [13] is for the unpolarized structure functions of proton $F_2^p$ and neutron $F_2^n$. From Eq.2.32, Eq.2.40 - Eq.2.42 we obtain the relation:

$$S_G = \int_0^1 \frac{1}{x} (F_2^p(x) - F_2^n(x)) dx,$$

$$= \frac{1}{3} + \frac{2}{3} \int_0^1 u_s(x) - d_s(x). \quad (2.58)$$
If the unpolarized sea quark distributions is symmetric, \( S_C = 1/3 \) should be obtained. The measured value by NMC is \( S_C = 0.235 \pm 0.026 \) [14], which means this assumption is not valid and an unpolarized sea quark flavor asymmetry exists.

### 2.4.2 Bjorken Sum Rule

In polarized case, the integral of \( g_1(x, Q^2) \) is derived from Eq.2.44:

\[
\Gamma_1(Q^2) = \int_0^1 g_1(x, Q^2) dx.
\]

(2.59)

With the assumption of iso-spin symmetry, the integrals of the proton and neutron spin structure functions \( \Gamma^p_1 \) and \( \Gamma^n_1 \) are expressed as follows:

\[
\Gamma^p_1 = \frac{1}{9}a_0 + \frac{1}{12}a_3 + \frac{1}{36}a_8,
\]

(2.60)

\[
\Gamma^n_1 = \frac{1}{9}a_0 - \frac{1}{12}a_3 + \frac{1}{36}a_8
\]

(2.61)

where \( a_0 \), \( a_3 \) and \( a_8 \) are the SU(3) flavor singlet, isor triplet and octet isosinglet combinations. They are expressed by the quark distributions:

\[
\begin{align*}
    a_0 &= (\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d}) + (\Delta s + \Delta \bar{s}), \\
    a_3 &= (\Delta u + \Delta \bar{u}) - (\Delta d + \Delta \bar{d}), \\
    a_8 &= (\Delta u + \Delta \bar{u}) + (\Delta d + \Delta \bar{d}) - 2(\Delta s + \Delta \bar{s}).
\end{align*}
\]

(2.62) (2.63) (2.64)

(2.65)

Furthermore, assuming SU(3) flavor symmetry, \( a_3 \) and \( a_8 \) can be obtained from the weak coupling constants \( F \) and \( D \):

\[
\begin{align*}
    a_3 &= F + D, \\
    a_8 &= 3F - D.
\end{align*}
\]

(2.66) (2.67)

The values of \( F, D \) are measured in neutron and hyperon \( \beta \)-decays. \( F \) and \( D \) are obtained as follows [15]:

\[
\begin{align*}
    F &= 0.459 \pm 0.008, \\
    D &= 0.798 \pm 0.008.
\end{align*}
\]

(2.68) (2.69)

From the difference of the two polarized structure functions, the Bjorken sum rule is derived as follows:

\[
\begin{align*}
    \Gamma^p_1(Q^2) - \Gamma^n_1(Q^2) &= \frac{1}{6} \left| \frac{g_A}{g_V} \right| \cdot Corr \\
    &= \frac{1}{6} \cdot (F + D) \cdot Corr,
\end{align*}
\]

(2.70) (2.71)

where \( g_A, g_V \) are the axial-vector and vector coupling constants. \( Corr \) is the correction factor with higher order QCD.

The Bjorken Sum Rule [16] [17] uses the assumption of isospin invariance only, and it is therefore a good tool to test QCD. No existing experimental data show a violation of this sum rule [18].
2.4.3 Ellis-Jaffe Sum Rule

If we assume that $\Delta s = 0$ in the Bjorken Sum Rule, we can derive the moment of $g_p^o$ and $g_n^o$ from Eq.2.60 and Eq.2.61:

\[
\Gamma^o_p = \frac{1}{12} \left| \frac{g_A}{g_V} \right| \left(1 + \frac{5}{3} \cdot \frac{3F - D}{F + D} \right) \cdot \text{Corr}
\]
\[
\Gamma^o_n = \frac{1}{12} \left| \frac{g_A}{g_V} \right| \left(-1 + \frac{5}{3} \cdot \frac{3F - D}{F + D} \right) \cdot \text{Corr}
\]

At $Q^2 = 5\text{GeV}^2$, the following values are obtained for Ellis-Jaffe sum rule[19]:

\[
\Gamma^o_p = 0.179 \pm 0.009,
\]
\[
\Gamma^o_n = -0.019 \pm 0.009.
\]

These calculated values are not consistent with the values from the experiment [18]. The measured values are clearly smaller than the prediction. This disagreement shows a polarization of the strange quarks. The violation of Ellis-Jaffe Sum Rule is the starting point of the study of the nucleon spin structure (the Spin Crisis). One of the motivations for the HERMES experiment is to solve this puzzle.

2.5 Semi-Inclusive Measurement

Now we return the consideration of the DIS process again. The DIS process of lepton $l$ on nucleon $N$ is shown in the previous section as follows:

\[
l + N \rightarrow l' + h + X,
\]

where $l'$ is the scattered lepton, $h$ is the detected hadron, $X$ are other not detected particles. In inclusive measurements, we detect the scattered lepton only. In semi-inclusive measurements, we detect produced hadrons $h$ in coincidence with the scattered lepton $l'$. In this measurement we can extract the quark spin distributions of each flavor. The diagram of semi-inclusive DIS is shown in Fig.2.7. The virtual photon emitted by the incident lepton is absorbed by one quark in the target and some hadrons are produced. We assume that the hadron with the largest momentum is produced from the fragmentation of the struck quark in the target. This hadron is called “leading hadron”.

There are two types of produced hadrons, one is the “current fragment” which is produced from the fragmentation of the struck quark and another is the “target fragment” which is produced from the target remnant. These two can be kinematically separated by the Feynman scaling variable $x_F$. We need the current fragments and therefore select the hadron with $x_F > 0$.

The fragmentation processes of the struck quark cannot be calculated exactly, so it is parameterized using the experimental data [20] [21], which are so called fragmentation function $D^h_f(z)$. These functions define the probability that the hadron $h$ is with momentum fraction $z$ produced from a struck quark with flavor $f$. 
Figure 2.7: Diagram of semi-inclusive DIS. \( P_h \) is the momentum of a produced hadron in the DIS process. \( X \) shows other produced particle which is not identified.

<table>
<thead>
<tr>
<th>The semi-inclusive DIS variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{</td>
</tr>
<tr>
<td>( x_F = \frac{P_{</td>
</tr>
<tr>
<td>( z = \frac{P \cdot P_{h \text{ lab}}}{P \cdot q} \frac{E_h}{\nu} )</td>
</tr>
</tbody>
</table>

Table 2.2: Definition of semi-inclusive kinematical values.

2.5.1 Fragmentation Functions

In the QPM, the hadron cross section \( \sigma_h \) in semi-inclusive DIS is expressed with fragmentation functions and the unpolarized quark distributions:

\[
\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma^h}{dz}(x, Q^2, z) = \frac{\sum_f e_f^2 q_f(x, Q^2) D^h_f(x, Q^2, z)}{\sum_f e_f^2 q_f(x, Q^2)}, \tag{2.75}
\]

where \( \sigma_{\text{tot}} \) is the cross section of inclusive DIS, \( e_f \) is the quark charge and \( q_f \) are the unpolarized PDF. The fragmentation functions are normalized by the multiplicity \( n_h \) of all produced hadron:

\[
\sum_f \int_0^1 D^h_f(Q^2, z) \, dz = n_h(Q^2), \tag{2.76}
\]
and energy momentum conservation leads to:
\[
\sum_k \int_0^1 z D_f^k(Q^2, z) \, dz = 1. \tag{2.77}
\]

The fragmentation functions of quarks with flavor \( f \) are determined for all hadrons. Then the \( n_f \times n_h \) types fragmentation functions exist, where \( n_f \) is the number of the quark types and \( n_h \) is that of the hadron types. We assume iso-spin symmetry and charge symmetry to reduce the number of \( D_f^k(x) \). If we use these two assumptions, for example for pion production, three types of fragmentation functions are obtained as follows:
\[
\begin{align*}
D^+(z) &\equiv D_u^+(z) = D_d^+(z) = D_s^+(z) = D^+_a(z), \tag{2.78} \\
D^-(z) &\equiv D_d^-(z) = D_u^-(z) = D_s^-(z) = D^-_a(z), \tag{2.79} \\
D_s(z) &\equiv D_s^+(z) = D_s^-(z) = D_s^+(z) = D_s^-(z), \tag{2.80}
\end{align*}
\]

where the \( Q^2 \) dependence of the fragmentation functions has been omitted. Here we consider only three quark types \( (u, d, s) \) and \( \pi \) as produced hadron. \( D^+(z) \) is called favored fragmentation functions and \( D^-(z) \) is called unfavored. In the case of strange quarks, we define \( D_s(z) \) separately because of the mass of strange quark. In theory, some fragmentation models are established. These models are shown in the next section.

### 2.5.2 Fragmentation Models

Fragmentation functions are one of the most important variables when we study semi-inclusive DIS processes, but they are not calculated in perturbative QCD. So some phenomenological fragmentation models have to be used for example in Monte Carlo programs. Three fragmentation models are described below, all of which are based on the QPM.

#### Independent Fragmentation Model

The independent fragmentation model was considered to describe the fragmentation process in 1978, which is one of the first models of fragmentation by Field and Feynmann [22]. This model is based on the simple assumption that every quark fragments independently. When a quark \( q_0 \) is struck, it combines with other antiquark \( \bar{q}_1 \) and a \( q_0\bar{q}_1 \) pair is created which is the first meson with energy fraction \( z_0 \). The energy left \( z_1 = (1 - z_0) \) is used for another pair \( q_1\bar{q}_2 \). These meson creation process is iterated down to a certain energy cutoff \( E_{\text{min}} \).

One of the parameters of this model is \( f(1 - z)dz \) which gives the probability that a particular meson is created first. A further parameter is the probability \( \gamma_f \) for the creation of a \( q_f\bar{q}_f \) quark pair with same flavor. The isospin invariance gives \( \gamma_u + \gamma_d + \gamma_s = 1 \) and \( \gamma \equiv \gamma_u = \gamma_d \). According to the measured \( K/\pi \)-ratio the probability of \( s\bar{s} \) creation is \( \gamma_s = 0.3\gamma \).

In this model the ratio between the favored and unfavored fragmentation functions is obtained as follows:
\[
\frac{D^-(z)}{D^+(z)} = \frac{\gamma(1 - z)}{z + \gamma(1 - z)}. \tag{2.81}
\]
If we neglect the strange quarks \((\gamma = 0.5)\) this ratio is expressed as \(D^-/D^+ = (1 - z)/(1 + z)\).

The independent fragmentation can describe many properties of hadron fragmentation, but there are some inherent problems like the violation of color and flavor quantum number conservation. Therefore other models have been developed and the independent fragmentation model is usually not used any longer.

**LUND String Model**

The LUND String model [23] is the most successful model today, which is based on the QCD prediction. It is similar to the independent fragmentation model, but now the fragmentation is not independent, but the quarks are connected to the color field. With growing distance between the quarks, the color field is sketched as a tube and is called a string. The energy is stored in this string of color field. When the energy exceeds the mass of a \(q\bar{q}\)-pair, a new pair is produced and the string is divided in two strings. The fragmentation proceeds until the last pair is close to the mass of a colorless hadron.

In this model, there are main three parameters in the function which is called LUND symmetric fragmentation function:

\[
f(z) = \frac{(1 - z)^a}{z} \cdot \exp\left(\frac{-bm_+^2}{z}\right),
\]

where \(m_+\) is the transverse mass squared \(m_+^2 = m^2 + p_\perp^2\), \(a\) and \(b\) are parameters which are determined from experiment.

The LUND string model is used by the JETSET [24] library which is the basis for Monte Carlo studies for HERMES. These parameters have to be tuned for the conditions at HERMES [25]. The numbers are shown below (Fig.2.8 and Tab.2.3). In the present studies, these values are used for all MC productions.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>(&lt;P_\perp^2&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default Value</td>
<td>0.30</td>
<td>0.58</td>
<td>0.36</td>
</tr>
<tr>
<td>Fitted Value</td>
<td>0.82</td>
<td>0.24</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Table 2.3: Fitted parameters of LUND model for HERMES. The parameters of the LUND are tuned for HERMES experiment. These fitted valued are determined to reproduce the spectra from all hadrons.

**Cluster Model**

The Cluster model describes the fragmentation processes by the QCD perturbation theory. The highly virtual partons go into color-singlet clusters after the interaction between the photon and the parton which is called “preconfinement”. The clusters decay into hadrons from simple phase space arguments. The Cluster model has the advantage that it is most theoretical and the adjustable parameters are the QCD scale parameters.
Figure 2.8: Charged hadron spectra corrected for detector influences. The solid line shows the MC data with the fitted parameters in Tab.2.3. The dashed line is the MC data with the default parameters which is before the tuning of MC.

This model is used in the HERWIG Monte Carlo generator [26]. The parameters of this model are fitted for the HERMES experiment already. However the Cluster model fragmentation are not consistent with the HERMES data. Since the LUND model has better agreement than the Cluster model, we use the LUND model with fitted parameters for HERMES in our study.

2.5.3 Hadron Asymmetry

In the semi-inclusive measurement, $A_{h}^h$ is defined as a semi-inclusive asymmetry for hadron $h$ according to Eq.2.46:

$$A_{h}^h = \frac{\sigma_h(180^\circ) - \sigma_h(0^\circ)}{\sigma_h(180^\circ) + \sigma_h(0^\circ)} = \frac{N_{h}^{\uparrow\downarrow} - N_{h}^{\uparrow\uparrow}}{N_{h}^{\uparrow\downarrow} + N_{h}^{\uparrow\uparrow}}, \quad (2.83)$$

where $N_{h}^{\uparrow\downarrow}$ is the number of detected hadrons in the case of the beam and target polarizations anti-parallel and $N_{h}^{\uparrow\uparrow}$ is for the case of parallel polarizations.

The semi-inclusive structure functions are defined as:

$$g_{i}^{h}(x, Q^2, z) = \frac{1}{2} \sum_{f} \epsilon_f^2 D_{f}^{h}(Q^2, z) \Delta q_{f}(x, Q^2), \quad (2.84)$$

$$F_{i}^{h}(x, Q^2, z) = \frac{1}{2} \sum_{f} \epsilon_f^2 D_{f}^{h}(x, Q^2, z) q_{f}(x, Q^2), \quad (2.85)$$

where the assumption that the fragmentation process is spin-independent is used.
From Eq. 2.51 and Eq. 2.50, we obtain the semi-inclusive spin asymmetry $A_1^h(x, Q^2)$ from $g_1^h(x, Q^2, z)$ and $F_1^h(x, Q^2, z)$ under the assumption of $g_2 = 0$:

$$A_1^h(x, Q^2) \approx \frac{\int dz \, g_1^h(x, Q^2, z)}{\int dz \, F_1^h(x, Q^2, z)} = \frac{1 + R(x, Q^2)}{1 + \gamma^2} \cdot \frac{\sum_f e_f^2 \Delta q_f(x, Q^2) \int dz \, D_f^h(Q^2, z)}{\sum_f e_f^2 q_f(x, Q^2) \int dz \, D_f^h(Q^2, z)}, \quad (2.86)$$

where we have to apply the relation of Eq. 2.50 at HERMES kinematic range. This hadron spin asymmetry $A_1^h(x, Q^2)$ provides us the possibility to extract quark polarizations.
Chapter 3

HERMES Experiment

3.1 Experimental Setup

The HERMES experiment is carried out at DESY in Hamburg, Germany. The detector is located in the East Hall of the HERA lepton-proton collider ring. The HERMES uses only the polarized lepton beam of HERA, together with a polarized internal gas target. The interaction between a polarized target and a polarized beam provides the possibility to study the quark spin. Details of the HERA lepton storage ring including the locations of the other three HERA experiments, ZEUS, H1 and HERA-B, are shown in Fig 3.1.

3.2 The HERA Storage Ring

The Fig.3.1 shows the HERA lepton (electron or positron) ring. The ring can be operated with lepton at an energy of 27.5 GeV. The beam becomes polarized transverse to the beam plane due to the asymmetry in the emission of synchrotron radiation (Sokolov-Ternov mechanism [27]). The beam polarization \( P(t) \) is given as function of time \( t \):

\[
P(t) = p_{\text{max}} (1 - e^{-t/\tau}),
\]

where \( \tau \) is the polarization build up time and \( p_{\text{max}} \) is the equilibrium polarization. At HERA the beam reaches 60% polarization in about 30 minutes after the beam injection.

Because it is necessary to obtain the longitudinal beam polarization at HERMES, two spin rotators are located up and downstream of the HERMES Spectrometer. The first rotates the lepton spin to the longitudinal direction the second one rotates back to transverse. The beam polarization is measured by a transverse polarimeter in the West Hall of HERA and the polarization is constant in the whole storage ring. The longitudinal polarization of the HERA lepton beam is shown in Fig. 3.2.
Figure 3.1: The HERA storage ring. HERMES are located with the other three experiments, ZEUS, H1 and HERA-B. Because the longitudinal beam polarization is needed for HERMES, the two spin rotator is located up and down stream of the HERMES experiment. In 1995 - 1997, the positron beam is used and the electron beam is operated during 1998.

Figure 3.2: The longitudinal polarization of HERA beam. The polarization reaches up to 60% in about 30 minutes.
3.3 The Internal Gas Target

HERMES uses a polarized internal gas target, applying the novel technique of an internal storage cell. The target gas is injected in an open-ended aluminum tube inside the lepton storage ring. The thickness of this aluminum is about 100 μm. The polarized gas coming from an atomic beam source (ABS), where the polarization is achieved by Stern-Gerlach separation, is injected into the storage cell via a small feed tube. The leaking gas from both open ends is pumped away by a differential pump system. Fig 3.3 shows the design of the target chamber.

We used a helium3 (³He) target in 1995, a proton (p) target in 1996 and 1997 and a deuteron (D) target from 1998 to 2000. In this system the target density is $1.2 \times 10^{15}$ nucleons/cm$^2$ for $³$He and $7.5 \times 10^{13}$ nucleons/cm$^2$ for p and D.

![Diagram of internal gas target](image)

Figure 3.3: The internal target of HERMES.
3.4 The HERMES Spectrometer

HERMES spectrometer has a large acceptance and is a forward angle detector. This spectrometer is shown in Fig.3.4 and Fig.3.5 [28]. With the HERMES detector, scattering angles of particles are accepted within $\pm 170$ mrad in the horizontal direction and between $+/(-)40$ mrad and $+/(-)140$ mrad in the vertical direction.

A spectrometer magnet and several detectors for particle tracking allow the determination of particle momenta. The tracking system consists of microstrip gas chambers, drift chambers and three proportional chambers in the magnet.

For semi-inclusive measurements in DIS, HERMES needs a very good particle identification (PID), not only to separate lepton and hadrons, but also to identify the hadrons. To achieve this high quality particle identification, the HERMES spectrometer is equipped with four special PID detectors: an electro magnetic lead-glass calorimeter, a preshower hodoscope, a transition radiation detector (TRD) and Cherenkov Counter. The original threshold Cherenkov has been replaced in 1998 by a Ring Imaging Cherenkov Counter (RICH) to allow a clean hadron identification over the momentum range of $2 \sim 15$ GeV.

![HERMES Spectrometer Diagram](image)

Figure 3.4: The HERMES Spectrometer. Two identical detectors are set upper and lower of the beam line. The Cherenkov Counter is operated as a threshold Cherenkov Counter during 1995 - 1997 data taking. Since 1998, a Ring Imaging Cherenkov Counter are replaced to identify produced hadrons.
3.4. THE HERMES SPECTROMETER

3.4.1 The Tracking System

Spectrometer Magnet

The HERMES spectrometer has a H-type magnet with a deflecting power of 1.5 Tm and it is usually operated at 1.3 Tm for the momentum measurement. The lepton beam lines are protected by a horizontal septum plate with compensator coils which are arranged in the beam plane. The angular acceptance is 40 mrad \(< |\theta_y| < 140\) mrad vertically, and \(|\theta_x| < 170\) mrad horizontally, where the lower vertical limit is given by the septum plate. The total acceptance of scattering angles is 40 mrad < |\theta| < 220 mrad.

Vertex and Front Chambers

In upper region of the spectrometer magnet, the micro-strip gas chambers which is called vertex chambers VC1/2 and the drift vertex chambers DVC and the drift front chambers FC1/2 are installed to reconstruct the particle vertex and to determine the scattering angle. The VC's consisted of 2 modules each with 6 planes, these planes are made by glass bases of 300 \(\mu m\) thickness which are covered with 7 \(\mu m\) aluminum strips. These strips are separated by 193 \(\mu m\) and arranged in 3 mm distance from a cathode plane. The gap is filled with DiMethylEther(DME)/Ne mixture (50/50\%) gas. The efficiencies of vertex chambers were around 95% for 1997 running, and the resolution of about 65 \(\mu m\) (\(\sigma\)) per plane was obtained.

The FC's is a pair of drift chambers, each chamber has one module with 6 planes.
The size of drift cell is 7 mm (3.5 mm maximum drift length), and the used gas is Ar/CO₂/CF₄ (Ar:CO₂:CF₄ 90:5:5) like in all other HERMES drift chambers.

The DVC’s were constructed and installed downstream of the VC, because of problems with the VC’s. The DVC acceptance is vertically ± 35 mrad ~ ± 270 mrad and horizontally ± 200 mrad. But the DVC drift cell is smaller than the FC’s. The gas composition of DVC is Ar/CO₂/CF₄, the same as for the FC’s.

**Magnet Chambers**

The proportional wire chambers MC1/2/3 are mounted in the gap of the spectrometer magnet. These were originally installed to analyze multiple tracks in high multiplicity events. It has turned out that they are also particularly useful for the analysis of particles with low energy which are deflected by the magnet so that they do not come in the acceptance, pion from the Λ-decays, for example. Each chamber consists of 3 modules. The distance between anode and cathode planes is 4±0.03 mm. The gas is the same as the drift chamber’s but the proportions of the mixture is different (Ar:CO₂:CF₄ 65:30:5).

**Back Chambers**

In the region downstream of the spectrometer magnet two sets of the back drift chambers BC1/2 and BC3/4 are mounted. The difference between BC1/2 and BC3/4 is the detector size. The active areas of these modules depend on their z-position and the acceptance of the spectrometer. The BC1/2 active area is horizontally 1880 mm and vertically 520 mm, and the BC3/4’s is 2890 mm and 710 mm. Each set of chambers has two modules with 6 planes. The efficiency of the drift chamber depends on the drift distance and the typical efficiency is well above 99%. The details are shown in [29]. These detectors are shown in Fig.3.6.

![Figure 3.6: Schematic view of the HERMES BC module.](image)
3.4.2 The Particle Identification System

As I mentioned before, a clean particle identification is very important at HERMES. The PID is performed in two steps: first a lepton-hadron separation and second the identification of the hadron. Especially the second point is essential for the study of the quark spin distribution. As for the lepton-hadron separation, the hadron rejection factor (HRF) of the system is designed to be $10^4$ at least, where HRF is defined as the total number of hadrons divided by the number of hadrons misidentified as leptons.

This value depends on the efficiency for the lepton identification. The contamination of the scattered lepton sample by hadrons is below 1% for the whole kinematic range. This hadron rejection is done by the calorimeter, the preshower hodoscope and the TRD. The second step, the hadron identification is done by the fourth PID detector, the RICH. These four detectors are described in the following.

Calorimeter

The calorimeter (Fig.3.7) provides a first level trigger for scattered leptons. It consists of two modules which have 420 blocks each above and below the beam line. All 840 blocks are made by lead-glass blocks with size of $9 \times 9 \times 50$ cm$^3$ and are arranged in a $42 \times 10$ array for each module. Each block is readout by a photomultiplier tube (PMT). Both the calorimeter walls can be moved 50 cm vertically from the beam pipe to prevent radiation damage during beam injection or dump. The energy resolution of this detector is parameterized by the following function:

$$\frac{\sigma E}{E} = 1.5 \pm 0.5 + \frac{5.1 \pm 1.1}{\sqrt{E[GeV]}}.$$  \hspace{1cm} (3.2)

where $E$ is energy of leptons measured by the calorimeter. Combined with the preshower counter, the efficiency for the detection of lepton is about 95%. The calorimeter is described in detail in [30].

Figure 3.7: Isometric view of the HERMES Calorimeter.
Hodoscopes

There are two hodoscopes H1 and H2, which are plastic scintillators. H2 is behind the 11 mm Pb radiator (two radiation lengths) sandwiched between two 1.3 mm stainless steel sheets and used as a preshower counter in front of the calorimeter. The first level trigger is obtained by a combination with the calorimeter and H1, where H2 gives a discrimination between leptons and hadrons. Both the hodoscopes consist of 42 vertical scintillator modules each in the upper and lower detector. The size of the modules is $9.3 \times 91 \times 1$ cm$^3$ and they are read out by PMTs of 5.2 cm diameter.

Transition Radiation Detector

The transition radiation detector is used for the discrimination between leptons and hadrons [31]. Only leptons emit transition radiation in the HERMES energy region.

The HERMES TRD is a multiwire proportional chamber constructed by 6 modules above and below the beam. Each module has polyethylene/polypropylene fibers as radiators and Xe/CH$_4$ gas (90:10) in the proportional chamber. The fibers used for the radiators are 17-20 µm in diameter and have a material density of 0.039 g/cm$^3$. The radiators are 6.35 cm thick, and Xe/CH$_4$ has been chosen because of its efficient X-ray absorption.

The chambers are 2.54 cm thick and the wire diameter was used to be unusually 75 µm large to allow operation at high voltage while limiting the gas gain to about $10^4$. The original purpose of the TRD is to obtain the pion rejection factor (PRF) of at least 100 for 90% lepton efficiency at 5 GeV and above, accordingly the energy averaged PRF is about 300 [31] for 90% lepton efficiency. When we analyze the data as the function of momentum, the PRF of 130 is obtained at 5 GeV. The design goal is therefore accomplished.

Ring Imaging Cherenkov Counter

To study the quark spin structure, it is necessary that produced hadrons (pion, kaon, proton) are identified separately. Until 1998, threshold Cherenkov Counter was used to identify pions which are produced in DIS scattering. This threshold Cherenkov Counter had good performance for pion separation from hadrons. The efficiency of this counter is above 99%. In 1998 a Ring Imaging Cherenkov Counter (RICH) has been installed as a substitute for the threshold Cherenkov Counter. The HERMES RICH is designed to identify pions, kaons and protons in a momentum region of $2 \sim 15$ GeV. This RICH has two radiators, aerogel and C$_4$F$_{10}$ gas. Measuring two ring images produced by Cherenkov photons on the PMT plane, we can identify and separate each particle. The RICH is described in detail in the next chapter.
Chapter 4
Ring Imaging Cherenkov Counter

In its original version the HERMES spectrometer had a threshold Cherenkov Counter to detect produced hadrons, especially pions. Although this threshold counter had good performance for the pion detection, a new Ring Imaging Cherenkov Counter (RICH) was proposed in 1997 [32] to identify also other hadrons produced in DIS. The RICH was installed in 1998 and is operating since then. This RICH is designed to detect three types of hadrons: pions ($\pi^+$, $\pi^-$), kaons ($K^+$, $K^-$) and (anti-)protons ($p$, $\bar{p}$).

The HERMES RICH has a very special property that it is working with two radiator, silica aerogel and C$_4$F$_{10}$. Thanks to recent developments, silica aerogel is available with high transmission and low refraction index. If a hadron passes the RICH, it emits Cherenkov light at different angles for silica aerogel and C$_4$F$_{10}$ Gas. Therefore the Cherenkov light forms two rings on the photomultiplier plane. We can identify the hadron by determining the Cherenkov angle of the two rings. The details are discussed in the following.

4.1 Detector Design

In this section, the fundamental principle of the RICH is described. If a charged particle is moving with velocity $v$ in an optical medium with refractive index $n$ and its velocity $v$ exceeds the velocity of light in the medium $c/n$, it emits Cherenkov light with the Cherenkov radiation angle $\theta$. The Cherenkov radiation angle is determined as:

\begin{align*}
  v &= c\beta, \\
  \cos \theta &= \frac{1}{n\beta}. \quad (4.1) \quad (4.2)
\end{align*}

From this relation, we can get the particle velocity $v$ if we can measure the Cherenkov radiation angle $\theta$ and know the refractive index $n$ of the material. Since the particle momentum is measured by the spectrometer magnet, it is possible to identify the particle which passes the RICH if we can measure the Cherenkov angle $\theta$. The refractive index of the radiator material determines the Cherenkov angle produced by the charged particles. At HERMES the RICH had to be installed in a predetermined place where the threshold Cherenkov Counter was mounted. The
The diagram of Cherenkov radiation. A charged particle with the velocity $v$ which exceeds the velocity of light in the medium can emit Cherenkov light with the Cherenkov angle $\theta$.

The whole design of RICH is shown in Fig.4.2. As radiators, silica aerogel and C$_4$F$_{10}$ gas were chosen because their reflective indices satisfy our demand.

The Cherenkov angles from aerogel and C$_4$F$_{10}$ gas are shown in Fig.4.3 as a function of particle momentum. If the particle momentum is above the threshold of the Cherenkov radiation of this particle, it emits the Cherenkov lights and makes ring images on the photon detector plane which is then read out by arrayed photomultipliers. One of the parameters which determines the performance of RICH is $p_{\text{max}}$, which is the maximum separation momentum. This is the maximum momentum for which the average Cherenkov angle of two particles is separated by standard deviations $n_\sigma$. The $p_{\text{max}}$ parameter of RICH is shown below [33]:

$$p_{\text{max}} = \sqrt{\frac{m_2^2 - m_1^2}{2kF n_\sigma}},$$  \hspace{1cm} (4.3)

where $m_1, m_2$ are masses of two particles and $kF = \tan \theta \cdot \sigma_\theta/\sqrt{N}$ is the RICH detector constant, $N$ is the number of detected photons, $\theta$ is the Cherenkov angle and $\sigma_\theta$ is the standard deviation of the Cherenkov angle distribution. The RICH at HERMES is designed as $n_\sigma = 4.652$.

This RICH can identify particles from two ring images made by Cherenkov photons. Besides this it can act also as a threshold Cherenkov Counter in a particular momentum region. This momentum region is indicated in Fig.4.4. The light shaded region in Fig.4.4 indicates the region where the RICH acts as a threshold Cherenkov Counter, and the dark shaded region indicates the region where we need to reconstruct the Cherenkov angle to identify particles.

As the reconstruction of the Cherenkov angles is most important for PID, we use different PID algorithms and compare the reconstructed angles. All the algorithms need Monte Carlo simulations of RICH. Therefore a realistic MC is important. So we have to study the properties of the RICH in detail and must include these into the MC.

In the following section, we describe the properties and the MC of the RICH.
Figure 4.2: Schematic view of Ring Imaging Cherenkov Counter at HERMES. Two identical RICH is located upper and lower of the beam line, these figures show the upper RICH. The bottom figure is the side view of RICH. The aerogel tiles are assembled back of the entrance window. The C$_4$F$_{10}$ gas are filled in the RICH. A charged particle emits Cherenkov photons by these two radiators, and the photons are reflected by the mirror to detect the ring images on the PMT array. The mirror are divided by eight segments for one RICH.
Figure 4.3: The relation between Cherenkov angle and particle momentum for electrons, pion, kaon and protons. The solid lines describe the Cherenkov angle from aerogel and the dotted lines are for that from \( \text{C}_4\text{F}_{10} \) gas. Due to using two radiators, the particle identification in large momentum region becomes possible.

Figure 4.4: Momentum ranges for hadron separation. Between the dashed lines the hadrons can be separated.
4.2 Study of RICH Property

4.2.1 Aerogel Radiator

As mentioned before, the RICH has two radiators which are silica aerogel made from $\text{SiO}_2$ (Fig.4.5 and Fig.4.6) and $\text{C}_4\text{F}_{10}$ gas. Especially, silica aerogel is used as a radiator for a RICH for the first time. Seeing Fig.4.2, there is a container for aerogel which is next to the entrance window. The entrance window is 187.7 cm wide and 46.4 cm high, and the container size is 200 cm wide and 60 cm high. As exit window a 3.2 mm thick UVT-lucite is placed. The container is made by 1mm thick aluminum and is filled with dry $\text{N}_2$ gas. Aerogels in the shape of small tiles are stacked in it. The aerogel tiles are stacked in 5 layers, 5 horizontal rows and 17 vertical columns for the top and the bottom part of the RICH. These assemblies of 425 aerogel tiles in total for one RICH acts as radiator wall with a thickness of about 5.5 cm as shown in Fig.4.9. To cut the diffused reflection at the sides of the aerogel tiles, black plastic spacers are mounted between the tiles.

These aerogel tiles are produced at Matsushita Electric Works [34] [35]. The typical size of aerogel tile is $11 \times 11 \times 1.1 \text{ cm}^3$, which is determined by the production process. Besides, the form of one aerogel tile is not perfect rectangular as shown in Fig.4.8. The refractive index of aerogel is around 1.0304, but it has some deviations, which has effects on the Cherenkov angle produced by particles passing through aerogel. The transmittance of aerogel is measured already [36] [37]. From this study, the transmittance is determined to be above 90 %. The transmittance $T$ can be parameterized as a function of wavelength $\lambda$ and aerogel thickness $t$ as follow:

$$ T = A \cdot \exp \left( \frac{C \cdot t}{\lambda^4} \right), $$

(4.4)

where $C$ and $A$ are called the Hunt parameters depending on the aerogel properties. $C$ describes the amount of Rayleigh scattering and $A$ is included to account for surface reflections due to the difference in the refractive indices between aerogel and gas. The fitted parameters for HERMES are $C \cdot t = 0.0094 \mu\text{m}^4$ and $A = 0.964$ [36].

The lucite of the aerogel exit window absorbs photons which have wavelengths less than 280 nm to remove Rayleigh scattered Cherenkov photons, which can become the source of background. As can be seen Eq.4.4, the number of Rayleigh scattered photons increases as the wavelength $\lambda$ becomes shorter. Since photons with short wavelength have to be removed. This efficiency is shown in Fig.4.10.

To modify the RICH Monte Carlo, we studied the details of the main aerogel properties: the aerogel size and the refractive indices of the different tiles, and the description of the surface. We measured the relations between the temperature and the index, and the dependence of the index on the position within one aerogel tile, because good knowledge on the refractive index is most important to identify particles [38]. The main results of aerogel property measurement are following.

The Distribution of the Refractive Index

The refractive index of aerogel is not exactly constant, but differs slightly from tile to tile. To use aerogel as a radiator for the RICH, there are some conditions which
Figure 4.5: Composition of silica aerogel [34]. The aerogel is made from SiO₂ chemically and has highly porous structure. The typical particle size is 1 ~ 2 nm.

Figure 4.6: Hydrophobic process for silica aerogels. Due to this new productions techniques, clear and hydrophobic aerogels became available.
4.2. STUDY OF RICH PROPERTY

Figure 4.7: Photo of aerogel tile. Silica aerogel produced with the new techniques has a high transparency and transmittance. The typical size is shown in Fig.4.8.

Figure 4.8: Typical size of one aerogel tile. The typical size of one aerogel tile is $11 \times 11 \times 1$ cm. The height of a aerogel edge is higher than the central part because of surface tension in the process produced silica aerogels.
Figure 4.9: Aerogel wall as radiator in RICH. The assembly of aerogel tiles as Cherenkov radiator is shown for one RICH. 850 aerogel tiles are assembled.

Figure 4.10: Efficiencies for transmittance through lucite. The light with the wavelength $\lambda$ less than 280 nm cannot be go through lucite, which is included in MC as a $\lambda$ dependence.
should be satisfied. One condition is that the average of refractive index $n_{\text{mean}}$ is between 1.028 and 1.032, and another is that the values of all aerogel refractive indices are in the region of $n_{\text{mean}} \pm 0.002$. In addition to these, there is a condition for the standard deviation $\sigma$ of refractive indices which comes from the resolution of the Cherenkov angles. The effect on the resolution can be estimated as:

$$\frac{\Delta \theta}{\theta} \leq 0.5\%.$$  \hfill (4.5)

From this relation, the resolution of the refractive indices are calculated as:

$$\frac{\Delta (n - 1)}{n - 1} = 2 \frac{\Delta \theta}{\theta} \leq 1\%.$$  \hfill (4.6)

The distribution of refractive index has to be less than 1%. The average of refractive index is about 1.03, therefore the standard deviation $\sigma$ of the refractive index has to be less than $3.0 \times 10^{-4}$ [38] [39] [40].

To select the aerogel tiles which can be used for the RICH, we measured the refractive indices of 1332 aerogel tiles [38]. The result of this measurement is that the standard deviation $\sigma$ is $4.1 \times 10^{-4}$, which didn’t satisfy the above condition. But this problem was solved by separating the sample in two groups, because there are two independent RICH detector in the upper and lower half of the HERMES spectrometer. Fig.4.11 shows the distribution of the refractive index for 1040 selected aerogel tiles. The refractive index for the two groups is shown in Fig.4.12, where the original distribution has been divided at the central value of refractive index. In this case the standard deviations are $2.5 \times 10^{-4}$ and $2.6 \times 10^{-4}$, which satisfy the expected conditions. This procedure allowed us to use the aerogel as radiator material of the RICH.
Figure 4.11: Distribution of the refractive index of 1040 selected aerogels. The horizontal axis shows the value of \( n - 1 \), where \( n \) is the refractive index of aerogels. The vertical axis is for the number of aerogel tiles.

Figure 4.12: The distribution of refractive index \( n \) of aerogel which are divided into two groups. The horizontal axis shows the value of \( n - 1 \) as Fig.4.11. The left figure is for the aerogel tiles with 1.0290 \( \leq n \leq 1.0302 \) and the right is for that with 1.0303 \( \leq n \leq 1.0313 \).
The Geometry

The condition on the size and the surface of the aerogel tiles is that the distributions of the longitudinal and transverse sizes are within ± 0.5 mm [38]. We measured the size of the aerogel tiles, and the distributions for all three dimensions for the 1040 selected aerogel tiles are shown below in Fig.4.13.

The Description of Surface and Edge

The aerogel surface has to be flat to obtain complete ring images from Cherenkov photons, which means that the deviation of the surface from flat has to be less than 1.0 mm. Because the aerogel tile is made from sol, the edge of the tile is higher than the center of tile which was due to the surface tension in the production processes. We measured the shape of the surface of aerogel. We selected one aerogel tile to study the surface. The result of this measurement is shown in Fig.4.14 together with the fitted function. From this figure, we can see that the distance between the edge and the center of the tile is about 1.5 mm. We need to tiles which the deviation of the surface is less than 1.0 mm, although it seems that the tile is not satisfy this condition. But there is not so large influence to detected rings images, because almost all part of aerogel surface is within 1.0 mm from flat and only the part which is distant in 5 mm from the edge has more deviation up to 1.5 mm. Beside, the black plastic spacers which are mounted between the tiles cut the photons which pass through the edge of tile. Therefore the aerogel tiles can be satisfy our demand and the function in Fig.4.14 is used as the typical model of the surface of aerogel tile.
Figure 4.13: Distributions of the size of the aerogel tiles. The distributions for three dimensions are shown, the definition of the dimensions are in Fig.4.9.
4.2. STUDY OF RICH PROPERTY

4.2.2 Mirrors

The mirror of of the both RICH detector halves is divided into eight segments which are arrayed to make a part of spherical surface. The size of one mirror segment is 252.4 cm in the horizontal and 79.4 cm in the vertical. It weights less than 13 kg. The mirror reflectivity was already measured before the RICH was installed. That study showed that the reflectivity is above 85 % for light in the 300 ~ 600 nm range of wavelength (Fig.4.15). This reflectivity was measured for only one mirror segment and we adapted this reflectivity to other segments. By this method, no position dependence of the mirror reflectivity is available. Besides, the mirror has only a simple alignment because of the short time scale of the RICH installation. But the reflectivity and alignment of the mirror are important, because we need the number of fired PMTs to identify particles, and this number depends among others on the reflectivity. Therefore we need to know the details of the mirror alignment and have to check the effects on the PID.

I studied the details of the mirror reflectivity, including the effects of the mirror alignment and put the measured data into the MC. I discuss my analysis of the mirror reflectivity in Sec.4.3.1.

4.2.3 Photon Detector Plane

The photomultiplier plane has a size of 120 cm × 60 cm and contains an array of PMTs, which are Philips XP1911 with a diameter of 18.6 mm (0.75 inch). There are 1934 PMTs in each RICH half in 73 columns of alternately 26 and 27 PMTs each. The minimum active photo-cathode diameter is 15 mm.

This PMT has a broad quantum efficiency curve that matches the Cherenkov
Figure 4.15: The wavelength dependence of the mirror reflectivity for Cherenkov photon. This dependence is shown as functions which is used in MC. Before MC modification, this dependence is included for the mirror property.

light spectrum of aerogel well. It is also good for Cherenkov light from the gas because the efficiency extends into the UV region. The PMT elementary cell of the array is a hexagon with a PMT is at the center. The distance between two adjacent cell centers is 23.3 mm (Fig. 4.16).

The PMT photo-cathodes cover only 38% of the area of the focal plane. To increase the coverage of PMT, a light-gathering cone, made of aluminized plastic foil funnel was inserted onto each photo-cathode. This PMT photo-cathodes with foil funnels then cover 91% of the surface. These funnels minimize the dead space between the PMTs. They have a high reflectivity above $\lambda = 200$ nm. The photon collection efficiency increased to about 80% which was only 40% before inserting the funnels. Their effects on the collection efficiency are shown in Fig. 4.17.

4.3 Monte Carlo Simulation for RICH

For particle identification, we need to reconstruct the Cherenkov angles, using the location fired PMTs, the information of the particle tracks and the photon hit position on PMTs. As the PMT plane does not completely match the true mirror focal surface, the photon collection efficiency is not 100%. We need the comparison of Monte Carlo with experimental results to identify the particle type with these input parameters. So it is very important that the MC can reproduce the experimental data exactly. We can do accurate particle identification if we have a reliable MC data set. Therefore I studied the MC simulation of the RICH and included certain additional effects in the MC code.
Figure 4.16: Schematic picture of PMTs with funnels. The minimum active photo-cathode diameter is 15 mm. The funnels are attached to minimize the dead space between the PMTs. The effect of these funnels are shown in Fig.4.17.

Figure 4.17: The photon collection efficiency verses angle of incidence. The total efficiency is marked by the solid points, the other points show the components of the efficiency as the number of “bounces”, which is reflection from the foil surface. The influence of the funnels is found from the difference between the total efficiency and the efficiency for “direct” detection.
4.3.1 Development of MC for RICH

Though the MC for the RICH had been available, it was not realistic enough because the description of the properties of aerogel and the mirror has been much too simple. For example, the refractive index and the size of all the aerogel tiles were treated as constant, the surface of the aerogel was flat and the mirror alignment was assumed to be ideal. The Cherenkov angles of photons from aerogel and gas were not consistent with experimental data, the Cherenkov angle from MC being smaller than that of the experimental data. Moreover, the number of fired PMTs from MC events was less than that of the experimental data. The Cherenkov angle and the number of the fired PMTs are primary values for the PID. Therefore I analyzed the different effects of the RICH properties on the PID and included these effects in the MC in order to reproduce the experimental data.

The properties which affect the PID are the following:

1. The aerogel properties
   - The realistic size \((x, y, z)\) and the surface of the aerogel tile
   - The actual refractive index of all aerogel tiles

2. The Rayleigh scattering and the boundary corrections
   - The Rayleigh scattering which affects the number of the fired PMTs
   - The location of the gap between the aerogel wall and the exit window of the aluminum container

3. The position dependence of the mirror reflectivity
   - The mirror reflectivity of each segment which contains the corrections due to the mirror misalignment

In the next section, I describe the correction of the MC and results after the above mentioned properties are included.

The Aerogel Tile Properties

In Sec.4.2.1, the properties of the aerogel tiles are described. As mentioned before, 425 aerogel tiles are used for one RICH, but the size of all aerogel tiles is not exactly the same for all the tiles. Therefore, the measured sizes in all three dimensions \((x, y, z)\) of 850 aerogel tiles are included in the MC code. In the same way, the values of the refractive indices are included. As the result the Cherenkov angle became dependent on the refractive index of each aerogel tile. Because the aerogel radiator wall is assembled by a large number of aerogel tiles, there are gaps between the tiles. These gaps are filled with dry \(N_2\) gas, but there are no Cherenkov photon produced when particles go through this gaps. As they cause a decrease in the number of fired PMTs, I included these boundary effects in the MC code (Fig.4.18). After the measured sizes are included, a more realistic value for the surface of the tiles had also to be included. So, the function from Fig.4.14 has been implemented in the MC code.
Figure 4.18: The RICH in the MC. The aerogel wall as radiator is assembled by 425 tiles, which are shown in the right figure. The Cherenkov photons produced by aerogel wall and gas are reflected by the mirror.

Before including these effects, the standard deviation of the MC Cherenkov angle from particles which passed through the aerogel was smaller than that angle in the experimental data.

**Rayleigh Scattering**

The Cherenkov photon from a particle which goes through aerogel is scattered by Rayleigh scattering. The scattered photons are detected as background events, so the number of the fired PMTs becomes smaller. The number of scattered photons is proportional to the thickness of the radiator. The thickness of one tile is defined about 1.1 cm in MC, however detailed measurement shows that the thickness of the main part of aerogel tile is 0.9 cm, which is smaller than the edge thickness, as explained in Fig.4.8. Therefore the effective distance for Rayleigh scattering is about 4.5 cm. In the original MC version, the effective distance was 5.5 cm which reduced the number of fired PMTs too much. Therefore I modified this value of the thickness of the aerogel wall in the MC code, and compared the number of fired PMTs with and without correction.

**Position Dependence of the Mirror Reflectivity**

The Cherenkov photons are reflected by the mirror of the RICH and they enter the PMT arrays. If the mirror reflectivity is 100 %, all the produced Cherenkov photons which hit the mirror can reach the PMTs. The mirror is not perfect in reality, and the number of fired PMTs depends on the mirror reflectivity. Therefore this reflectivity affects on the performance of the PID. As was mentioned before, the wavelength dependence of the mirror reflectivity is included in MC code, which was measured before the RICH installation, but this reflectivity is the result of only one mirror segment and is applied to all the segments. This was obviously
not enough to reproduce the experimental data. I studied the details of the mirror reflectivity of all 16 segments by comparing MC data with the experimental data. Then I included the position dependence of the mirror reflectivity. Here, I defined that the position dependence of the mirror reflectivity includes all mirror effects in terms of the number of fired PMTs. Especially it should be noticed that the position dependence of mirror reflectivity is not only all mirror effects but also other effects from the unknown factors, which is efficiency of funnels, light correction efficiency and PMT efficiency. Beside the effects among others are the mirror alignment of the 16 mirror segments, the slope of the track when a photon hits the mirror, and the real mirror reflectivities. We defined that the efficiency from all unknown factors are the position dependence of the mirror reflectivity to normalize for the experiment data in this analysis. These contribution of each effect should be analyzed more accurately in the next step.

The above effects on the PID, the aerogel properties and the Rayleigh scattering, have an impact on the Cherenkov angle of the photon which is produced by the aerogel. On the other hand, this position dependence of the mirror reflectivity is relevant to the Cherenkov photons produced by both aerogel and C₄F₁₀ gas. So both numbers of fired PMTs have been checked. In this analysis, the normalized number of fired PMTs by photons from C₄F₁₀ gas (N_{gas}) are used since the Cherenkov angle by photons from aerogel is larger than that from gas, therefore the diameter of the Cherenkov ring by aerogel photons is larger than the size of one mirror segment. While the diameter of the Cherenkov ring from gas are very small at the mirror, which average diameter about is 5 cm, hence the number of fired PMTs by gas photons are more suitable than that of by aerogel photons. The method of this analysis is described below [41].

1. divide the mirror surface into 5 cm × 5 cm bins

2. obtain the normalized number of fired PMTs by photons from C₄F₁₀ gas (N_{gas}) at the point where a track hits the mirror in both experimental (N_{gas}(EXP)) and MC (N_{gas}(MC)) data

3. take the ratio of N_{gas}(EXP) to N_{gas}(MC) in each bin

4. calculate the mean value of the ratio for each mirror segment

5. the mean value of the ratio is defined to be the reflectivity of this mirror segment

In this analysis, I used the number of N_{gas} to obtain the mirror reflectivity because the Cherenkov angle of photons coming from C₄F₁₀ gas is smaller than the Cherenkov angle of photons from aerogel (see Fig.4.3). Then almost all Cherenkov rings of photons from the gas are within one bin. I used N_{gas} for each bin to obtain the position dependence of the mirror reflectivity. In addition to this, N_{gas} is normalized as follows:

\[
N_{gas} = \frac{N_{pmt}}{L_{gas}/T3},
\]  

(4.7)
where $N_{pmt}^{\text{gas}}$ is the number of fired PMTs from the gas before normalization, $L_{\text{gas}}$ is the particle track length through the gas and 73 is the mean value of $L_{\text{gas}}$ which is obtained from the MC data. This is necessary because the number of produced Cherenkov photons is proportional to the particle track length through the radiator. The definition of $L_{\text{gas}}$ and the distribution of $L_{\text{gas}}$ are shown in Appendix.A (Fig.A.1).

There are some conditions for data selection. First, I selected $\pi^+$ and $\pi^-$ particles with momentum of $4.5 \sim 8.5$ GeV/c. The resulting Cherenkov angle from the gas is then $0.03 \sim 0.07$ rad. In addition, I used the particle with $N_{\text{gas}}$ larger than 2. The reason that $\pi^+$ and $\pi^-$ are selected is to avoid geometrical acceptances effect due to the particle charge. Furthermore $\pi^+$ and $\pi^-$ can be most accurately identified among all the detected hadrons. The selected momentum range, $4.5 \sim 8.5$ GeV/c, allows to obtain the purest $\pi^+$ and $\pi^-$ samples (see Fig.4.3). The photons from these particles are well distributed in a large region of the mirror. The selection cuts of the Cherenkov angle from the gas and $N_{\text{gas}}$ are made to assure that the detected photons are really generated by a pion.

To obtain the mirror reflectivity in each bin, I needed the ratio between the MC data, which have no photon absorption (all photon which hit the mirror are reflected, as the mirror reflectivity is 100 %) except for the wavelength dependence of the reflectivity (Fig.4.15) and the experimental data. These ratios are all obtained within the $5 \times 5$ cm bins of the mirror. I determined the mean values of the ratios in each mirror segment, which is the reflectivity of that segment including other efficiencies from unknown factors. Using this method, the results with the ratio of $N_{\text{gas}}(\text{EXP})$ to $N_{\text{gas}}(\text{MC})$ at the mirror surface are shown in Appendix.A (Fig.A.2). From these ratios, the mean value of the ratios for each mirror segment are evaluated, as shown in Appendix.A (Fig.A.3).

I included these value for each mirror segment as the position dependence of the reflectivity into the MC code to normalize the experimental data. The effects of this on the PID are shown in the next section (Sec. 4.3.2).
4.3.2 PID Efficiency after Modification of MC

In this section, the efficiency for the PID and the comparison between this efficiency before and after including the RICH properties, which I discussed above. The “efficiency” which is used in this section is defined below.

At first, the comparison of the Cherenkov angle between experimental data and MC is shown. The aerogel tile properties which are included in MC affect the Cherenkov angle. And the comparison of the number of fired PMTs, which is affected by the Rayleigh scattering and the Mirror reflectivities is discussed.

PID algorithm

The detected ring on the PMTs are not a complete circle because of the RICH geometry. Therefore the reconstructed particle tracks are needed to identify the particle type. At HERMES, a method which is called “Inverse Ray Tracing” (IRT) is used for the reconstruction of the Cherenkov angle from the track parameters and the position of the PMTs. A detailed discussion of IRT is beyond the subject of this thesis. It is found in [42] [43] [44].

The MC data is used for the estimation of the efficiency of IRT method. To do this estimation, it is necessary to compare the Cherenkov angle and the number of fired PMTs between MC and experimental data. I’ll show this comparison in the next section, and it becomes obvious that including the RICH properties is necessary to reproduce the experimental data.

Cherenkov Angle Resolution

The aerogel properties described in Sec.4.3.1 have been included step by step in several MC production to check their effect on the Cherenkov angle. The MC productions which I made are listed below.

1. Simplest simulation
   This production is the base for the others. In this MC, the aerogel radiator is considered as one wall, which is not divided into tiles.

2. Rayleigh scattering
   The Rayleigh scattering effect is added to the above.

3. Aerogel tile effects
   The aerogel tiles are included as 425 separate tiles, each different size and refractive index. The surface of the aerogel tiles is also simulated.

4. Boundary correction of photon refraction
   The effect of the gap between aerogel tiles and the exit window is considered.

5. Mirror reflectivity
   The position dependence of the mirror reflectivity which contain all mirror alignments effects is included.

The Cherenkov angles from aerogel and gas are shown Tab.4.1 for all above MC productions and the experimental data. In Tab.4.1, $\sigma_{\theta_{\text{aero}}}$ and $\sigma_{\theta_{\text{ref}}}$ is the
Table 4.1: Comparison of the Cherenkov angle for each MC production and the experimental data. The deviation are found between the type 1 and type 3 of MC data.

<table>
<thead>
<tr>
<th>type</th>
<th>$\theta_{aero} \pm \sigma_{aero}$ [mrad]</th>
<th>$\theta_{gas} \pm \sigma_{gas}$ [mrad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP</td>
<td>$244 \pm 9.3$</td>
<td>$49.0 \pm 9.2$</td>
</tr>
<tr>
<td>MC 1</td>
<td>$235 \pm 6.3$</td>
<td>$54.0 \pm 7.6$</td>
</tr>
<tr>
<td>MC 2</td>
<td>$233 \pm 6.0$</td>
<td>$54.2 \pm 7.5$</td>
</tr>
<tr>
<td>MC 3</td>
<td>$234 \pm 6.5$</td>
<td>$54.0 \pm 7.6$</td>
</tr>
<tr>
<td>MC 4</td>
<td>$241 \pm 7.1$</td>
<td>$54.2 \pm 7.7$</td>
</tr>
<tr>
<td>MC 5</td>
<td>$241 \pm 7.1$</td>
<td>$54.0 \pm 7.5$</td>
</tr>
</tbody>
</table>

standard deviation of the Cherenkov angle of photons from aerogel (gas). Because the properties of the RICH are mainly dominated by the aerogel, the Cherenkov angle from the gas was not affected by these MC modifications. The Cherenkov angle from aerogel became close to the experimental value after including all effects.

Fig.4.19 shows the comparison between the experimental data and the MC data. The dark shaded histogram are for the MC data, and the light shaded histogram is the experimental data. The MC data of the top figure of Fig.4.19 is the production of type 1, the second from the top shows type 2, the third shows type 3, the forth shows type 4 and the bottom shows type 5. We can see that the effect from the boundary correction (type 4) has large influence to Cherenkov angles. The MC production can reproduce the experimental data for the Cherenkov angle of the photon from aerogel and gas rather well though some discrepancy remains. We can see better agreement of the Cherenkov angle in the recent results, which is shown in Appendix A (Fig.A.4).
Figure 4.19: Comparison of the Cherenkov angles between the experimental data (light shaded histograms) and the MC data (dark shaded histograms). The left figure shows the Cherenkov angle from aerogel and the right figure shows that from the gas.
4.3. MONTE CARLO SIMULATION FOR RICH

Number of Fired PMTs

As same as the Cherenkov angle, the number of fired PMTs is an essential value for the PID. I checked the number of fired PMTs by photons from aerogel ($N_{\text{aerogel}}$) and from gas ($N_{\text{gas}}$) with MC and experimental data.

The used MC data are the same as in the previous section. The contribution of each effect to the number of fired PMTs is shown in Tab.4.2.

<table>
<thead>
<tr>
<th>type</th>
<th>$N_{\text{aerogel}}$</th>
<th>$N_{\text{gas}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EXP</td>
<td>11.3</td>
<td>11.4</td>
</tr>
<tr>
<td>MC 1</td>
<td>11.2</td>
<td>15.6</td>
</tr>
<tr>
<td>MC 2</td>
<td>7.0</td>
<td>13.2</td>
</tr>
<tr>
<td>MC 3</td>
<td>13.8</td>
<td>15.1</td>
</tr>
<tr>
<td>MC 4</td>
<td>16.0</td>
<td>15.2</td>
</tr>
<tr>
<td>MC 5</td>
<td>11.2</td>
<td>13.1</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of the number of fired PMTs for each MC production and experimental data.

Comparing the experimental data and the MC data of type 5, the MC data after including all effects are close to the experimental data, especially the effect of the mirror reflectivity (see the difference between type 4 and type 5) has largest influence on the number of PMTs. Fig.4.20 shows the comparison of the experimental data and the MC data for the number of PMTs. Again, the dark shaded histograms are the MC data. The order of figures are same as Fig.4.19.
Figure 4.20: Comparison the number of fired PMTs between the experimental data (light shaded histograms) and the MC data (dark shaded histograms). The left figure shows $N^{pmt}_{aero}$ and the right figure shows $N^{pmt}_{gas}$. 
4.3. **MONTE CARLO SIMULATION FOR RICH**

The mirror reflectivity affects not only $N_{aero}^{\text{pmt}}$ but also $N_{gas}^{\text{pmt}}$. In this respect the mirror reflectivity is different from other effects. I studied the contribution of the mirror reflectivity to $N_{aero}^{\text{pmt}}$ and $N_{gas}^{\text{pmt}}$ in more detail. I define the “Efficiency”, as the ratio between the number of fired PMTs in the experiment and in the MC. Therefore, the efficiency of aerogel is defined as $N_{aero}^{\text{pmt}}(\text{EXP})/N_{aero}^{\text{pmt}}(\text{MC})$ and the efficiency of gas is defined as $N_{gas}^{\text{pmt}}(\text{EXP})/N_{gas}^{\text{pmt}}(\text{MC})$. These efficiencies must be almost 1 if the MC reproduces the experimental data accurately. This value is one criterion for the quality of the MC. Additionally, I checked the $\theta_x$ and $\theta_y$ dependence of the efficiencies, where $\theta_x$ and $\theta_y$ are the slopes of the tracks at the target.

Fig.4.21 shows the efficiency versus the slopes for aerogel and gas before the MC modification, where the simple MC production is used for the composition with experimental data. In Fig.4.21, the regions for $\theta_x$ and $\theta_y$ are $-0.14 < \theta_x < 0.14$ and $0.04 < \theta_y < 0.16$. It can be seen from Fig.4.21 that the ratio of the number of fired PMTs is far from 1, about $0.4 \sim 0.7$, and there are large fluctuations in the efficiencies depending on $\theta_x$ and $\theta_y$. This shows that some effects which are on both $N_{aero}^{\text{pmt}}$ and $N_{gas}^{\text{pmt}}$ are missing in MC. Therefore I compared the experimental data and the MC production of type 5. Fig.4.22 shows this results.

Fig.4.22 shows that the mirror reflectivity has a strong effect to the number of fired PMTs. The fluctuations observed in the efficiencies before the MC modification became small, and the both efficiencies increased by the MC modification although their value is only $0.7 \sim 0.9$. Even after including the mirror reflectivity, there are some difference between $N_{aero(gas)}^{\text{pmt}}(\text{EXP})$ and $N_{aero(gas)}^{\text{pmt}}(\text{MC})$.

![Efficiency Graphs](image)

**Figure 4.21:** The $\theta_x$ and $\theta_y$ dependence of the efficiency for aerogel and gas before MC modification. The left figure shows efficiencies for aerogel and the right figure shows that for gas.
Figure 4.22: The $\theta_x$ and $\theta_y$ dependence of the efficiency for aerogel and gas after MC modification. The left figure shows efficiencies for aerogel and the right figure shows that for gas.

I considered that the cause of this is the limited acceptance of the PM tubes. I tried to use the Cherenkov photons from the gas to study the mirror reflectivity, but it is impossible to get the real number of detected photons from $N_{pmt}^{gas}$ because the Cherenkov ring made by the gas photons is very small (the average diameter is about 5cm at the mirror). As it is possible that some Cherenkov gas photons hit the same PMT, the number of fired PMTs is not directly proportional to the mirror reflectivities (see Fig.4.23).

To prove this speculation, I studied the relation between the number of fired PMTs and the mirror reflectivities with MC. I produced the MC data changing the mirror reflectivities of all mirror segments from 0 to 1, and made plots of the mean value of the number of fired PMTs versus the mirror reflectivity. In these productions, I used $\pi^+$ and $\pi^-$ particles. The results of this study are shown below in Fig.4.24. Fig.4.24 shows that $N_{pmt}^{aero}$ is proportional to the mirror reflectivity, but $N_{pmt}^{gas}$ isn’t. From these results, 2 or more photons have to hit the same PMT if the number of photons from gas is more than 12.

From the above detailed study on the number of fired PMTs, it is necessary that this non-linearity has to be considered to obtain the accurate mirror reflectivities. However, the accurate number of detected photons, which is not the number of fired PMTs, cannot be obtain from experimental data. Therefore this non-linearity has to be considered in future improvements of the MC. In spite of the last mentioned restriction, the MC have became close to the experimental one by including the RICH properties. This is beneficial for the analysis of the RICH data, especially for the extraction of the spin asymmetries which is discussed in the following chapter.
Figure 4.23: The Cherenkov rings of aerogel and gas on the PMT plane. The smaller ring is made by photon from gas. Some Cherenkov photons from gas can hit the same PMT. Therefore the mirror reflectivity is not proportional to the number of fired PMTs.

Figure 4.24: The mirror reflectivity versus the number of fired PMTs by photon from aerogel and gas with MC. The left figure shows that the reflectivity vs mean value of $N_{\text{aero}}^{\text{pmt}}$ and the right figure shows that the reflectivity vs mean value of $N_{\text{gas}}^{\text{pmt}}$. 

1.94 + 8.07*R

7.18 + 6.41*R

2.86 + 13.5*R

(Mirror Reflectivity)

(Mirror Reflectivity)
Chapter 5

Extraction of Polarized Quark Distributions

This chapter describes the formalism for the extraction of polarized quark distributions, especially the flavor asymmetry of the light sea quark: \( \Delta \bar{u} - \Delta \bar{d} \). In this analysis, spin asymmetries which are obtained from experiment and the parameterization of unpolarized quark distributions (Parton Distribution Function) which are used for the purity generation are needed. All available inclusive and semi-inclusive asymmetries measured by HERMES are used in this study and various types of quark distributions are extracted.

5.1 The Purity Method

5.1.1 Definition of Purity

For the extraction of quark polarizations, we need semi-inclusive spin asymmetries \( A_1^h(x) \) obtained from the experiment. In the limit of large \( Q^2 \), \( A_1^h(x) \) can be expressed from Eq.2.86 as follows:

\[
A_1^h(x, Q^2) = \frac{1 + R(x, Q^2)}{1 + \gamma^2} \cdot \frac{\sum_f e_f^2 \Delta q_f(x, Q^2) \int dz D_f^h(Q^2, z)}{\sum_f e_f^2 q_f(x, Q^2) \int dz D_f^h(Q^2, z)},
\]

where the spin asymmetry is related to the polarized and the unpolarized PDF. As several inclusive and semi-inclusive spin asymmetries are now available from HERMES, we can extract polarized quark distributions with help of the unpolarized PDF and fragmentation functions.

Spin asymmetries from experiment are obtained for each \( x_i \) bin and are considered to be independent of \( Q^2 \). \( F_1 \) and \( g_1 \) are dependent on \( Q^2 \). However, for the measured spin asymmetries which is approximate to the ratio of \( g_1 \) and \( F_1 \), no significant \( Q^2 \) dependence has been observed [45] [46]. Therefore the relation which is used in practical analysis reads:

\[
A_1^h(x_i) = C_i \cdot \sum_f P_f^h(x_i) \frac{\Delta q_f(x_i)}{q_f(x_i)},
\]

(5.2)
where $C_i$ is defined as $C_i = (1 + R(x_i))/\left(1 + \gamma_i^2\right)$ and the kinematic variables $R(x_i, Q^2)$ and $\gamma_i$ denote the mean value in the $x_i$ bin. $P_f^h(x_i)$ is defined as:

$$
P_f^h(x_i) = \frac{e^2 q_f(x_i) \tilde{D}_f^h(x_i, z)}{\sum_f e^2 q_f(x_i) \tilde{D}_f^h(x_i, z)},
$$

(5.3)

where $\tilde{D}_f^h(x_i, z)$ is the effective fragmentation function which includes the detector acceptance. They are the probability that one hadron $h$ with energy fraction $z$ is detected when the virtual photon is scattered by a quark with flavor $f$. The values of $z_i$ are determined for each $x_i$ bin. Therefore the purities depend on a single parameter $x_i$ in this definition. This is called the “quark flavor purity” introduced in [47]. Introducing this purities $P_f^h(x_i)$, we can express the spin asymmetry $A_i^h(x_i)$ as a linear combination of quark polarizations. They represent the probabilities that a hadron $h$ is produced from a quark with flavor $f$. According to the definition of purity, the relation holds for every $x_i$ bin:

$$
\sum_f P_f^h(x_i) = 1.
$$

(5.4)

Actually, we use the equation Eq.5.2 to extract the quark polarizations. If we measure spin asymmetries for several types of hadrons, the quark polarizations can be calculated by solving the matrix equation:

$$
A = C_i \cdot P \cdot Q,
$$

(5.5)

where $A$ is the vector of measured spin asymmetries, $Q$ is the vector of each quark polarization and $P$ is the purity matrix which contains the purities of each quark flavor as matrix elements. They are defined as:

$$
A = \begin{pmatrix}
A_1^h(x_i) \\
\vdots \\
A_m^h(x_i)
\end{pmatrix}, \quad Q = \begin{pmatrix}
\Delta q_{f_1}/q_{f_1}(x_i) \\
\vdots \\
\Delta q_{f_n}/q_{f_n}(x_i)
\end{pmatrix},
$$

(5.6)

$$
P = \begin{pmatrix}
P_{f_1}^h(x_i) & \cdots & P_{f_m}^h(x_i) \\
\vdots & \ddots & \vdots \\
P_{f_1}^{h_m}(x_i) & \cdots & P_{f_m}^{h_m}(x_i)
\end{pmatrix}.
$$

(5.7)

Here $f_i$ is the quark flavor: $f_i = \{u, \bar{u}, d, \bar{d}, s, \bar{s}\}$, and $h_i$ is the hadron type: $h_n = \{\pi^+, \pi^-, K^+, K^-, p, \bar{p}, \ldots\}$. In present measurements of HERMES pions, Kaon and protons can be detected, and we consider these three types of hadrons in the following. In the ideal case, Eq.5.5 can be solved when the number of hadron types (the number of asymmetries) $m$ is equal to the number $n$ of flavor types. But, as we will see later, we have to use more asymmetries than flavor types because there are correlations between some asymmetries. These correlations mean that the number of independent asymmetries are smaller than the number of available asymmetries $m$. In this analysis all available asymmetries for the extraction of quark distributions are used. A detailed discussion of the asymmetries will follow in Sec.5.5.
5.1. THE PURITY METHOD

The formalism of quark polarization extraction with purities can be extended to the inclusive measurement, where we detect the scattered lepton only. In this case the purity for the scattered lepton is described as:

\[ P_f^R(x_i) = \frac{e_f^2 q_f(x_i)}{\sum_I e_I^2 q_I(x_i)}, \tag{5.8} \]

because the fragmentation function of the inclusive electron \( D_f^R(x_i) \) is equal to 1 in every \( x_i \) bin: \( \int dz D_f^R(x_i, z) = 1 \). Therefore we can include the inclusive asymmetry in this analysis.

To extract the quark distributions we have to solve above matrix equation Eq.5.5. With a vector of measured asymmetries \( \mathbf{A} \), \( \chi^2 \) can be defined as follows:

\[ \chi^2 = (\mathbf{A} - C_i \cdot \mathbf{P} \mathbf{Q})^T \mathbf{V}_A^{-1} (\mathbf{A} - C_i \cdot \mathbf{P} \mathbf{Q}). \tag{5.9} \]

\( \chi^2 \) is minimized if \( \mathbf{Q} \) becomes a solution of the matrix equation Eq.5.5. In Eq.5.9, \( \mathbf{V}_A \) is a covariance matrix which is introduced to consider the correlations between the measured asymmetries in \( \mathbf{A} \), and is described in detail in Appendix B.1. \( \mathbf{V}_A \) can be expressed by correlation coefficients between asymmetries as follows:

\[ (\mathbf{V}_A)_{ij} = \rho(A_i^{h_i}, A_j^{h_j}) \delta A_i^{h_i} \delta A_j^{h_j}, \tag{5.10} \]

where \( \rho(A_i^{h_i}, A_j^{h_j}) \) is a correlation coefficient between asymmetry of hadron \( h_i \) (\( A_i^{h_i} \)) and hadron \( h_j \) (\( A_j^{h_j} \)). \( \delta A_i^{h_i} \) and \( \delta A_j^{h_j} \) denote the uncertainties of the asymmetries of types \( i \) and \( j \).

Under some assumptions, \( \rho(A_i^{h_i}, A_j^{h_j}) \) can be replaced by the respective correlation coefficient between the number of hadrons, which is described in Appendix B.1. According to this relation, the covariance matrix reads:

\[ (\mathbf{V}_A)_{ij} = \rho(N^{h_i}, N^{h_j}) \delta A_i^{h_i} \delta A_j^{h_j}, \tag{5.11} \]

\[ \rho(N^{h_i}, N^{h_j}) = \frac{\left< n^{h_i} n^{h_j} \right>}{\sqrt{\left< (n^{h_i})^2 \right> \left< (n^{h_j})^2 \right>}}, \tag{5.12} \]

where \( \left< n^{h_i} \right> \) is an averaged particle multiplicity for hadron \( h_i \), which is the number of hadrons \( h_i \) in one DIS event.

The statistical errors on the extracted quark distributions \( \mathbf{Q} \) is expressed by covariance matrix of \( \mathbf{Q} \). This is shown below:

\[ \mathbf{V}_Q = C_i^{-1} (\mathbf{P}^T \mathbf{V}_A^{-1} \mathbf{P})^{-1}. \tag{5.13} \]

With this covariance matrix, the statistical errors on quark distributions are found to be:

\[ \delta \left( \frac{\Delta q_i}{q_i} \right) = \sqrt{(\mathbf{V}_Q)_{ii}}. \tag{5.14} \]

The quark distributions are extracted with this method which I have mentioned above. To do this analysis, the Singular Value Decomposition (SVD) method [48]
is used, which is the method for solving most linear least-squares problems and obtaining singular values. In this analysis the number of asymmetries is not stable because I tried to use several sets of input asymmetries for extraction of quark distributions. In the case where the number of input asymmetries is larger than that of the extracted quark distributions, SVD can solve the matrix equation Eq.5.5. Therefore this is a powerful tool for solving this matrix. The detail of the SVD method is described in Appendix.B.2.

An accurate determination of the systematic errors has been omitted in this study, but it will be mentioned only summarily later in Sec.6.4.

5.2 Evaluation of Purities

To solve the matrix equation Eq.5.5, I need measured spin asymmetries and purity matrix. In this analysis, asymmetries which have been obtained by other studies [49] [50] are used as input, and the details of all used asymmetries are described in Sec.5.5. The purities consist of unpolarized quark distributions and fragmentation functions which include of the acceptance the HERMES detectors. They are produced by MC simulation of the unpolarized DIS process, which is made by three steps. The first two steps generate the actual DIS events and the unpolarized PDF $q(x)$. The fragmentation functions $D^p_i(x; z)$ which are independent of the acceptance of HERMES are necessary as input for MC. The third step gives effects from the detector acceptance. In the following, the detail of each step is described.

In a first step, the PEPSI [51] generator, which is based on LEPTO [52], produced DIS events with unpolarized PDF, based on the CTEQ4LQ [53] and the GRV95 [54] parameterizations. In the main analysis, CTEQ4LQ parameterizations are used. The data which have been produced with GRV95 are used to estimate the systematic errors from the purities. LEPTO selects the DIS kinematics plane according to the lepton-nucleon scattering cross section and decides which quark is scattered by using the unpolarized PDF.

In the second step, the JETSET generator [24], which is part of LEPTO [52], processes hadronization and gives the final state of hadrons in DIS. JETSET contains different kind of hadronization programs which are based on the Independent model (Sec.2.5.2), the LUND String model (Sec.2.5.2) and the Cluster model (Sec.2.5.2) and there are default model parameters for each model. As I mentioned before in Sec.2.5.2, the LUND String model is the best to simulate hadronization nowadays, so it has been used to produce the final state hadrons in MC and the parameters for LUND model have been tuned for HERMES (Fig.2.8 and Tab.2.3) [25] to reproduce the experimental hadron spectra. The production with other hadronization models can be used to estimate systematic errors from fragmentation functions, but this hasn’t been done in this study.

In the last step, the simulation of effects from the HERMES detectors is performed. A detailed model of the HERMES detectors is available in the detector simulation program, which is named HMC [55] and is based on GEANT [56]. I modified some parts of this HMC for the RICH as mentioned in the previous sections, but to simulate the response of all detectors is time consuming, so it takes long to produce DIS events. Therefore I used a simple box acceptance model for
simulation of the detectors, which uses a momentum look-up table. It includes the momentum cut-offs, can generate accurate geometry of particle tracks behind the magnet, and the same cuts of event selection which is used in analysis of experimental data are applied.

From this MC data, the purities are obtained from the equation below:

$$P_f^h(x_i) = \frac{N_f^h(x_i)}{\sum_f N_f^h(x_i)}, \quad (5.15)$$

where $N_f^h(x)$ is the number of hadron $h$ which is produced when the quark in the nucleon with flavor $f$ is scattered. The $z$ dependence of $N_f^h(x)$ is ignored by summing up yields for this variable. This relation includes the fragmentation functions which contain also the detector acceptance of HERMES. This effective fragmentation functions are:

$$\bar{D}_f^h(x_i, z) = \frac{N_f^h(x_i, z)_{rec}}{\sum_h N_f^h(x_i, z)_{gen}}, \quad (5.16)$$

where $\sum_h N_f^h(x, z)_{gen}$ is the number of events which a quark with flavor $f$ is generated and $N_f^h(x, z)_{rec}$ is the number of reconstructed events with quark flavor $f$.

According to this relation, I generated purities $P_f^h$ for a proton and a neutron target, where the type of hadrons $h$ is $h = \{h^+, h^-, \pi^+, \pi^-, K^+, K^-, K^0, \bar{K}^0\}$ and the type of quark flavors $f = \{u, d, \bar{u}, \bar{d}, s, \bar{s}\}$. The hadron types of $h^+$ and $h^-$ include all produced hadrons as $\{\pi^+, \pi^-, K^+, K^-, p, \bar{p}, ..., \}$. Similarly the inclusive purities $P_f^p$ are generated. The reason that I need purities for two different targets proton and neutron, is to use all available asymmetries on proton, $^3$He and deuterium target. The details of this topic are described in Sec.5.4. As I have mentioned before, the unpolarized parameterization which is used for purity generation is CTEQ4LQ. Fig.5.1-Fig.5.7 (Fig.5.8-Fig.5.14) show the purities for proton (neutron) target generated with CTEQ4LQ, compared to the purities generated with GRV95.

These figures show the probability that one hadron $h$ is produced tagging a quark with flavor $f$ in each $x$ bin. For all hadrons, the high probabilities of hadron production from the valence quarks ($u, d$) are shown, in particular a high probability for $u$ quarks. Beside we can see which hadron is sensitive to the sea quarks: $\bar{u}, \bar{d}, s, \bar{s}$. For the case of the proton target, $\pi^+$ is sensitive to $\bar{d}$ because $\pi^+$ consists of $u$ and $\bar{d}$ (Fig.5.4). Similarly, $\pi^-$ is sensitive to $\bar{u}$ (Fig.5.5). Fig.5.6 and Fig.5.7 show that kaons are important to study of strange quarks. $K^+$, which contains $u$ and $\bar{s}$ quarks, has a high probability that they are produced from $\bar{s}$ quarks (Fig.5.6). Especially, $K^-$ is so sensitive to $\bar{u}$ and $s$ quarks than other hadrons (Fig.5.7) because it is made of two types sea quarks $\bar{u}$ and $s$.

For the case of the neutron target, the purities have a high value for $d$ quarks than that of the proton target. Fig.5.11 - Fig.5.14 show a high probability of $\bar{d}$ quarks. As the proton target, we can see that $K^-$ is so sensitive to $s$ quarks from Fig.5.14.

Comparing the purities generated with CTEQ4LQ and GRV, there are almost no difference for the $u, d, \bar{u}$ and $\bar{d}$ quarks in all the figures. On the other hand, the significant deviations are shown for the purities of $s$ and $\bar{s}$ quarks. These deviations come from the difference of $s$ and $\bar{s}$ quark distributions between CTEQ4LQ and
GRV. The CTEQ4LQ parameterization is a set with a low starting value of $Q^2$ and it has more information of the sea quarks than GRV. The deviations of the purities of sea quarks show these informations. The extraction of sea quark distributions is one of the main purposes, hence the purities generated with CTEQ4LQ are used in this study.

Figure 5.1: Inclusive ($\epsilon^+$) purities for proton target
5.2. EVALUATION OF PURITIES

Figure 5.2: $h^+$ purities for proton target

Figure 5.3: $h^-$ purities for proton target
Figure 5.4: $\pi^+$ purities for proton target

Figure 5.5: $\pi^-$ purities for proton target
5.2. EVALUATION OF PURITIES

Figure 5.6: $K^+$ purities for proton target

Figure 5.7: $K^-$ purities for proton target
Figure 5.8: Inclusive $(e^+) \text{ purities for neutron target}$
5.2. EVALUATION OF PURITIES

Figure 5.9: $h^+$ purities for neutron target

Figure 5.10: $h^-$ purities for neutron target
Figure 5.11: $\pi^+$ purities for neutron target

Figure 5.12: $\pi^-$ purities for neutron target
5.2. EVALUATION OF PURITIES

Figure 5.13: $K^+$ purities for neutron target

Figure 5.14: $K^-$ purities for neutron target
5.3 Assumptions on Sea Quark Polarizations

As I mentioned before, I need at least six asymmetries which must be independent of each other in order to extract quark distributions of all flavors: \( f = \{ u, d, \bar{u}, \bar{d}, s, \bar{s} \} \). At HERMES the asymmetries from 1995 \( \sim \) 1998 data taking are available, but the number of independent asymmetries is not sufficient to extract all the types of quark polarizations accurately. Therefore, two kinds of assumptions on the sea quark polarizations or distributions are used to decrease the number of extracted quark types and avoid this problem. The first type assumes to be the same polarizations for all sea quark flavors, which has already been used in other thesis [57] [49] [58] [59]. The second type, which is introduced first time in this analysis, is based on the Chiral Quark Soliton Model [6]. The details are shown in Sec 5.3.2.

In the following, I will describe these two kinds of assumptions and types of quark polarizations or distributions which are extracted in each case.

5.3.1 Assumption of Polarization Symmetric Sea

At first, the term of “quark polarization” and “quark distribution” have to be defined clearly. The quark polarization is defined that the ratio between the unpolarized quark distributions and the polarized quark distributions as \( \Delta q_f / q_f(x) \). The quark distribution expresses the \( x \) distribution of the polarized quarks as \( \Delta q_f(x) \).

The first assumption is that all sea quark polarizations are the same as follows:

\[
\frac{\Delta q_s(x)}{q_s(x)} = \frac{\Delta \bar{u}(x)}{\bar{u}(x)} = \frac{\Delta \bar{d}(x)}{\bar{d}(x)} = \frac{\Delta s(x)}{s(x)} = \frac{\Delta \bar{s}(x)}{\bar{s}(x)}. \tag{5.17}
\]

With this assumption, we can extract two types of quark polarizations. One is the extraction of the valence quark polarizations, named “valence decomposition” and another is the extraction of each quark flavor polarizations, named “flavor decomposition”. They are shown next.

Valence Decomposition

The spin asymmetry of hadron \( h \) can be decomposed into contributions from valence quarks with Eq.5.17:

\[
A_1^h(x) = P_1^h(x) \cdot \frac{\Delta u_v(x)}{u_v(x)} + P_2^h(x) \cdot \frac{\Delta d_v(x)}{d_v(x)} + P_3^h(x) \cdot \frac{\Delta s_v(x)}{s_v(x)}, \tag{5.18}
\]

where the coefficient \( C_i = (1 + R)/(1 + \gamma_i^2) \) which is multiplied to the purity matrix (Eq.5.5) is omitted for brevity. The purities in Eq.5.18 are defined as:

\[
P_1^h(x) = P_u^h(x) \cdot \frac{u(x) - \bar{u}(x)}{u(x)}, \tag{5.19}
\]

\[
P_2^h(x) = P_d^h(x) \cdot \frac{d(x) - \bar{d}(x)}{d(x)}, \tag{5.20}
\]

\[
P_3^h(x) = P_u^h(x) \cdot \frac{\bar{u}(x)}{u(x)} + P_d^h(x) + P_s^h(x) + P_\bar{s}^h(x) \cdot \frac{\bar{d}(x)}{d(x)} + P_\bar{d}^h(x)

+ P_s^h(x) + P_\bar{s}^h(x). \tag{5.21}
\]
5.3. ASSUMPTIONS ON SEA QUARK POLARIZATIONS

where $P^h_q(x) (q = u, \bar{u}, d, \bar{d}, s, \bar{s})$ are the original purities defined in Eq.5.3.

The extracted quark polarization vector $\mathbf{Q}$ is as follows:

$$\mathbf{Q} = \left( \frac{\Delta u_v(x)}{u_v(x)}, \frac{\Delta d_v(x)}{d_v(x)}, \frac{\Delta q_s(x)}{q_s(x)} \right).$$  \hspace{1cm} (5.22)

Therefore with these definition of purities of Eq.5.19-Eq.5.21, I obtained the valence quark polarizations: $\Delta u_v / u_v$ and $\Delta d_v / d_v$.

Flavor Decomposition

Similarly, the spin asymmetry can be decomposed into contributions from different quark flavors with Eq.5.17:

$$A^h_1(x) = P^h_1(x) \cdot \frac{\Delta u(x) + \Delta \bar{u}(x)}{u(x) + \bar{u}(x)} + P^h_2(x) \cdot \frac{\Delta d(x) + \Delta \bar{d}(x)}{d(x) + \bar{d}(x)} + P^h_3(x) \cdot \frac{\Delta q_s(x)}{q_s(x)},$$  \hspace{1cm} (5.23)

where the purities are defined as:

$$P^h_1(x) = P^h_u(x) \cdot \frac{u(x) + \bar{u}(x)}{u(x)},$$  \hspace{1cm} (5.25)

$$P^h_2(x) = P^h_d(x) \cdot \frac{d(x) + \bar{d}(x)}{d(x)},$$  \hspace{1cm} (5.26)

$$P^h_3(x) = -P^h_u(x) \cdot \frac{\bar{u}(x)}{u(x)} + P^h_\bar{u}(x) - P^h_d(x) \cdot \frac{\bar{d}(x)}{d(x)} + P^h_\bar{d}(x) + P^h_s(x).$$  \hspace{1cm} (5.27)

In this case, the extracted quark polarization vector $\mathbf{Q}$ is as follows:

$$\mathbf{Q} = \left( \frac{\Delta u(x) + \Delta \bar{u}(x)}{u(x) + \bar{u}(x)}, \frac{\Delta d(x) + \Delta \bar{d}(x)}{d(x) + \bar{d}(x)}, \frac{\Delta q_s(x)}{q_s(x)} \right).$$  \hspace{1cm} (5.28)

With these relations, the contributions of the quark flavors $u$ and $d$ to the spin of the nucleon can be extracted.

Together with the assumption on the sea quark polarization of Eq.5.17, three independent asymmetries are needed at least.

5.3.2 Assumption of Sea Quark from CQSM

Another new assumption is about the sea quark distributions, which is based on the Chiral Quark Soliton Model (CQSM). A simple relation between the polarized anti-quark distributions is:

$$\Delta \bar{u}(x) + \Delta \bar{d}(x) - 2 \Delta \bar{s}(x) = \frac{3F - D}{F + D} \left[ \Delta \bar{u}(x) - \Delta \bar{d}(x) \right],$$  \hspace{1cm} (5.29)

where $F$ and $D$ are the weak coupling constants in neutron and hyperon $\beta$-decays (Eq.2.69), and which is based on CQSM in the large-$N_c$ limit ($N_c$ is the number of colors) [6].
One of the main purposes of this study is the extraction of flavor the asymmetry of the light sea quark \( \Delta \bar{u} - \Delta \bar{d} \). Therefore it is not desirable to use the assumptions in Eq.5.17 to extract \( \Delta \bar{u} - \Delta \bar{d} \). I introduced the first part of Eq.5.30 as a new assumption to substitute Eq.5.17.

So the new assumptions are as follows:

\[
\Delta \bar{s}(x) = \frac{1}{2} \left( \Delta \bar{u}(x) + \Delta \bar{d}(x) - \frac{3F - D}{F + D} \left[ \Delta \bar{u}(x) - \Delta \bar{d}(x) \right] \right),
\]

\[
\frac{\Delta s(x)}{s(x)} = \frac{\Delta \bar{s}(x)}{\bar{s}(x)}. \tag{5.30}
\]

With these new assumptions, I can extract quark distributions directly, especially the flavor asymmetry of the light sea quarks.

**Flavor Asymmetry of Light Sea Quarks**

With Eq.5.30, the spin asymmetry can be expressed as follows:

\[
A_1(x) = P_{u}^{h}(x) \left[ \Delta u(x) + \Delta \bar{u}(x) \right] + P_{s}^{h}(x) \left[ \Delta d(x) + \Delta \bar{d}(x) \right] + P_{d}^{h}(x) \left[ \Delta \bar{u}(x) + \Delta \bar{d}(x) \right],
\]

where the purities are defined as:

\[
P_{u}^{h}(x) = X_{u}^{h}(x), \tag{5.31}
\]

\[
P_{s}^{h}(x) = X_{s}^{h}(x), \tag{5.32}
\]

\[
P_{d}^{h}(x) = X_{d}^{h}(x) + \frac{3}{2} X_{0}^{h}(x), \tag{5.33}
\]

\[
P_{4}^{h}(x) = X_{4}^{h}(x) - \frac{3F - D}{F + D} \left( X_{4}^{h}(x) + \frac{1}{2} X_{0}^{h}(x) - X_{8}^{h}(x) \right), \tag{5.34}
\]

\[
P_{6}^{h}(x) = X_{6}^{h}(x), \tag{5.35}
\]

where coefficients of \( X^{h}(x) \) are shown below:

\[
X_{u}^{h}(x) = \frac{P_{u}^{h}(x)}{u(x)}, \tag{5.36}
\]

\[
X_{d}^{h}(x) = \frac{P_{d}^{h}(x)}{d(x)}, \tag{5.37}
\]

\[
X_{0}^{h}(x) = \frac{1}{3} \left\{ \frac{P_{u}^{h}(x)}{u(x)} + \frac{P_{s}^{h}(x)}{\bar{u}(x)} - \frac{P_{d}^{h}(x)}{d(x)} + \frac{P_{d}^{h}(x)}{s(x)} - \frac{P_{s}^{h}(x)}{\bar{s}(x)} \right\}, \tag{5.38}
\]

\[
X_{3}^{h}(x) = \frac{1}{2} \left\{ \frac{P_{u}^{h}(x)}{u(x)} + \frac{P_{s}^{h}(x)}{\bar{u}(x)} - \frac{P_{d}^{h}(x)}{d(x)} - \frac{P_{d}^{h}(x)}{s(x)} \right\}, \tag{5.39}
\]

\[
X_{8}^{h}(x) = \frac{1}{6} \left\{ \frac{P_{u}^{h}(x)}{u(x)} + \frac{P_{s}^{h}(x)}{\bar{u}(x)} - \frac{P_{d}^{h}(x)}{d(x)} + \frac{P_{d}^{h}(x)}{s(x)} \right\} + 2 \left( \frac{P_{u}^{h}(x)}{s(x)} - \frac{P_{s}^{h}(x)}{\bar{s}(x)} \right). \tag{5.40}
\]

The ratio of \( F \) and \( D \) are obtained from [60]: \( F/D = 0.492 \pm 0.083 \). The \( P^{h}(x) \) in the definitions of \( X^{h}(x) \) above are the purities that are defined originally in Eq.5.3.
5.3. ASSUMPTIONS ON SEA QUARK POLARIZATIONS

In this analysis, the elements of the purity matrix $\mathcal{P}$ is a combination of the original purities (Eq.5.3) and the unpolarized PDF.

Here the extracted quark vector $Q$ is as follows:

$$Q = \left( \Delta u(x) + \Delta \bar{u}(x), \Delta d(x) + \Delta \bar{d}(x), \Delta \bar{u}(x) + \Delta \bar{d}(x), \Delta \bar{u}(x) - \Delta \bar{d}(x) \right)$$ (5.41)

One of the characteristic of this analysis with the new assumptions is that the quark distributions can be extracted directly, not only the polarization which is the ratio between the polarized and the unpolarized distributions. Additionally, using the results of the flavor asymmetry extraction Eq.5.41, it is possible to extract almost all flavor quark distributions: $\Delta u, \Delta d, \Delta \bar{u}, \Delta \bar{d}, \Delta s = \Delta \bar{s}$. This analysis is described in the next.

It has to be mentioned that due to the combination with the unpolarized PDF, which are $Q^2$ dependent, also the matrix $\mathcal{P}$ and the quark vector $Q$ become $Q^2$ dependent in this flavor asymmetry extraction. As the measured asymmetries are considered to be in the first order $Q^2$ independent, the PDF in the combination of the original purities (Eq.5.36 - Eq.5.40) at a fixed $Q^2$ of $Q^2=2.5$ GeV$^2$ has been used to get the $Q$ vector also at this fixed $Q^2$. The $Q^2$ dependence of the purities which are used to extract the flavor asymmetry (Eq.5.31 - Eq.5.34) are shown in Fig.5.15 and Fig.5.16. The purities are calculated at fixed value $Q^2$ of $Q^2=1.2, 2.5$ and 10 GeV$^2$. The solid circle shows the purities extracted using the mean value of $Q^2$ in each $x$ bin. At small $x$, there are no $Q^2$ dependence for all combination purities $P^h_1 \sim P^h_4$. The small deviation can be observed in the high $x$ bins, although these deviations don’t have influence on the extracted quark distributions in this HERMES precision.

**Single Quark Distribution**

With results of the above analysis, as has been suggested, I can obtain the quark distributions of almost all flavors, where the distribution of $\Delta s(x)$ and $\Delta \bar{s}(x)$ are defined to be the same.

First, I extract $\Delta u(x), \Delta \bar{u}(x), \Delta d(x), \Delta \bar{d}(x)$ from the results of Eq.5.41. With $\Delta \bar{u} + \Delta \bar{d}$ and $\Delta \bar{u} - \Delta \bar{d}$ of Eq.5.41 and the first relation of Eq.5.30 the strange quark distributions can be extracted: $\Delta \bar{s}(x) = \Delta s(x)$. With this method, the quark distributions of each other flavor can be obtained.
Figure 5.15: The $Q^2$ dependence of the purities for flavor asymmetry extraction for the proton target. These purities are combination of the original purities (Eq.5.3) and unpolarized PDF, which is used CTEQ4LQ.
Figure 5.16: The $Q^2$ dependence of the purities for flavor asymmetry extraction for the neutron target. These purities are combination of the original purities (Eq.5.3) and unpolarized PDF, which is used CTEQ4LQ.
5.4 Interpretation of Asymmetries on the $^3$He and D Targets

At HERMES, proton (p), helium3 ($^3$He) and deuteron (D) have been used as targets. The quark distributions are defined on the proton and on the neutron are related by isospin symmetry: $u_p = d_n$ and $d_p = d_n$. As a polarized neutron target is not available, $^3$He and D targets are used as neutron targets. To analyze the asymmetries on the $^3$He and the D targets, we have to translate these experimental asymmetries into the asymmetries on the neutron target. In this study, two methods are used to translate the measured asymmetries on the $^3$He and the D target, which are based on the same relation and are identical essentially. The number of input asymmetries which can be used in each method is different. The details are described in the following.

5.4.1 Asymmetry on the $^3$He Target

The $^3$He consists of two protons and one neutron. If the protons have anti-parallel spins so that the total proton spin is zero (S-state), which is the most probable configuration, the spin of $^3$He is entirely carried by the neutron. But the effective spin of the neutron in $^3$He seen in the measurement is diluted by the two protons because of the angular momentum in the $^3$He. So for a completely polarized $^3$He, the effective polarizations of the protons and the neutron have to be considered. The effective polarization of the proton is $p^p_{^3\text{He}} = -0.028$ and that of the neutron is $p^n_{^3\text{He}} = 0.86$ [61].

With the value of these effective polarizations, the measured asymmetry on the $^3$He target are expressed by the asymmetry on the proton and the neutron in every $x$ bin as:

$$A_{1;^3\text{He}}^h = f_{p;^3\text{He}}^h p_{^3\text{He}}^p A_{1;p}^h + f_{n;^3\text{He}}^h p_{^3\text{He}}^n A_{1;n}^h$$

(5.42)

where $f_{p;^3\text{He}}^h$ and $f_{n;^3\text{He}}^h$ are the dilution factors which give the probability of scattering on one of the two protons or the neutron. From this definition, $f_{p;^3\text{He}}^h + f_{n;^3\text{He}}^h = 1$ is obvious. These factors can be expressed as follows:

$$f_{p;^3\text{He}}^h = \frac{2\sigma_p^h}{\sigma_{^3\text{He}}^h} = 2 \frac{n_p^h}{n_{^3\text{He}}^h} \frac{\sigma_{^3\text{He}}}{\sigma_p},$$

$$f_{n;^3\text{He}}^h = 2 \frac{n_n^h}{n_{^3\text{He}}^h} \cdot \frac{F_2^p}{F_2^p + F_2^n},$$

(5.43)

where $\sigma_p^h$ and $\sigma_{^3\text{He}}^h$ are the integrated unpolarized cross sections of the hadron $h$ on the proton target and on the $^3$He target, respectively. $\sigma_p$ and $\sigma_{^3\text{He}}$ are the inclusive cross sections on each target. $n_p^h$ and $n_{^3\text{He}}^h$ are the multiplicities of hadron $h$ on each target. In Eq.5.43, these relations are used:

$$\sigma_{^3\text{He}}^h = 2\sigma_p^h + \sigma_n^h,$$

$$F_2^n = 2F_2^D - F_2^p,$$

(5.44)  (5.45)
where $\sigma_h^h$ is the integrated unpolarized cross section of the hadron $h$ on the neutron target.

With the relation of Eq.5.42 and the coefficients from Eq.5.43, we can translate the asymmetry on the $^3$He target to a combination of that on the proton and the neutron. In the followings, I will describe the practical way of using the $^3$He asymmetries as input asymmetries with Eq.5.42.

### 5.4.2 Asymmetry on the D Target

In case of the D target, we can consider the translation of the asymmetry on the D target similarly to the case of the $^3$He target. When the deuteron is in the S-state, the spins of the proton and the neutron are aligned parallel with no angular momentum. But the proton and the neutron polarization are diluted by the existence of D-state in which one of the nucleons is anti-aligned. This effective polarizations of the proton and the neutron are $p_D^p = p_D^n = 0.925\,^1$ [62] [63]. This effect has to be considered in the treatment of the asymmetry on the D target. Here $f_{p,D}^h + f_{n,D}^h = 1$ is also valid. In this case, the asymmetry is expressed by the asymmetry on the proton and the neutron as:

$$A_{L,D} = f_{p,D}^h p_D^h A_{L,p}^h + f_{n,D}^h p_D^n A_{L,n}^h,$$

and the dilution factor $f_{p,D}^h$ can be defined as:

$$f_{p,D}^h = \frac{\sigma_P^h}{\sigma_D^h} = \frac{n_D^h}{n_D^n} \cdot \frac{\sigma_P}{\sigma_D} = \frac{n_D^h}{n_D^n} \cdot \frac{F_2^p}{2F_2^D},$$

where $\sigma_D^h$ is the integrated unpolarized cross section of the hadron $h$ on the D target and $\sigma_D$ is the inclusive cross section. $n_D^h$ is the multiplicity of hadron $h$ on the D target. Similarly, Eq.5.45 and this relation are used:

$$\sigma_D^h = \sigma_P^h + \sigma_n^h,$$

which is the substitute for Eq.5.44.

These relations are used in the next section.

### 5.4.3 Method to use Asymmetry on $^3$He and D

In the previous sections, the expressions of the asymmetry on the $^3$He and the D target are shown. Here I describe how to apply these relations to extract the quark distributions.

When I use two types of asymmetries on the proton and the neutron target as input of Eq.5.5, the matrix equation is expanded as follows:

$$\begin{pmatrix} A_p \\ A_n \end{pmatrix} = C_1 \cdot \begin{pmatrix} P_p \\ P_n \end{pmatrix} \cdot Q,$$

$^1$They are almost equal the D-State probability, $\omega_D = 0.058 \pm 0.01$. 


where $A_p$ and $A_n$ are asymmetries on the proton and the neutron target respectively, and $\mathcal{P}_p$ and $\mathcal{P}_n$ are purity matrices on the respective target. Since the asymmetries on a free neutron target are not available experimentally, as mentioned before, asymmetries on the $^3\text{He}$ and the D target should be used, which are substitutes for that on the neutron with the relations of Eq.5.42 and Eq.5.46. Because the MC generator on the $^3\text{He}$ and the D target cannot reproduce the experimental data, the asymmetries have to be calculated for that on the neutron.

There are two methods to apply the above relations to Eq.5.49. The one is the method to use the mixing matrix which is shown in the following [57] [49] [58] [59]. Another is the new method, which is introduced in this study, making new definitions of the purity for the $^3\text{He}$ and the D target. Both methods are based on the same relation, however, the kind of asymmetries which can be used in each method is different. With the mixing matrix method, the inclusive, $h^+$ and $h^-$ asymmetries on all target (p, $^3\text{He}$ and D) can be used. On the other hand, with the new definition of purity, $\pi^+$, $\pi^-$, $K^+$ and $K^-$ asymmetries on the D target also can be included to extract quark distributions. Therefore, we can use up to 13 types of asymmetries with the new definition of purity. Although there is a condition to use the new definition of purity, which is described later. To study of quark distributions, in particular sea quarks, the pion and kaon asymmetries are considered to be important, because they contain sea quarks. Especially $K^-$ consists of two sea quarks, $\bar{n}$ and $s$, so it is seems to be sensitive to sea quark distributions. Therefore, we select the method with the new definition of purity to calculate moment finally, but both results are shown as follows. The details of both methods are described in the following sections.

The Mixing Matrix

From Eq.5.42 and Eq.5.46, the asymmetries on the neutron target are obtained from the measured asymmetries as follows:

- the case of the $^3\text{He}$ target
  
  \[
  A_{1,n}^h = \frac{1}{f_{n,^3\text{He}}^h p_{n,^3\text{He}}^h} \cdot \left( A_{1,^3\text{He}}^h - f_{p,^3\text{He}}^h p_{p,^3\text{He}}^h A_{1,p}^h \right) 
  = \frac{1}{p_{n,^3\text{He}}^h \left(1 - f_{p,^3\text{He}}^h \right)} \cdot \left( A_{1,^3\text{He}}^h - f_{p,^3\text{He}}^h p_{p,^3\text{He}}^h A_{1,p}^h \right). \quad (5.50)
  \]

- the case of the D target
  
  \[
  A_{1,n}^h = \frac{1}{f_{n,D}^h p_{D}^h} \cdot \left( A_{1,D}^h - f_{p,D}^h p_{p,D}^h A_{1,p}^h \right)
  = \frac{1}{p_{D}^h \left(1 - f_{p,D}^h \right)} \cdot \left( A_{1,D}^h - f_{p,D}^h p_{p,D}^h A_{1,p}^h \right). \quad (5.51)
  \]

If I use Eq.5.50 and Eq.5.51 in the matrix equation of Eq.5.49, the following expansion is achieved:

\[
\begin{pmatrix}
A_p \\
A_{^3\text{He}} \\
A_D
\end{pmatrix} = C_i \cdot \mathcal{N} \cdot 
\begin{pmatrix}
\mathcal{P}_p \\
\mathcal{P}_{^3\text{He}} \\
\mathcal{P}_D
\end{pmatrix} \cdot Q,
\quad (5.52)
\]
5.4. INTERPRETATION OF ASYMMETRIES ON THE $^3$He AND D TARGETS

where $A_{^3He}$ and $A_D$ are the asymmetries on the $^3$He and the D target, and $\mathcal{N}$ is called “the mixing matrix” which is introduced to calculate $A_{^3He}$ and $A_D$ to $A_n$ with Eq.5.50 and 5.51.

The elements of the mixing matrix $\mathcal{N}$ are shown below:

$$
\mathcal{N} = \begin{pmatrix}
1 & \cdots & 0 \\
\cdots & \ddots & \cdots \\
0 & \cdots & 1
\end{pmatrix}
$$

(5.53)

where

$$
a_{^3He}^h = f_{p,^3He}^p p_{^3He}^p A_{^3He}^h, \quad b_{^3He}^h = (1 - f_{p,^3He}) p_{^3He}^n A_{^3He}^h, \\
a_D^h = f_{p,D}^p P_D^p, \quad b_D^h = (1 - f_{p,D}) P_D^n
$$

(5.54)

(5.55)

where $h_i$ is the hadron type of asymmetries. With the mixing matrix $\mathcal{N}$, asymmetries on the $^3$He and the D target can be included in this analysis, although there is a condition to use this method. This method is allowed only if the corresponding asymmetries on the proton target exists for all used asymmetries on the helium 3 and deuteron target. Now the available data on the proton target are the inclusive and the charged hadron ($h^+, h^-$) asymmetries, and there is no data for identified pions, kaons and protons, since the RICH had not yet been installed during the data taking time in the proton target. However, since the $\pi, K$ and $p$ asymmetries are considered to be sensitive to the sea quark polarization, the inclusion of deuteron data, where $\pi, K$ and $p$ are identified with RICH, are quite important. Because these pion and kaon asymmetries cannot be used in this mixing matrix method, I considered another method to use these additional asymmetries. This new method is described in the next part.

The New Definition of Purity for the $^3$He and the D Target

Here a new definition of the purities on the $^3$He and the D target is introduced to use the purities for the proton and the neutron target. As the mixing matrix method, the Eq.5.42 and Eq.5.46 are based on this new method and expressed as follows:

- the case of the $^3$He target

$$
A_{^3He}^h = f_{p,^3He}^p P_{^3He}^p A_{^3He}^{h_1} + f_{n,^3He}^n P_{^3He}^n A_{^3He}^{h_2}
$$

$$
= a_{^3He}^h A_{^3He}^{h_1} + b_{^3He}^h A_{^3He}^{h_2}
$$

$$
= a_{^3He}^h \sum_f P_{f,p}^h \frac{\Delta q_f}{q_f} + b_{^3He}^h \sum_f P_{f,n}^h \frac{\Delta q_f}{q_f}
$$

$$
= \sum_f \left( a_{^3He}^h \cdot P_{f,p}^h + b_{^3He}^h \cdot P_{f,n}^h \right) \frac{\Delta q_f}{q_f}
$$

\[ P_{f,H}^{h} = \sum_{f} P_{f,H}^{h} \frac{\Delta q_{f}}{q_{f}} \tag{5.56} \]

where \( P_{f,P}^{h}, P_{f,n}^{h} \) and \( P_{f,^{3}He}^{h} \) are purities of particle \( h \) for the proton, the neutron and the helium3 target respectively. To obtain Eq.5.56, the purities for the helium3 target are defined as:

\[ P_{f,^{3}He}^{h} = (a_{^{3}He}^{h} \cdot P_{f,P}^{h} + b_{^{3}He}^{h} \cdot P_{f,n}^{h}). \tag{5.57} \]

- the case of the D target is similarly to the case of the \( ^{3}\text{He} \) target

\[ A_{1,D}^{h} = f_{P,D}^{h} P_{D}^{h} A_{1,p}^{h} + f_{n,D}^{h} P_{D}^{h} A_{1,n}^{h} \]

\[ = a_{D}^{h} A_{1,p}^{h} + b_{D}^{h} A_{1,n}^{h} \]

\[ = \sum_{f} (a_{D}^{h} \cdot P_{f,P}^{h} + b_{D}^{h} \cdot P_{f,n}^{h}) \frac{\Delta q_{f}}{q_{f}} \]

\[ = \sum_{f} P_{f,D}^{h} \frac{\Delta q_{f}}{q_{f}} \tag{5.58} \]

where \( P_{f,D}^{h} \) are the purities for the deuteron target, and they are defined as:

\[ P_{f,D}^{h} = (a_{D}^{h} \cdot P_{f,P}^{h} + b_{D}^{h} \cdot P_{f,n}^{h}). \tag{5.59} \]

With the Eq.5.56 and Eq.5.58, all available asymmetries including pion and kaon asymmetries can be used. We need to know the multiplicities of particle \( h \) on all targets to estimate the new redefined purities on the \( ^{3}\text{He} \) and the D target. However, since there are no pion and kaon data for the proton target, we assume on the multiplicities on the proton as follows:

\[ \frac{n_{p}^{h}}{n_{^{3}He}^{h}} = \frac{n_{p}^{h}}{n_{D}^{h}} = 1. \tag{5.60} \]

Using Eq.5.60, the coefficients are changed like below with Eq.5.43 and Eq.5.47:

\[ a_{^{3}He}^{h} = \frac{2F_{2}^{p}}{2F_{2}^{D} + F_{2}^{n}} P_{^{3}He}^{p}. \tag{5.61} \]

\[ b_{^{3}He}^{h} = \left( 1 - \frac{2F_{2}^{p}}{2F_{2}^{D} + F_{2}^{n}} \right) P_{^{3}He}^{n}. \tag{5.62} \]

\[ a_{D}^{h} = \frac{F_{2}^{p}}{2F_{2}^{D}} \cdot P_{D}^{p}. \tag{5.63} \]

\[ b_{D}^{h} = \left( 1 - \frac{F_{2}^{p}}{2F_{2}^{D}} \right) \cdot P_{D}^{n}. \tag{5.64} \]

These above coefficients don’t depend on the experimental variables, but they depend on the parameterizations of the structure functions: \( F_{2}^{p} \) and \( F_{2}^{D} \). There is
5.4. INTERPRETATION OF ASYMMETRIES ON THE $^3$He AND D TARGETS

no fundamental difference from the original method with the mixing matrix, but this method has more flexibility. In this study, the structure functions are obtained from [64].

Fig. 5.17 shows the multiplicity ratios for the proton and the deuteron targets. The ratios are close to one, which justifies the assumptions in Eq. 5.60. As a comparison of the result with this method and that with the mixing matrix method shows, the deviations of the ratios from one can be neglected. In addition, with the preliminary data of the pion and kaon multiplicities on the proton target, which are obtained from the unpolarized data [65], the results with the assumption Eq. 5.60 are compared with that using real multiplicities. The deviation between two results are not observed and they are almost identical in the present statistics.

With the assumption Eq. 5.60, the new definition of the purity for the $^3$He and the D target is available for all hadron asymmetries. Now the asymmetries of pion and kaon on the deuteron target which have been identified by the RICH can be used to extract quark polarizations. The results of the analysis performed with all available asymmetries on all the targets for both assumptions on the sea quark distributions (Eq. 5.17 and Eq. 5.30) are shown in Sec. 6.1.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$n_p^{h^+}/n_p^{h^+}$</th>
<th>$n_p^{h^-}/n_p^{h^-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-2}$</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>1</td>
<td>1.0</td>
<td>1.0</td>
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<tr>
<td>1.2</td>
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<td>1.6</td>
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</tr>
<tr>
<td>1.8</td>
<td>1.8</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Figure 5.17: The ratio of the multiplicities on the proton and the deuteron target. The top figure shows the ratio of $h^+$ multiplicities and the bottom that of $h^-$. The data on the proton target are from 1997 [66] and on the D target from 1998 [67].
5.5 The Set of Analyzed Asymmetries

In this section, all asymmetries measured by HERMES for different particles and different targets which have been used as an input in this analysis are presented.

5.5.1 1995 Asymmetries on the $^3$He Target

In 1995, only a threshold Cherenkov Counter with a limited momentum range for the hadron identification was operated. Therefore, inclusive, $h^+$ and $h^-$ asymmetries on the $^3$He target are available. They are shown in Fig.5.18. At HERMES, the binning in $x$ consists of nine bins and the definition of the binning in $x$ is shown in Appendix.C.1. Here, the values obtained from [49] have been used, which are given in Tab.C.2.

![Graph](image)

Figure 5.18: The inclusive and semi inclusive asymmetries on the $^3$He target from 1995. The charged hadron asymmetries are shown as the semi inclusive data. For each point the error bars represent the statistical uncertainty. The systematic uncertainty is not shown in this figure.

In 1995, the positron beam are used at HERA. The charged hadron asymmetries on the $^3$He target are measured for the first time. The inclusive asymmetry in Fig.5.18 shows slightly negative, and the hadron asymmetries are almost zero. The S-state, the two proton spin is zero, is dominated the wave function of the $^3$He nucleus, therefore the measured asymmetries are almost caused by the neutron.
5.5.2 1996 and 1997 Asymmetries on the Proton Target

In 1996 and 1997, the threshold Cherenkov Counter was also used, so only inclusive, $h^+$ and $h^-$ asymmetries are available on the proton target. Figs. 5.5.2 and Fig. 5.5.2 show the 1996 and 1997 asymmetries. These asymmetries have been also obtained from [49]. The numerical values of these asymmetries are shown in Tab. C.3 and Tab. C.4. Both asymmetries are on the proton target, although, they are treated separately because their systematic uncertainty are different.

![Graphs showing 1996 and 1997 asymmetries on the proton target](image)

Figure 5.19: The inclusive and semi inclusive asymmetries on the proton target from 1996 and 1997. The charged hadron asymmetries are shown as the semi inclusive data. For each point the error bars represent the statistical uncertainty. The systematic uncertainty is not shown in this figure.

The inclusive asymmetries on the proton target show positive value and increase as $x$ increase. In the second bin from the high $x$, the $h^+$ asymmetry in 1996 shows a strange behavior, however, it is not physical phenomenon. This behavior doesn’t be shown in 1997, the statistic in 1997 is more than that in 1996. The same is true for $h^-$ asymmetries in high $x$ bins. The negative hadron asymmetries show smaller positive value than that of corresponding positive hadron asymmetries. It gives a hint that $u$ quarks have positive and $d$ quarks have negative polarizations.
5.5.3 1998 Asymmetries on the D Target

In 1998, the RICH was started to detect the scattered hadrons. So inclusive, $h^+$, $h^-$, $\pi^+$, $\pi^-$, $K^+$ and $K^-$ asymmetries are available on the D target. Fig.5.5.3 shows these asymmetries. These asymmetries have been extracted in [50]. The numerical values of these asymmetries are shown in Tab.C.5 and Tab.C.6.

![Graphs showing the inclusive and semi inclusive asymmetries on the D target from 1998.](image)

Figure 5.20: The inclusive and semi inclusive asymmetries on the D target from 1998. The charged hadron, pion and kaon asymmetries are shown as the semi inclusive data. For each point the error bars represent the statistical uncertainty. The systematic uncertainty is not shown in this figure. The charged kaon asymmetries are measured first time at HERMES.

The inclusive asymmetry on the D target shows positive and slightly increasing as $x$ increase. The positive value of the inclusive asymmetry is smaller than that on the proton target. As the data in 1996, the positive hadron asymmetry shows a small value in the second high $x$ bin, although, it is caused by the statistics. The positive hadron, pion and kaon asymmetries show a slightly positive, and they with negative charge are near to zero. The pion asymmetries are similar to that of hadrons, it shows that the contribution of pions to hadrons are most dominate. The proton and anti-proton asymmetries on the D target can be measured with RICH, and they are already obtained. Though they are not used to extract the quark distributions, since the knowledge of the fragmentation function of proton and anti-proton are poor and the protons which are identified with RICH have a contamination by the proton decayed from the lambda particle. Additionally the proton and anti-proton is not so large statistics, hence we don’t use the proton and anti-proton asymmetries in this study.
5.5.4 The Sets of Input Asymmetries

The following shows the set of asymmetries which has been used for each method on the proton, the $^3\text{He}$ and the D target. In 1995-1997, HERA has been operated with positron and in 1998 with electron.

The Method with the Mixing Matrix

In the mixing matrix method, the asymmetries of particle $h$ on the proton target is necessary for that on the $^3\text{He}$ and the D target, respectively. As I mentioned before, because only the inclusive, $h^+$ and $h^-$ asymmetries on the proton target are available, I can use only the asymmetries of these particle for all targets. With the expanded matrix of Eq.5.52, the sets of input asymmetries are shown below:

\[ \mathbf{A}_p = \begin{pmatrix} A_{1,p}^+(96) \\ A_{1,p}^-(96) \\ A_{1,p}^+(96) \\ A_{1,p}^-(97) \\ A_{1,p}^+(97) \\ A_{1,p}^-(97) \end{pmatrix}, \quad \mathbf{A}_{^3\text{He}} = \begin{pmatrix} A_{1,^3\text{He}}^+(95) \\ A_{1,^3\text{He}}^-(95) \end{pmatrix}, \quad \mathbf{A}_D = \begin{pmatrix} A_{1,D}^+(98) \\ A_{1,D}^-(98) \end{pmatrix}, \quad (5.65) \]

where $A_{1,i}^j(k)$ is the asymmetry $A_1$ of particle $j$ on the target $i$ of the year $k$. The asymmetries of the year 1996 and 1997 are treated separately because the systematic errors coming from the target polarization are different.

The Method with the New Definition of Purity for the $^3\text{He}$ and D Target

With the new definition of the purities for the $^3\text{He}$ and the D targets, I can use more asymmetries as input because it is not necessary to have the asymmetries on the proton target of all the particle types. This gives the following set of asymmetries:

\[ \mathbf{A}_p = \begin{pmatrix} A_{1,p}^+(96) \\ A_{1,p}^-(96) \\ A_{1,p}^+(96) \\ A_{1,p}^-(97) \\ A_{1,p}^+(97) \\ A_{1,p}^-(97) \end{pmatrix}, \quad \mathbf{A}_{^3\text{He}} = \begin{pmatrix} A_{1,^3\text{He}}^+(95) \\ A_{1,^3\text{He}}^-(95) \end{pmatrix}, \quad \mathbf{A}_D = \begin{pmatrix} A_{1,D}^+(98) \\ A_{1,D}^-(98) \end{pmatrix}, \quad (5.66) \]

The sets of asymmetries on the proton and the $^3\text{He}$ targets are the same as the previous ones, but additionally I can use the asymmetries of $\pi^+$, $\pi^-$, $K^+$ and $K^-$ on the D target. Here I didn’t use the $h^+$ and $h^-$ asymmetries of the D target because there are large correlations between the asymmetries of the hadrons and that of pions and kaons.
Chapter 6

Analysis and Results

6.1 Results for the Polarized Quark Distributions

In this section, the results for quark polarizations $\Delta q/q$ and distributions $\Delta q$ are shown which have been obtained with the two methods described in Sec.5.4.3. There are four types of extracted quark polarizations and distributions (see Sec.5.3.1 and Sec.5.3.1). Before we extract quark polarization from experimental data, we checked the extraction method with MC data. This means that the quark polarizations extracted with this method should reproduce the polarized PDF which is put into MC as input data. First, the input data such as inclusive and charged hadron asymmetries is produced by using polarized PDF with MC. We use GRSV parameterization [68] as polarized PDF. Next, the purity matrix is produced by using CTEQ4LQ [53] or GRV [54] as unpolarized PDF. The quark polarization is determined from the simulated asymmetries and purities with this method. If the method is correct, the obtained results from MC data should reproduce the GRSV polarized PDF which is used as input of MC.

Additionally, the comparison between the mixing matrix method and the new definition of purity are shown, where inclusive and charged hadrons asymmetries on all targets are used as input (see Eq.5.65) for both method. Comparing the both results, the influence from the assumption of Eq.5.60 can be checked, and we can see that the assumption on the ratio of the multiplicities (Eq.5.60) does not have large influence on the extracted quark distributions. The results of this study is shown in the following.

Here Fig.6.1 shows the results of the flavor decomposition from MC. There are two types of quark polarizations, which are the result with the mixing matrix and that with the new definition of purity. In Fig.6.1, the solid line shows the GRSV parameterization [68] which is used for production of the MC data. We can see that the extracted quark distributions in Fig.6.1 reproduce the input GRSV parameterizations for the both results. From the comparison between the results with the mixing matrix method and the new definition of purity, it becomes clear that the deviations between these two methods are very small. Therefore the result with the assumption on the ratio of the multiplicities is consistent with that without this assumption, and almost can reflect the realistic quark distributions.

The figure of the extracted flavor asymmetry from the MC is shown in Fig.6.2.
Figure 6.1: The flavor decomposition from MC. The two results with the mixing matrix method (Sec.5.4.3) and the new definition of purity (Sec.5.4.3) are shown for comparison. \( q_s \) is defined as sea quarks: \( q_s = \{ \bar{u}, \bar{d}, s, \bar{s} \} \).

From this result, it is shown that the reproduction of the input GRSV PDF is achieved for the quark distribution extraction. For both the flavor decomposition and the flavor asymmetry extraction, there is no large difference between the results with the mixing matrix and the new definition of purity.

From Fig.6.1 and Fig.6.2, the methods which are described in Sec.5.3.1 and Sec.5.3.2 are reliable to obtain the quark distributions and polarizations. In addition, it is clear that the systematic uncertainties which come from the analysis method is small. In the next section, the extracted quark distributions from the experimental results are shown.
Figure 6.2: The flavor asymmetry extraction of light sea quarks from MC. The two results with the mixing matrix method (Sec.5.4.3) and the new definition of purity (Sec.5.4.3) are shown for comparison.
6.1.1 The Valence Decomposition

Fig.6.3 and Fig.6.4 are the results of the valence decomposition which are discussed in Sec.5.3.1. Fig.6.3 is obtained by solving the matrix equation Eq.5.5. Fig.6.4 shows the values times x and unpolarized PDF. The purities which were produced from CTEQ4LQ PDF are used as input for Eq.5.5, like for all the following results. The numerical values of these results are listed in Tab.C.14 and Tab.C.16.

![Graph](image_url)

Figure 6.3: The extracted quark polarizations in valence decomposition from experimental data. The solid circles show the results with the mixing matrix using inclusive and charged hadron asymmetries. The open squares is that with the new definition of purities using inclusive, charged hadron, charged pion and charged kaon asymmetries. In this decomposition, assumption of polarization symmetric sea is used.
Figure 6.4: The quark distributions obtained from the valence decomposition result at $Q^2 = 2.5$ GeV$^2$. The solid lines express the positivity limits which are unpolarized PDF, where CTEQ4LQ parameterizations are shown. The two results with the mixing matrix and the new definition of purity are obtained.

The solid circle shows the result with the mixing matrix to treat asymmetries on the $^3$He and D target (Sec.5.4.3), and the open square shows the result with the new definition of purity (Sec.5.4.3). In Fig.6.4, the solid curves show the positivity limit $|\Delta q/q| \leq C_t^{-1}$ obtained from Eq.5.2, the quark polarizations don’t exceed this limit. $q_o$ expresses the sea quarks which include $\{\bar{u}, \bar{d}, \bar{s}, \bar{\bar{s}}\}$.

Fig.6.3 shows that the polarization of $u_o$ quarks is positive over the full $x$ region, and there is a slight increase with increasing $x$. The polarization of $d_o$ quarks is
negative in the small $x$ region, but it is not obvious in the highest $x$ bins. In case of the sea quarks $q_s$, the polarization is flat and consistent with zero over the whole in $x$ range. Because of the decreasing contribution of the sea quarks in the high $x$ range, our data are not sensitive in this region and therefore the highest two $x$ bins have been omitted.

Comparing the two methods of treating the asymmetries on the $^3$He and the D target (solid circle and open square), there are only small differences, which are in the same order of magnitude as the difference seen in the MC simulation. It is no large effect on the valence quarks to use the asymmetries of pions and kaons instead of general hadron asymmetries. For the polarization of the valence quarks, the extracted values are stable and not sensitive to the extraction method.
6.1. RESULTS FOR THE POLARIZED QUARK DISTRIBUTIONS

6.1.2 The Flavor Decomposition

The results for the flavor decomposition are presented in Fig.6.5 and Fig.6.6, where the purities obtained from CTEQ4LQ [53] have been used. The numerical values of these results are listed in Tab.C.18 and Tab.C.20. We can obtain the contributions of each quark flavor in this decomposition especially that of \( u(= u + \bar{u}) \) and \( d(= d + \bar{d}) \) quarks. As same as the valence decomposition results, they show a comparison of the two extraction methods.

![Figure 6.5](image-url)

Figure 6.5: The extracted quark polarizations in flavor decomposition from experimental data. The solid circles show the results with the mixing matrix using inclusive and charged hadron asymmetries. The open squares is that with the new definition of purities using inclusive, charged hadron, charged pion and charged kaon asymmetries. In this decomposition, assumption of polarization symmetric sea is used.
Looking at the results of $u$ quark in Fig.6.5, the positive value of the $u$ quark polarization is shown and there is almost no difference between the two methods. The statistic errors are also small for both results, therefore it is clear that the polarization of the $u$ quark is positive and increasing as $x$ increases.

For the $d$ quark polarization, the two results are almost the same and there is not obvious difference. They show negative values in almost all $x$ bins, but it is not clear for the highest bin of $x$. From Fig.6.6, the $d$ quark distribution almost exceeds the positivity limit in the highest bin of $x$.

In this case, $q_s$ are defined as $q_s = \bar{u} + \bar{d} + s + \bar{s}$. About the sea quarks of $q_s$, it shows zero or a slightly negative value in the small $x$ region. The valence quark polarization can be extracted without depending the extraction method and the type of the used asymmetries. I also calculated the moments of $\Delta u + \Delta \bar{u}$ and $\Delta d + \Delta \bar{d}$. They are shown in Sec.6.2.

In the valence and the flavor decomposition, the contributions of the valence quarks to the nuclear spin can be extracted separately, but that of the sea quarks cannot be obtained clearly. Hence I analyzed the sea quark distributions with the new assumptions of Eq.5.30. The results are shown in the following sections.
Figure 6.6: The quark distributions obtained from the flavor decomposition result at $Q^2 = 2.5$ GeV$^2$. The solid lines express the positivity limits which are unpolarized PDF, where CTEQ4LQ parameterizations are shown. The two results with the mixing matrix and the new definition of purity are obtained.
6.1.3 Flavor Asymmetry of Light Sea Quarks

Here I show the results for the flavor asymmetry extraction of light sea quarks, which are the main subject of this study based on the new extraction method assuming CQSM, we obtained the polarized sea quark distribution including $\Delta \bar{u} - \Delta \bar{d}$ as shown in Fig.6.7 and Fig.6.8. It should be noticed that the results with the new definition of purity include pion and kaon asymmetries measured with RICH of the first time at HERMES, although that of the mixing matrix only use the inclusive and charged hadron asymmetries. These results are obtained by solving Eq.5.1, although in this case the purities are different between the previous analysis of the valence and the flavor decomposition. The purities which are used here include the unpolarized PDF from CTEQ4LQ (see Eq.5.31-Eq.5.40), therefore it is possible to extract the polarized quark distributions directly, not the quark polarizations. As I mentioned earlier, the quark distributions are extracted at $Q^2=2.5$ GeV$^2$.

When we look at the distributions of the $u$ and $d$ quarks, $\Delta u + \Delta \bar{u}$ and $\Delta d + \Delta \bar{d}$, of Fig.6.7, they are consistent in two extraction methods and there is almost no difference between the solid circle and the open square points. The distribution of $\Delta u + \Delta \bar{u}$ is positive values in all measured region, and which decreases with the $x$ increasing. While the result of the $\Delta d + \Delta \bar{d}$ is negative, which is opposite to the case of $u$ quark, and it increases with the $x$ increasing. From Fig.6.8, all valence and sea quark distributions, multiplied $x$, lie within the positivity limit.

However, there are some differences in the distributions of the sea quarks in Fig.6.7 between the two extraction methods (see the scale of Fig.6.7 carefully, which is large comparing the figures for the valence quarks). For the distribution of the $\Delta \bar{u} + \Delta \bar{d}$, there might be a deviation between the two methods in small $x$ region. The result with the mixing matrix, which is shown with the solid circle, tells us that this distribution is slightly negative in the middle $x$ region but it increases to zero in the low $x$ bin. The result with the new definition of purities which is indicated with the open square. However, it is almost negative in the middle $x$ and does not increase in lowest $x$ bin.

The flavor asymmetry of light sea quark $\Delta \bar{u} - \Delta \bar{d}$ can be extracted directly in this analysis. The distribution of $\Delta \bar{u} - \Delta \bar{d}$ has a slight positive value. Comparing the solid circle and the open square, the small difference is observed. In the lowest $x$ bin, the both result shows almost zero for the flavor asymmetry, although using the new definition of purity, the flavor asymmetry increases slightly in the lowest $x$ bin. Using the pion and kaon asymmetries which contain additional information about the sea quarks seems to have the biggest influence in the low $x$ region as expected. From the result with the new definition of purity (the open square), we can see that $\Delta \bar{u} - \Delta \bar{d}$ is almost zero, but slightly positive. However, statistic precision is not enough yet to really find a significant difference between the two approaches.

For both the distribution of $\Delta \bar{u} + \Delta \bar{d}$ and $\Delta \bar{u} - \Delta \bar{d}$, it might be possible that the asymmetries of pion and kaon cause these difference of the sea quark distribution between the two extraction methods in the low $x$ region. However, the more number of asymmetries and high statistics are necessary to obtain accurate sea distributions, in particular for pions and kaons.
Figure 6.7: The extracted quark distributions including flavor asymmetry of light sea quarks with the new assumptions based on CQSM (Sec.5.3.2) from experimental data, which is obtained at $Q^2 = 2.5$ GeV$^2$. The solid circles show the results with the mixing matrix using inclusive and charged hadron asymmetries. The open squares is that with the new definition of purities using inclusive, charged hadron, charged pion and charged kaon asymmetries.
Figure 6.8: The quark distributions times $x$ from the flavor asymmetry extraction, which is obtained at $Q^2 = 2.5$ GeV$^2$. The solid lines express the positivity limits which are unpolarized PDF, where CTEQ4LQ parameterizations are shown. The two results with the mixing matrix and the new definition of purity are obtained.
6.1.4 Single Quark Distribution

From the results of flavor asymmetry extraction, which is shown in the previous section (Sec.6.1.3), the quark distribution for each single quark flavor can be extracted (see Sec.5.3.2). The results are shown in Fig.6.9. For the valence quarks, $\Delta u$ and $\Delta d$, there is almost no difference between the two extraction methods. The moments of the both results are consistent, which is shown in Sec.6.2.

It makes clear that the absolute value of $\Delta u$ is positive in the measured $x$ region and that of $\Delta d$ is negative, in the same $x$ region. From Fig.6.10, the $d$ quark distribution times $x$ exceeds the positivity limit in the second highest $x$ bin. Therefore we can say that the second point of $\Delta d$ in Fig.6.9 is a little off the statistical fluctuation. The statistical deviation of the data in this $x$ bin can already be seen in the asymmetries on all targets. The systematic uncertainty needs to be evaluated in the next step. Even with this uncertainty, the distribution of $u$ and $d$ quarks can be extracted stable.

The distribution of $\Delta \bar{u}$ is almost zero over the measured $x$ region. The comparison of the two methods shows that there is almost no difference in any $x$ bin. The distributions of $\Delta \bar{d}$ and $\Delta \bar{s}$ seem to be a little different for the two extraction methods, especially there might be difference in low $x$ region, which shows that the difference in $\Delta \bar{u} - \Delta \bar{d}$ and $\Delta \bar{u} + \Delta \bar{d}$ between the methods is totally driven by the difference in $\Delta \bar{d}$. It is also clear that the contribution of the $\bar{d}$ quark dominates the flavor asymmetry, $\Delta \bar{u} - \Delta \bar{d}$, since $\Delta \bar{u}$ itself is almost zero in all the measured $x$ region.

There is another point to mention concerning the $\Delta \bar{s}$ quark distribution. In this study, it is assumed that the distributions of $\Delta s$ and $\Delta \bar{s}$ are equal. Hence the result for the $\Delta \bar{s}$ distribution also shows that of $\Delta s$ quarks. From the results with the new definition of purity (the open square), we can see that it is slightly negative in all $x$ bins (neglecting the three bins of high $x$). The moment of $\Delta s$ quark distribution shows this effect clearly, which is shown in Sec.6.2. Fig.6.10 shows that no data points, within their errors, lie outside the positivity limit.

There are some previous determinations of the distribution of $\Delta \bar{s}$ [69] before our analysis. Some results show slight negative values of $\Delta \bar{s}$ distribution and moments. In the present study, it also shows the slight negative value of the $x$ dependence and moments which are described in Sec.6.2. With the two methods, the mixing matrix method and the new definition of purity, I obtained the result consistent with the previous studies. The comparison between this results and the theoretical prediction are shown in the following section (Sec.6.2.3).
Figure 6.9: The extracted single distributions from the results of the flavor asymmetry, which is obtained at $Q^2 = 2.5$ GeV$^2$. The solid circles show the results with the mixing matrix using inclusive and charged hadron asymmetries. The open squares is that with the new definition of purities using inclusive, charged hadron, charged pion and charged kaon asymmetries.
Figure 6.10: The single quark distributions times $x$, which is obtained at $Q^2 = 2.5$ GeV$^2$. The solid lines express the positivity limits which are unpolarized PDF, where CTEQ4LQ parameterizations are shown. The two results with the mixing matrix and the new definition of purities are obtained.
6.1.5 Results for the different Purities

In the analysis described above, I used the purities which are produced from CTEQ4LQ [53] unpolarized PDF. However, it is important to check the stability of the result for extracted quark polarization. Therefore, I tested with the value of the purities which is produced by several different unpolarized PDF, such as CTEQ4LQ and GRV. Then quark polarization is determined by using two purities based on CTEQ and GRV. Fig.6.11 shows the result of the flavor decomposition with the mixing matrix by the CTEQ4LQ and the GRV purities. There is almost no deviation which comes from the difference of PDF because the used purities by CTEQ4LQ and GRV are consistent, especially for the valence quarks. This comparison tells us that the produced purities don’t give an significant contribution to the systematic uncertainty. This is also clear for Fig.6.12, which shows that the flavor asymmetry of light sea quarks does not depend on the purities used.

![Graph showing comparison of flavor decomposition using purities from CTEQ4LQ and GRV.](image)

Figure 6.11: Comparison of the flavor decomposition using purities from CTEQ4LQ and GRV. The solid circles are results using purities produced from CTEQ4LQ. The open circles are that from GRV.
Figure 6.12: Comparison of the flavor asymmetry extraction using purities from CTEQ4LQ and GRV. The solid circles are results using purities produced from CTEQ4LQ. The open circles are that from GRV.
6.2 Moments of Polarized Quark Distributions

In the previous sections, the results of the extraction of the quark polarizations and distributions are shown for each extraction type. To determine the moments of the extracted quark distributions, they are fitted with parameterized functions. The used functional forms are as follows:

\[ \Delta q(x) = N_q x^{\alpha_q} \cdot q(x), \]
\[ \Delta q_s(x) = N_{q_s} x^{\alpha_{q_s}} (1-x)^{\beta_{q_s}} \cdot q_s(x), \]

(6.1)

(6.2)

where \( q \) is the valence quark: \( u, u', d, d', \) and \( q_s \) is the sea quark: \( \bar{s}, \bar{d}, s, \bar{s} \). These forms are for the GRSV parameterizations [68]. \( N_q, \alpha_q \) and \( \beta_q \) are fit parameters. The CTEQ4 PDF at \( Q_0^2 = 2.5\) GeV \( ^2 \), which is the \( Q^2 \) mean value at HERMES, are used for the unpolarized PDF in Eq.6.1 and Eq.6.2.

The \( n \)-th moment is defined as:

\[ \Delta^{(n)} q_f(Q_0^2) = \int_0^1 x^{n-1} \Delta q_f(x, Q_0^2) dx. \]

(6.3)

Eq.6.3 is integral over the full \( x \) range, \( 0 < x < 1 \), though the experimental data are available only in a limited \( x \) region which is \( 0.023 < x < 0.6 \) at HERMES.

The practical calculation of the moments is done as follows. There is a little difference between the case of the valence and flavor decomposition (first case) and that of flavor asymmetry and each quark distribution extraction (second case), because in the first case, the results are obtained as the quark polarizations \( \Delta q_f/q_f \) in the second case, the results are the quark distributions \( \Delta q_f \).

The first moment in the measured \( x \) region is obtained as:

\[ \Delta q_f(Q_0^2) = \sum_i \left( \left. \frac{\Delta q_f}{q_f} \right|_{x_i} \int_{x_i}^{x_{i+1}} q_f(x, Q_0^2) dx \right), \]

(6.4)

where, in the first case, \( \left. \frac{\Delta q_f}{q_f} \right|_{i} \) is the results from the experiment, and in the second case, it is the value of the results divided by the unpolarized CTEQ4LQ PDF [53]. \( \left. \frac{\Delta q_f}{q_f} \right|_{i} \) is constant within each bin \( (x_i, x_{i+1}) \) and \( q_f(x) \) is obtained by the CTEQ4LQ parameterization. Outside the measured region, the fitted parameters of \( N_q, \alpha_q \) and \( \beta_q \) are used to determine the moments. In the high \( x \) region of \( 0.6 < x < 1 \), extrapolations of the fitted functions (Eq.6.1 or Eq.6.2) are required, and the moments are calculated as this integrals. In the low \( x \) region of \( 0 < x < 0.023 \), there is no clear prediction to determine polarized quark distributions. Therefore we applied a simple Regge parameterization [70] [71] of \( \Delta q_f(x) \propto x^{-\alpha} \) with \( \alpha = 0 \) fitted to the data for \( x < 0.075 \). The moments in the low \( x \) region are calculated as the integral of extrapolations with this Regge parameterization, which is constant in this study.

To obtain the moments for the sea quarks, there is a point that we should notice. As I have mentioned before, almost all sea quark polarizations and distributions exceed the positivity limit in the high \( x \) region, hence I neglected three points in the high \( x \) region to calculate the moment. Therefore the method of determination of the moment in high \( x \) region, which is described above, is only applied to the moment calculation for valence quarks. In this study, the results with the new definition of
purities are used to calculate the moments for extracted quarks because these results include informations from pion and kaon asymmetries.

The parameterized quark distributions and moments are shown in the next sections for the results with the new definition of purity, and the comparison with the theoretical prediction will be discussed. Only the fitted functions by Eq.6.1 are used to obtain the moments in the high \( x \) region, \( 0.6 < x \). Therefore the best fit for valence quarks by Eq.6.1 are shown in the following.

### 6.2.1 The Valence and Flavor Decomposition

The fit results for valence quarks extracted from the valence and flavor decomposition are shown in Fig.6.13, which are the results with the new definition of purity. \( \Delta u + \Delta \bar{u} \) and \( \Delta d + \Delta \bar{d} \) extracted from the flavor decomposition include the sea quarks, although they are fitted by Eq.6.1 because \( u \) and \( d \) quarks are dominant.

For the results with the mixing matrix, the moments can be seen in Appendix.C.5. And the numerical values of the first moment, extracted from the results of Fig.6.13 by using Eq.6.4, are shown in Tab.6.1 and Tab.6.2, respectively. The fitted parameters, \( N_q, \alpha_q \) and \( \beta_q \) which are used to calculate the moments are shown in Appendix.C.5 (Tab.C.29 and Tab.C.33).

![Figure 6.13: The polarized valence quark distributions which are fitted by the function of Eq.6.1. The left figures are shown the valence quarks extracted by the valence decomposition. The right figures are extracted by the flavor decompositions. The thick lines are the result of a best fit and the thin lines are obtained from GRSV parameterization corrected \( (1 + R) \).](image)

In Fig.6.13, the thick line shows the result of a fit with the functions of Eq.6.1 and the thin line represents the GRSV PDF [68] which is corrected by the term \( (1 + R) \) for the comparison with this results. The fit of the extracted \( u \) quark distributions, \( x\Delta u \) and \( x(\Delta u + \Delta \bar{u}) \) show good agreement with the corrected GRSV PDF, especially the \( \Delta u + \Delta \bar{u} \) is consistent in the full \( x \) region. For the \( d \) quark distributions, \( x(\Delta d + \Delta \bar{d}) \), which is extracted by the flavor decomposition, is consistent with GRSV, but \( x\Delta d \)
does not agree in the case of the valence decomposition. They are still compatible with GRSV within the statistic errors.

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-$x$</th>
<th>high-$x$</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u_v$</td>
<td>0.56 ± 0.09</td>
<td>0.04</td>
<td>0.02</td>
<td>0.62 ± 0.09</td>
</tr>
<tr>
<td>$\Delta d_v$</td>
<td>-0.14 ± 0.15</td>
<td>-0.02</td>
<td>0.00</td>
<td>-0.17 ± 0.15</td>
</tr>
<tr>
<td>$\Delta q_s$</td>
<td>-0.03 ± 0.05</td>
<td>-0.00</td>
<td>0.00</td>
<td>-0.04 ± 0.05</td>
</tr>
</tbody>
</table>

Table 6.1: The first moment for the valence decomposition

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-$x$</th>
<th>high-$x$</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u + \Delta \bar{u}$</td>
<td>0.52 ± 0.04</td>
<td>0.04</td>
<td>0.02</td>
<td>0.57 ± 0.04</td>
</tr>
<tr>
<td>$\Delta d + \Delta \bar{d}$</td>
<td>-0.21 ± 0.07</td>
<td>-0.03</td>
<td>0.00</td>
<td>-0.24 ± 0.07</td>
</tr>
<tr>
<td>$\Delta q_s$</td>
<td>-0.03 ± 0.05</td>
<td>-0.00</td>
<td>0.00</td>
<td>-0.04 ± 0.05</td>
</tr>
</tbody>
</table>

Table 6.2: The first moment for the flavor decomposition

The moments of the valence and the flavor decomposition results are calculated for three $x$ regions, one is the measured $x$ region $0.023 < x < 0.6$, others are the low $x$ and high $x$ region. The moments for each region are given by the method which is explained before. The total moment in the region of $0 < x < 1$, are obtained summing up the moments of the three regions. These total moments can be compared with the theoretical predictions and the result by SMC [72]. This is discussed in Sec.6.2.3.

6.2.2 Flavor Asymmetry and Single Quark Distribution

Here, the fit results for valence quarks from the flavor asymmetry extraction and the single quark distribution are shown in Fig.6.14, which have been achieved with the new definition of the purity. For the results with the mixing matrix, the numerical values of the first moment and the fitted parameters are shown Appendix.C.5. The fitted parameters for this results also are shown in Appendix.C.5 (Tab.C.37 and Tab.C.41).

The definitions of the curves in Fig.6.14 are the same as for the figure for the previous valence and the flavor decomposition. In this case, the fits to the $u$ quark distributions $\Delta u$ and $\Delta u + \Delta \bar{u}$ show consistency with the GRSV PDF in the full $x$ region. $\Delta d$ and $\Delta d + \Delta \bar{d}$ are in agreement with GRSV PDF within the statistic errors, similarly to the fit for the valence and flavor decomposition.

The calculated moments for the flavor asymmetry and all quark distributions are shown for three $x$ regions in Tab.6.3 and Tab.6.4. The total moments of $\Delta u + \Delta \bar{u}$ and $\Delta d + \Delta \bar{d}$ show a good consistency with the moments extracted with the flavor decomposition in Tab.6.2. These moments are compared with the theoretical predictions in the following section.
6.2. MOMENTS OF POLARIZED QUARK DISTRIBUTIONS

Figure 6.14: The polarized valence quark distributions which are fitted by the function of Eq. 6.1. The left figures show the valence quarks extracted by the flavor asymmetry extraction. The right figures are results for the single quark distribution. The thick lines are the result of a best fit and the thin lines are obtained from GRSV parameterization corrected by $(1 + R)$.

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-$x$</th>
<th>high-$x$</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u + \Delta \bar{u}$</td>
<td>$0.53 \pm 0.05$</td>
<td>$0.04$</td>
<td>$0.02$</td>
<td>$0.59 \pm 0.05$</td>
</tr>
<tr>
<td>$\Delta d + \Delta \bar{d}$</td>
<td>$-0.19 \pm 0.08$</td>
<td>$-0.03$</td>
<td>$0.00$</td>
<td>$-0.22 \pm 0.08$</td>
</tr>
<tr>
<td>$\Delta \bar{u} + \Delta \bar{d}$</td>
<td>$-0.09 \pm 0.08$</td>
<td>$-0.01$</td>
<td>$0.00$</td>
<td>$-0.10 \pm 0.08$</td>
</tr>
<tr>
<td>$\Delta \bar{u} - \Delta \bar{d}$</td>
<td>$0.09 \pm 0.09$</td>
<td>$0.02$</td>
<td>$0.00$</td>
<td>$0.11 \pm 0.09$</td>
</tr>
</tbody>
</table>

Table 6.3: The first moment for flavor asymmetry extraction

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-$x$</th>
<th>high-$x$</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u$</td>
<td>$0.54 \pm 0.12$</td>
<td>$0.04$</td>
<td>$0.02$</td>
<td>$0.60 \pm 0.12$</td>
</tr>
<tr>
<td>$\Delta d$</td>
<td>$-0.18 \pm 0.13$</td>
<td>$-0.03$</td>
<td>$0.00$</td>
<td>$-0.21 \pm 0.13$</td>
</tr>
<tr>
<td>$\Delta \bar{u}$</td>
<td>$0.00 \pm 0.06$</td>
<td>$0.00$</td>
<td>$0.00$</td>
<td>$0.00 \pm 0.06$</td>
</tr>
<tr>
<td>$\Delta \bar{d}$</td>
<td>$-0.09 \pm 0.06$</td>
<td>$-0.02$</td>
<td>$0.00$</td>
<td>$-0.10 \pm 0.06$</td>
</tr>
<tr>
<td>$\Delta s$</td>
<td>$-0.06 \pm 0.04$</td>
<td>$-0.01$</td>
<td>$0.00$</td>
<td>$-0.07 \pm 0.04$</td>
</tr>
</tbody>
</table>

Table 6.4: The first moment for single quark distribution
6.2.3 Comparison of Results with Predictions

The obtained moments of quark distributions can be compared to the theoretical predictions or other experimental results. Here the moment of this study which has been extracted with the new definition of purity is used.

There is one experimental measurement of polarized quark distributions and moments by SMC [72]. From this experiment, the results of the valence decomposition are available, with the assumption of SU(3) symmetric sea: 
\[ \Delta \bar{u}(x) = \Delta \bar{d}(x) = \Delta s(x) = \Delta \bar{s}(x). \]
It is possible to compare our results and the SMC results which are extrapolated to \( Q^2 = 2.5 \text{ GeV}^2 \) and integrated over the HERMES \( x \) range, \( 0.023 < x < 0.6 \). This comparison is shown in Tab.6.5, where the systematic errors are omitted. Only the statistical errors are shown. For this study in Tab.6.5, \( \Delta u_v \) and \( \Delta d_v \) are obtained by the results of the valence decomposition in Tab.6.1, in addition to \( \Delta \bar{u} \) and \( \Delta \bar{d} \) are the results from Tab.6.4. There is good agreement with the SMC results, especially for \( \Delta u_v \). There are some deviations in the sea quark distribution, \( \Delta \bar{u} \) and \( \Delta \bar{d} \), because the assumptions on the sea quarks which are used in each analysis are significantly different.

In the next, the comparison with the theoretical predictions is discussed. There are two predictions which can be compared with our results, which are achieved by the lattice QCD and the SU(3) analysis within errors. Tab.6.6 shows these comparisons. Here the values of the lattice QCD are calculated for \( Q^2 = 5 \text{ GeV}^2 \) [73]. The present results show a lower value than the prediction by the lattice QCD. The calculation of lattice QCD is performed in the quenched approximation.

<table>
<thead>
<tr>
<th></th>
<th>This study</th>
<th>SMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta u_v )</td>
<td>0.56 ± 0.09</td>
<td>0.59 ± 0.08</td>
</tr>
<tr>
<td>( \Delta d_v )</td>
<td>-0.14 ± 0.15</td>
<td>-0.33 ± 0.11</td>
</tr>
<tr>
<td>( \Delta \bar{u} )</td>
<td>0.00 ± 0.06</td>
<td>0.02 ± 0.03</td>
</tr>
<tr>
<td>( \Delta \bar{d} )</td>
<td>-0.09 ± 0.06</td>
<td>0.02 ± 0.03</td>
</tr>
</tbody>
</table>

Table 6.5: Comparison of the first moment between this study and the results by SMC. These values are for the HERMES measured region, \( 0.023 < x < 0.6 \), and obtained at \( Q^2 = 2.5 \text{ GeV}^2 \).

<table>
<thead>
<tr>
<th></th>
<th>This study</th>
<th>Lattice QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta u_v )</td>
<td>0.62 ± 0.09</td>
<td>0.84 ± 0.05</td>
</tr>
<tr>
<td>( \Delta d_v )</td>
<td>-0.17 ± 0.15</td>
<td>-0.25 ± 0.02</td>
</tr>
</tbody>
</table>

Table 6.6: Comparison of the first moment between this study and the prediction by the lattice QCD. The theoretical value by the lattice QCD is obtained at \( Q^2 = 5 \text{ GeV}^2 \).
6.2. MOMENTS OF POLARIZED QUARK DISTRIBUTIONS

<table>
<thead>
<tr>
<th></th>
<th>This study</th>
<th>SU(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\Delta u + \Delta \bar{u}$</td>
<td>$0.57 \pm 0.04$</td>
</tr>
<tr>
<td></td>
<td>$\Delta d + \Delta \bar{d}$</td>
<td>$-0.24 \pm 0.07$</td>
</tr>
<tr>
<td>2.</td>
<td>$\Delta u + \Delta \bar{u}$</td>
<td>$0.59 \pm 0.05$</td>
</tr>
<tr>
<td></td>
<td>$\Delta d + \Delta \bar{d}$</td>
<td>$-0.22 \pm 0.08$</td>
</tr>
<tr>
<td>3.</td>
<td>$\Delta u + \Delta \bar{u}$</td>
<td>$0.60 \pm 0.13$</td>
</tr>
<tr>
<td></td>
<td>$\Delta d + \Delta \bar{d}$</td>
<td>$-0.31 \pm 0.14$</td>
</tr>
<tr>
<td></td>
<td>$\Delta s + \Delta \bar{s}$</td>
<td>$-0.14 \pm 0.06$</td>
</tr>
</tbody>
</table>

Table 6.7: Comparison of the first moment between this study and the prediction by the SU(3) analysis. These theoretical values are obtained at $Q^2 = 2.5$ GeV$^2$.

For the moments of the flavor decomposition results, the prediction of the SU(3) analysis can be compared. In Tab.6.7, the extraction method is shown in the first column. The moments in row No.1 are obtained with the flavor decomposition results (Tab.6.2), and in row No.2 the results from the flavor asymmetry extraction are shown (Tab.6.3). In row No.3 the moments calculated from the extracted results of the single quark distribution (Tab.6.4) are shown. For row No.3, the moment of the strange quarks is listed, which is obtained from the $\Delta \bar{s}$ moment in Tab.6.4. In the extraction of single quark distributions, the assumption is used that the distributions of $\Delta s$ and $\Delta \bar{s}$ are equal, hence $\Delta s + \Delta \bar{s} = 2 \Delta \bar{s}$, which is used to calculate the value in Tab.6.7.

For row No.1 of the flavor decomposition results, both moments are consistent with the prediction of SU(3) analysis within errors. The same is true for the moments extracted by the flavor asymmetry extraction method. Additionally, the results obtained by single quark distribution in row No.3 are in good agreement with the SU(3) predictions. Especially the valence quarks show the best agreement, and the moment of the strange quarks shows a significantly negative value and this is consistent with the value of the prediction. From these, the sum of the quark spin contributions $\Delta \Sigma = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} = 0.23 \pm 0.11$ in the case of No.3.
6.3 Flavor Asymmetry Extraction with Another Method

In the previous section, the flavor asymmetry of light sea quarks with the purity matrix method is extracted. This flavor asymmetry was predicted by the CQSM. Besides it was extracted by another method from the asymmetries of HERMES [9] [10]. In the other method, we don’t need to use assumptions on the polarized quark distributions, and use the relations which is based on QPM only (Eq.2.78-Eq.2.80 and Eq.2.86). We estimated the flavor asymmetry of light sea quarks with this method using the asymmetries of HERMES, and compared it with the results obtained in the previous section. Additionally, the dependence of the flavor asymmetry on the fragmentation functions is studied. Only the parameterized fragmentation functions defined as the favored one and the unfavored one are available. Therefore the fragmentation functions of strange quarks $D_s$ are considered to be the same as the unfavored ones, but I estimated the contribution of $D_s$ on the flavor asymmetry with the theoretical model of the fragmentation functions in [74].

6.3.1 Another Method for Flavor Asymmetry Extraction

According to [9] [10], the asymmetry of hadron $h$ on the proton target is written by:

$$A_{1,h}^h(x) = \frac{1 + R(x)}{1 + \gamma^2} \cdot \frac{\sum_f e_f^2 \left\{ \Delta q_f(x) D_f^h + \Delta q_{\bar{f}}(x) D_{\bar{f}}^h \right\}}{\sum_f e_f^2 \left\{ q_f(x) D_f^h + q_{\bar{f}}(x) D_{\bar{f}}^h \right\}}, \quad (6.5)$$

where $f$ shows the quark flavor : $f = \{ u, d, s \}$. In Eq.6.5, the flavor of quarks and anti-quarks is shown separately. $D_f^h(x)$ is the integrated fragmentation function in the measured region, which is $D_f^h = \int_{x_{min}}^1 dz D_f^h(z)$.

The fragmentation functions are classified in the following:

$$(D^\pi)^+ \equiv D_u^\pi = D_d^\pi = D_s^\pi = D_s^{\bar{\pi}} = D_s^{\bar{\pi}} = D_s^{\bar{\pi}} = D_s^{\bar{\pi}} = D_s^{\bar{\pi}}, \quad (6.6)$$

$$(D^\pi)^- \equiv D_d^{\bar{\pi}} = D_s^{\bar{\pi}} = D_s^{\bar{\pi}} = D_s^{\bar{\pi}} = D_s^{\bar{\pi}} = D_s^{\bar{\pi}} = D_s^{\bar{\pi}} = D_s^{\bar{\pi}}, \quad (6.7)$$

$$(D^K)^+ \equiv D_u^K = D_d^K = D_s^K = D_s^{K^*} = D_s^{K^*} = D_s^{K^*} = D_s^{K^*} = D_s^{K^*}, \quad (6.8)$$

$$(D^K)^- \equiv D_d^{K^*} = D_s^{K^*} = D_s^{K^*} = D_s^{K^*} = D_s^{K^*} = D_s^{K^*} = D_s^{K^*} = D_s^{K^*}, \quad (6.9)$$

$$(D^p)^+ \equiv D_u^p = D_d^p = D_s^p = D_s^p = D_s^p = D_s^p = D_s^p = D_s^p, \quad (6.10)$$

$$(D^p)^- \equiv D_d^p = D_s^p = D_s^p = D_s^p = D_s^p = D_s^p = D_s^p = D_s^p, \quad (6.11)$$

where $(D^h)^+$ and $(D^h)^-$ are the favored and the unfavored fragmentation functions of hadron $h$, respectively. Returning to Eq.6.5, this relations can be rewritten as:

$$\sum_f e_f^2 \left\{ \Delta q_f(x) D_f^h + \Delta q_{\bar{f}}(x) D_{\bar{f}}^h \right\}$$

$$= \frac{1 + \gamma^2}{1 + R(x)} \cdot A_{1,h}^h(x) \left[ \sum_f e_f^2 \left\{ q_f(x) D_f^h + q_{\bar{f}}(x) D_{\bar{f}}^h \right\} \right]$$

$$\equiv \Delta N_p^h(x), \quad (6.12)$$
where $\Delta N^h_p(x)$ refers to the spin-dependent production processes of charged hadrons with the proton target. Similarly, we can get $\Delta N^h_n(x)$ for the neutron target. Forming a combination of $\Delta N^h_p(x)$ and $\Delta N^h_n(x)$, we obtain the following relation:

$$\Delta \bar{u}(x) - \Delta \bar{d}(x) = \frac{\Delta N^h_p(x) - \Delta N^h_n(x)}{2I_1} + \frac{\Delta N^h_p(x) - \Delta N^h_n(x)}{2I_2},$$

$$\Delta \bar{u}(x) - \Delta \bar{d}(x) = \frac{\Delta N^h_p(x) - \Delta N^h_n(x) + \Delta N^h_{p+}(x) - \Delta N^h_{n-}(x)}{2I_1}$$

where $I_1$ and $I_2$ are defined as the combination of the fragmentation functions:

$$I_1 = 3(D^+)^+ + 4(D^K)^+ + 3(D^0)^+ + 3(D^-)^+ + 2(D^K)^- + 3(D^0)^-$$

$$I_2 = 5(D^+)^+ + 4(D^K)^+ + 3(D^0)^+ - 5(D^-)^- - 4(D^K)^- - 3(D^0)^-.$$

From Eq.6.13, if we have $N^h_p(x)$ and $N^h_n(x)$ which can be obtained by experimental asymmetries on the respective target, the flavor asymmetry of light sea quarks can be extracted without the help of MC and the additional assumptions.

Experimentally, I have to use the asymmetry on the D target to obtain $\Delta N^h_n$, so it is necessary to change Eq.6.12 for the process on the D target. In case of the D target, Eq.6.12 reads as follows:

$$\sum_f e^2_f \left( A_f(x)D^h_f + A_f(x)D^b_f \right)$$

$$\equiv \Delta N^h_D(x),$$

where $\omega_D$ is the $D$-state probability, $\omega_D = 0.058$ [62] [63]. Using Eq.6.16, Eq.6.13 is rewritten as:

$$\Delta \bar{u}(x) - \Delta \bar{d}(x) = \frac{2 \Delta N^h_p(x) - \Delta N^h_{p+}(x) + 2 \Delta N^h_{p-}(x) - \Delta N^h_{n-}(x)}{2I_1}$$

$$\Delta \bar{u}(x) - \Delta \bar{d}(x) = \frac{2 \Delta N^h_p(x) - \Delta N^h_{p+}(x) - \Delta N^h_{p-}(x) + \Delta N^h_{n-}(x)}{2I_2}.$$
the unfavored fragmentation functions of pion, kaon and proton: \((D^{\pi})^+, (D^{\pi})^-, (D^{K})^+, (D^{K})^-, (D^{p})^+, (D^{p})^-\). The fragmentation functions of strange quarks are considered to be the same as the unfavored fragmentation functions. They are not obtained from experiment yet because the poor statistics of data. But the contributions of strange quark to the fragmentation functions should be treated separately from the unfavored fragmentation functions, like in Eq.2.80. Therefore the spectator model for the strange quark fragmentation function is used which will be described in the next section. In spite of using the strange quark fragmentation function, Eq.6.14 and Eq.6.15 are not changed.

The Fragmentation Functions in the Spectator Model

Because the fragmentation functions of strange quarks are not obtained from the experiment, I used that from the spectator model described in [74]. In this paper, the model parameters of pion fragmentation are obtained by fitting the pion data of EMC [21]. This way, the fragmentation functions of the strange quarks are obtained which are different from the unfavored fragmentation functions. In this model, the fragmentation functions of pion and kaon are estimated. These fragmentation functions are shown in Fig.6.3.2.

With these parameterized fragmentation functions, I studied the flavor asymmetry of light sea quarks from Eq.6.17. To study the contribution of the strange quarks to the flavor asymmetry, I used several sets of fragmentation functions as input. These sets are shown below:

1. The EMC parameterizations
   \((D^{\pi})^+, (D^{\pi})^-, (D^{K})^+, (D^{K})^-, (D^{p})^+, (D^{p})^-\) : [EMC parameterization]

2. The Spectator model parameterizations
   \((D^{p})^+, (D^{p})^-\) : [EMC parameterization]
   \((D^{\pi})^+, (D^{\pi})^-, (D^{K})^+, (D^{K})^-, (D^{p})^+, (D^{p})^-\) : [Spectator model]

3. The contribution of the strange quarks
   \((D^{\pi})^+, (D^{\pi})^-, (D^{K})^+, (D^{K})^-, (D^{p})^+, (D^{p})^-\) : [EMC parameterization]
   \((D^{\pi})^-, (D^{K})^+, (D^{K})^-, (D^{p})^-\) : [Spectator model]

4. The pion contributions
   \((D^{K})^+, (D^{K})^-, (D^{p})^+, (D^{p})^-\) : [EMC parameterization]
   \((D^{\pi})^+, (D^{\pi})^-\) : [Spectator model]

5. The kaon contributions
   \((D^{\pi})^+, (D^{\pi})^-, (D^{p})^+, (D^{p})^-\) : [EMC parameterization]
   \((D^{K})^+, (D^{K})^-\) : [Spectator model]

where \((D^{\pi})_s\) and \((D^{K})_s\) are the pion and kaon fragmentation functions of the strange quarks respectively. Comparing the set 1 and the set 2, we can estimate the fragmentation model dependence of the flavor asymmetry, and comparing the set 1 and the set 3, the dependence on the fragmentation functions of the strange quarks is obtained. Additionally, comparing the set 1 and set 4(5), we obtain the contributions of the pion(kaon) fragmentation functions. This analysis is useful to estimate the systematic error from the fragmentation model.
Using the asymmetries on the proton (’97) and the deuteron (’98) and the above sets of the fragmentation functions, the flavor asymmetry with Eq.6.17 is studied. The result are shown in the following and comparison with that from the purity method with the mixing matrix. It should be noted that the geometrical acceptance of HERMES detector, which could be z dependent, is not taken into account in the present analysis.

Figure 6.15: Fragmentation functions in the Spectator model. The pion and kaon fragmentation functions of strange quarks are extracted by fitting model parameters to EMC data.
6.3.3 Results and Comparison with the Purity Matrix Method

When I extracted the flavor asymmetry by solving Eq.6.17, I applied 5 sets of fragmentation functions, which have been already described. The extracted $\Delta \bar{u} - \Delta \bar{d}$ distributions are shown in Fig.6.16.

![Diagrams showing the extracted flavor asymmetry](image)

Figure 6.16: Flavor asymmetry of light sea quarks extracted from Eq.6.17. These figures are obtained by different sets of fragmentation functions.

In this analysis, only the asymmetries of the two years are used as input. Therefore the statistic errors became larger than the results of the purity methods. From Fig.6.16, in spite of using difference fragmentation functions, these figures don’t show any difference, but they are very consistent. From these comparisons, it can be concluded that the fragmentation functions don’t have large influence on the flavor asymmetry of light sea quarks. It also shows that the systematic errors which come from the uncertainty on the fragmentation functions is not dominant in the total systematic errors.

In Fig.6.17, the comparison of the result of the purity method with the mixing matrix based on the inclusive and hadron asymmetries and this direct extraction is shown. From Fig.6.17, the uncertainty from the MC and from the assumptions on the sea quarks is estimated in the purity method because the direct extraction method does not depend on the MC or any assumptions, only on fragmentation functions.

Fig.6.17 indicates that the flavor asymmetry extracted by the direct extraction shows good agreement with the result of the purity method. For the direct extraction
Figure 6.17: Comparison between the direct extraction method and the purity matrix method. The result of the direct extraction method with the EMC parameterization are shown. The upper figure is shown as flavor asymmetry of light sea quarks extracted two different method. The lower figure is the results times $x$.

results, the statistic errors are larger than the purity method. Because the direct extraction is less statistics than the purity matrix method. The bottom figure of Fig.6.17 shows $x$ times the flavor asymmetry. In two bins of the high $x$, they exceed the positivity limit for the result of the direct extraction method. Hence the neglect of the sea quark distribution in high $x$ bins can be justified by the other analysis.

Comparing the result of the purity method and that of the direct extraction method, the existence of a flavor asymmetry of light sea quarks $\Delta \bar{u} - \Delta \bar{d}$ is confirmed. It is indicated that it is positive and there are no large contributions to the systematic uncertainties by MC, the fragmentation functions and the assumptions on the sea quark distributions.
6.4 Simple Estimation of Systematic Errors

In this study, the accurate extraction of the systematic uncertainty has been skipped, although the simple estimations are studied. In this section, these estimations are summarized.

The systematic errors of the extracted quark polarizations and distributions can be separated into two parts, one comes from the uncertainty of the experiment and the other comes from that of the analysis method. The experimental uncertainty arises by different causes as follows:

- Beam polarization
- Target polarization
- Yield fluctuations
- Cross section ratio $R$
- Spin structure function $g_2$
- Smearing corrections
- Radiative corrections
- Error on the measured asymmetries

The analysis method uncertainty are given as follows:

- Unpolarized PDF
- Method of using asymmetries on the $^3$He and the D target
- Fragmentation Functions

In this study, we only discussed the analysis method uncertainty. For the uncertainty from the unpolarized PDF, we can estimate by comparing the purities generated with CTEQ4LQ and GRV. Additionally, the results for the different purities are shown in Fig.6.11 and Fig.6.12. From these comparisons, the uncertainty from PDF are not observed significantly. Some deviations can be seen in the comparison of the purities. However, the extracted quark distributions are almost identical.

The uncertainty from the interpretation of asymmetries on the $^3$He and the D target is already discussed. There is almost no influence on the systematic uncertainty from the different method, the mixing matrix method and the new definition of purity. We can see it comparing the both results with MC data in Fig.6.1 and Fig.6.2. The deviation cannot be observed clearly.

In Sec.6.3, the uncertainty from the fragmentation models are shown. The results with the different fragmentation functions are almost the same. We can see that the difference of fragmentation functions doesn’t have large contribution to the systematic uncertainty. This is shown in Fig.6.16. From these observations, it can be concluded that the systematic uncertainty from the analysis method are small.

The main contribution to the systematic uncertainty comes from the experimental uncertainty. Therefore the range of the systematic errors are shown from the previous analysis [69] for reference. In [69], the systematic errors are obtained as:

- for valence decomposition
  \[
  0.026 < \frac{\Delta u_v}{u_v} < 0.071, \quad (6.18)
  \]
  \[
  0.098 < \frac{\Delta d_v}{d_v} < 0.437. \quad (6.19)
  \]
6.4. SIMPLE ESTIMATION OF SYSTEMATIC ERRORS

- for flavor decomposition

\[ 0.007 < \frac{\Delta u + \Delta \bar{u}}{u + \bar{u}} < 0.039, \quad (6.20) \]

\[ 0.016 < \frac{\Delta d + \Delta \bar{d}}{d + \bar{d}} < 0.129. \quad (6.21) \]

For the present analysis, the systematic uncertainty can be expected to be not so different from these values.
Chapter 7

Conclusions and Summary

The main subject of this thesis is the extraction of the flavor asymmetry of light sea quarks, which is the asymmetry between the polarized $u$ and $d$ quark distribution: $\Delta u - \Delta d$. Another subject is to extract of the sea quark distribution.

The purity matrix method has been used to extract the quark distributions, which needs several spin asymmetries from the experiment as input. The quark distributions have been studied by this method, especially for the valence quarks. To study the sea quark distributions, more input asymmetries are necessary. Fortunately at HERMES, there are several inclusive asymmetry, $h^+$ and $h^-$ asymmetries on the proton and helium3 target available. Since 1998, a Ring Imaging Cherenkov Counter replaced the threshold Cherenkov Counter and more asymmetries became available. In the present study, these pion and kaon asymmetries on the deuteron target in 1998 have been used.

In this work, the efficiencies of the HERMES RICH have been studied first. This RICH has two Cherenkov radiators, silica aerogel and C$_4$F$_{10}$ gas. I estimated the effects coming from the radiators and detector geometries. The results for the efficiencies have been included in the Monte Carlo simulation, which is essential to study the experimental data accurately. It became possible to reproduce the experimental data by the updated MC much better than before. To achieve a more accurate analysis of the experimental data, it is necessary to understand the RICH performance in detail. This update is an important contribution to the extraction of the pion and kaon asymmetries in 1998.

Secondly, using the asymmetries available at HERMES, I extracted the quark distributions by the purity matrix method. To use this method, I introduced new assumptions to study the sea quark distributions, which are based on the Chiral Quark Soliton Model. Additionally, I proposed a new method to treat the asymmetries on the helium3 and the deuteron target. Therefore it became possible to include the pion and kaon asymmetries in this analysis. Finally, the valence and the sea quark distributions, the flavor asymmetry of the light sea quarks and their moments were obtained. It is confirmed that $\Delta u$ and $\Delta u_v$ are positive and $\Delta d$ shows negative values. For the flavor asymmetry, $\Delta u - \Delta d$ is slightly positive and its moment has also a slight positive value. $\Delta u$ is almost zero, and both $\Delta d$ and $\Delta s (= \Delta s)$ show slight negative values. The moments of these results are almost consistent with the theoretical prediction and the other experimental results. However, more statistics is needed to really clarify the sea quark distributions. The present
method to extract the flavor asymmetry is a good tool for the study of the nucleon spin content and gave a hint to several quark distributions. Both results from the mixing matrix method and from the new definition of purities are consistent. The extracted results don’t depend on the extraction methods. It has been confirmed that the flavor asymmetry is not sensitive to the uncertainties which come from the parameterizations of the unpolarized quark distributions and the fragmentation functions. The main conclusions of this thesis are summarized below:

1. An Analysis method to extract flavor asymmetry of light sea quarks $\Delta \bar{u} - \Delta \bar{d}$ is proposed and examined. The single quark distributions $\Delta u$, $\Delta d$, $\Delta \bar{u}$, $\Delta \bar{d}$, $\Delta s = \Delta \bar{s}$ are extracted at the same time.

2. Flavor asymmetry of light sea quarks is extracted, which shows almost zero or slightly positive. Their first moments are also slight positive values.

3. Single quark distributions are obtained. Especially the $x$ dependence of $\Delta \bar{u}$, $\Delta \bar{d}$, and $\Delta s = \Delta \bar{s}$ are extracted. $\Delta \bar{u}$ is almost zero while $\Delta \bar{d}$ and $\Delta s = \Delta \bar{s}$ and their moments are slightly negative.

4. It is found that data of the pion and kaon asymmetries are important to study of the polarized sea quark distribution.

5. There are small systematic uncertainties which come from the unpolarized PDF, the extraction methods and the fragmentation functions.

As an outlook to future the following comments can be made. Because the data has been taken with RICH since 1998, asymmetries with more statistics will become available in the near future, especially for the pion and kaon asymmetries. These pion and kaon asymmetries are sensitive to the sea quarks. Therefore it is expected that an accurate study of the sea quark distributions becomes possible. Also the flavor asymmetry of light sea quarks will be determined more precisely. If enough different types of asymmetries are obtained, the single quark distributions can be extracted without any assumptions on the quark distributions. These studies based on the experimental data collected with the RICH will give more accurate results to the polarized quark distributions and have a large contribution to solve the spin problem of the nucleon.
Appendix A

Detail of MC Development

The details of MC development is described in following. To normalize the experimental data, the photon collection efficiencies are included as the position dependence of the mirror reflectivity. The analysis method to obtain the mirror reflectivity for each segment is described in Sec. 4.3.1. The $L_{gas}$ is used for normalization of the number of fired PMTs. The definition and distribution of $L_{gas}$ are shown in Fig. A.1.

Figure A.1: Definition and Distribution of $L_{gas}$ for MC data. The left figure shows the definition of $L_{gas}$, which is determined as the distance between the position that the particle track goes into gas and the hit point of the track on the mirror. The right figure is the distribution of $L_{gas}$. The mean value of $L_{gas}$ is 73, which is used for the normalization of the particle track length.
In Fig. A.2 the ratio between $N_{gas}$ (EXP) and $N_{gas}$ (MC) is shown, which is used to calculate the mirror reflectivity for each segment as the photon collection efficiency.

Figure A.2: The ratio between $N_{gas}$ (EXP) and $N_{gas}$ (MC). The particles are $\pi^+$ and $\pi^-$ with momentum of $4.5 < p < 8.5$ (GeV/c). This ratio gives the informations of efficiencies from the unknown factors, for example, the position dependence of the mirror reflectivity, the mirror alignment, the slope of the incident photon, the effect from funnels, the PMT efficiencies.
The mean value of the ratio which is shown in Fig.A.2 for each mirror segment calculated as Fig.A.3. These values are obtained in order to reproduce the experimental data. They can be considered as the photon collection efficiencies. These values are included in MC as the mirror reflectivities for each segment. The mirror reflectivities and alignments, the funnels and other unknown factors can contribute to these values.

Figure A.3: The position dependence of upper and lower mirror reflectivity. These values include not only the mirror reflectivities but also all other efficiencies from unknown factors.
The comparison between the experimental data and MC data after modification is shown in Sec. 4.3.2. We can see that the new MC gives a good reproduction comparing before modification. The latest comparison of the Cherenkov angle between the experimental data and the MC give a better result as shown in Fig.A.4.

![Figures](image.png)

Figure A.4: Recent Comparison of the Cherenkov angles between the experimental data (points) and the MC data (histogram). They shows the normalized distributions of reconstructed angles for single, low background electrons \((p > 5)\). The left figure shows a comparison of the Cherenkov angle from aerogel, and the right figure shows that from gas.

Fig.A.4 shows a good agreement in the central part of the distribution. The tails of the figure for aerogel angles are due to several effects from the background. The causes of backgrounds are discussed in [44] and are including in MC.
Appendix B

Detail of Analysis

B.1 Covariance Matrix $\mathcal{V}_A$

As the number of detected particles is Poisson distributed, the variance of the inclusive leptons $\sigma^2_{N_{e^+}}$ is calculated as:

$$\sigma^2_{N_{e^+}} = \langle N^2_{e^+} \rangle - \langle N_{e^+} \rangle^2 = N^2_{e^+}, \quad (B.1)$$

where $N_{e^+}$ is the number of inclusive leptons. For the semi-inclusive measurement, the variance of the semi-inclusive hadrons $\sigma^2_{N_h}$ is obtained as:

$$\sigma^2_{N_h} = \langle N^2_h \rangle - \langle N_h \rangle^2 = N^2_{e^+} \langle (n^h)^2 \rangle = N_h \frac{\langle (n^h)^2 \rangle}{\langle n^h \rangle}, \quad (B.2)$$

where $n^h_i$ is the number of hadrons $h$ per event and $N_h$ is the number of detected hadrons.

Using the variance, the statistical covariances $\text{Cov}(N_{h_1}, N_{h_2})$ for the particle count rates are calculated as:

$$\text{Cov}(N_{h_1}, N_{h_2}) \equiv (\mathcal{V}_N)_{12} = \langle N_{h_1} N_{h_2} \rangle - \langle N_{h_1} \rangle \langle N_{h_2} \rangle = N_{e^+} \langle n^{h_1} n^{h_2} \rangle, \quad (B.3)$$

$$\text{Cov}(N_{e^+}, N_h) \equiv (\mathcal{V}_N)_{eh} = N_{e^+} n^h, \quad (B.5)$$

where $n^h_i=1$ is used. The correlation coefficients of the particle count rate are:

$$\text{Cor}(N_{h_1}, N_{h_2}) \equiv \rho(N_{h_1}, N_{h_2}) = \frac{(\mathcal{V}_N)_{12}}{\sigma_{N_{h_1}} \sigma_{N_{h_2}}} \quad (B.6)$$

$$= \frac{\langle n^{h_1} n^{h_2} \rangle}{\sqrt{\langle (n^{h_1})^2 \rangle \langle (n^{h_2})^2 \rangle}}, \quad (B.7)$$

$$\text{Cor}(N_{e^+}, N_h) \equiv \rho(N_{e^+}, N_h) = \frac{n^h}{\sqrt{\langle (n^h)^2 \rangle}}. \quad (B.8)$$

The covariances $(\mathcal{V}_A)_{ij}$ of the asymmetries $A_i^{h_a}$ and $A_i^{h_b}$ are obtained from the covariances $(\mathcal{V}_N)_{ij}$ of the particle count rates as follows:
\[(V_A)_{ij} = \sum_{\mu,\nu} \frac{\partial A^h_i}{\partial N^i_{h\mu}} \frac{\partial A^h_j}{\partial N^j_{h\nu}} (V_N)_{\mu\nu}
\]
\[= \frac{\partial A^h_i}{\partial N^i_{h+}} \frac{\partial A^h_j}{\partial N^j_{h+}} (V_N)_{ij} + \frac{\partial A^h_i}{\partial N^i_{h-}} \frac{\partial A^h_j}{\partial N^j_{h-}} (V_N)_{ij}
\]
\[\simeq \frac{1}{2} \left[ \frac{\partial A^h_i}{\partial N^i_{h+}} \frac{\partial A^h_j}{\partial N^j_{h+}} + \frac{\partial A^h_i}{\partial N^i_{h-}} \frac{\partial A^h_j}{\partial N^j_{h-}} \right] (V_N)_{ij}
\]
\[\simeq \frac{\partial A^h_i}{\partial N^i_{h+}} \frac{\partial A^h_j}{\partial N^j_{h+}} (V_N)_{ij}, \quad (B.9)
\]

where Greek indices run over both spin states. In Eq.B.9, two facts are used, one is that the asymmetry for one particle type only depends on the count rates for this particle in both spin states which are denoted as \(N^h_{+}\) and \(N^h_{-}\), another is that the two spin states are statistically uncorrelated such as \((V_N)_{i+i}\). Furthermore two approximations are used which are \(N^h_{\pm} \approx \frac{1}{2} N^h\) and \(\left| \frac{\partial A^h_i}{\partial N^i_{h+}} \right| \approx \left| \frac{\partial A^h_i}{\partial N^i_{h-}} \right|\). With these assumptions, the errors on the asymmetries \(\sigma_{A^h_i}\) becomes:

\[\sigma^2_{A^h_i} = \left( \frac{\partial A^h_i}{\partial N^i_{h \pm}} \right)^2 \sigma^2_{N^h_i}, \quad (B.10)\]

where \(\sigma^2_{N^h_i}\) is the variance of the number of detected hadrons \(N^h_i\).

From Eq.B.10, the covariances of the asymmetries \((V_A)_{ij}\) is written as:

\[(V_A)_{ij} \simeq \frac{\sigma_{A^h_i}}{\sigma_{N^h_i}} \frac{\sigma_{A^h_j}}{\sigma_{N^h_j}} (V_N)_{ij}. \quad (B.11)\]

From the definition of correlations of the different types of asymmetries, \(\rho(A^h_i, A^h_j) \equiv (V_A)_{ij}/(\sigma_{A^h_i} \sigma_{A^h_j})\) and Eq.B.11, the relation that the correlations of asymmetries and particle count rates are equal:

\[\rho(A^h_i, A^h_j) = \rho(N^h_i, N^h_j), \quad (B.12)\]

has been deduced.

Therefore in the practical analysis, the correlations of particle count rates are substitutes for that of the asymmetries. The correlations of particle count rates which are used in this study are shown in the next Appendix.C.3, where the count rates are given for inclusive and semi-inclusive DIS events.
B.2 Singular Value Decomposition

“Singular Value Decomposition” (SVD) is the method for solving the matrix equation (Eq.5.5). This method can obtain the solution which minimizes $\chi^2$ in Eq.5.9. However the Cholesky decomposition is necessary to apply the SVD method. The Cholesky decomposition is expressed as:

$$V_A^{-1} = L^T L,$$

where $L$ is a triangular matrix. This decomposition can be applied because the covariance matrix $V_A$ is symmetric and positive. With $L$, $A'$ and $P'$ defined from $A$ and $P$ as follows:

$$A' = LA,$$
$$P' = LP.$$

With the relations Eq.B.14, $\chi^2$ is expressed as:

$$\chi^2 = (A' - C_i \cdot P' \cdot Q)^T (A' - C_i \cdot P' \cdot Q).$$

The SVD method is applied to Eq.B.15. From a mathematical theorem, any $m \times n$ matrix $P$ can be written as a product of an $m \times n$ column-orthogonal matrix $U$, an $m \times n$ diagonal matrix $W$ which has positive or zero elements. Therefore $P$ is expressed with a $n \times n$ orthogonal matrix $Z$ as:

$$P_{ij} = \sum_{k=1}^{n} U_{ik} W_{kj} Z_{jk},$$

where the diagonal of $W$ is denoted by the vector $W$. From Eq.B.16, the solution can be obtained as:

$$Q = Z \cdot \left[ \text{diag}(1/w_i) \right] \cdot (U^T \cdot A),$$

which minimizes $\chi^2$. This method can be applied to the case in which there are more equations than unknowns without modification.

The the covariances between the extracted quark distributions and the asymmetries are obtained as:

$$V_Q = C_i^{-1} \cdot (P^{-1} \cdot V_A \cdot (P^T)^{-1})$$
$$= C_i^{-1} \cdot (P^T \cdot V_A^{-1} \cdot P)^{-1}$$
$$= C_i^{-1} \cdot (P^{-1} \cdot (P^T)^{-1})^T,$$

which is obtained by Eq.B.13 and the following relation:

$$V_Mx = M \cdot V_x \cdot M^T,$$

where $V_Mx(V_x)$ is a covariance matrix of $Mx(x)$. And from Eq.B.17,

$$(V_Q)_{ij} = \sum_{k=1}^{n} \frac{Z_{ik}^I Z_{jk}^I}{W_k^I}. $$
Appendix C

Table of Results

C.1 Definition of the Binning in $x$

<table>
<thead>
<tr>
<th>Bin number</th>
<th>Range in $x$</th>
<th>$\langle x \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.023 $\sim$ 0.040</td>
<td>0.033</td>
</tr>
<tr>
<td>2</td>
<td>0.040 $\sim$ 0.055</td>
<td>0.047</td>
</tr>
<tr>
<td>3</td>
<td>0.055 $\sim$ 0.075</td>
<td>0.065</td>
</tr>
<tr>
<td>4</td>
<td>0.075 $\sim$ 0.100</td>
<td>0.087</td>
</tr>
<tr>
<td>5</td>
<td>0.100 $\sim$ 0.140</td>
<td>0.119</td>
</tr>
<tr>
<td>6</td>
<td>0.140 $\sim$ 0.200</td>
<td>0.168</td>
</tr>
<tr>
<td>7</td>
<td>0.200 $\sim$ 0.300</td>
<td>0.244</td>
</tr>
<tr>
<td>8</td>
<td>0.300 $\sim$ 0.400</td>
<td>0.342</td>
</tr>
<tr>
<td>9</td>
<td>0.400 $\sim$ 0.600</td>
<td>0.464</td>
</tr>
</tbody>
</table>

Table C.1: $x$ binning definition. The first column shows the number of each bin, their limits are defined in the second column. The mean value $\langle x \rangle$ of $x$ for the inclusive asymmetry on the D target is shown in the third column.
## C.2 Input Asymmetries

### C.2.1 1995 Asymmetries on the $^3$He Target

<table>
<thead>
<tr>
<th>Bin</th>
<th>$x$</th>
<th>$Q^2$</th>
<th>$A_1^{+} \pm $ stat. $\pm$ sys.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.033</td>
<td>1.220</td>
<td>-0.036 $\pm$ 0.013 $\pm$ 0.004</td>
</tr>
<tr>
<td>2</td>
<td>0.048</td>
<td>1.470</td>
<td>-0.009 $\pm$ 0.014 $\pm$ 0.004</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.730</td>
<td>-0.028 $\pm$ 0.015 $\pm$ 0.004</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.000</td>
<td>-0.025 $\pm$ 0.018 $\pm$ 0.004</td>
</tr>
<tr>
<td>5</td>
<td>0.118</td>
<td>2.310</td>
<td>-0.034 $\pm$ 0.019 $\pm$ 0.004</td>
</tr>
<tr>
<td>6</td>
<td>0.166</td>
<td>2.660</td>
<td>-0.038 $\pm$ 0.023 $\pm$ 0.005</td>
</tr>
<tr>
<td>7</td>
<td>0.239</td>
<td>3.070</td>
<td>-0.006 $\pm$ 0.030 $\pm$ 0.008</td>
</tr>
<tr>
<td>8</td>
<td>0.338</td>
<td>3.790</td>
<td>0.078 $\pm$ 0.051 $\pm$ 0.012</td>
</tr>
<tr>
<td>9</td>
<td>0.450</td>
<td>5.250</td>
<td>-0.024 $\pm$ 0.074 $\pm$ 0.019</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$x$</th>
<th>$Q^2$</th>
<th>$A_1^{-} \pm $ stat. $\pm$ sys.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.033</td>
<td>1.210</td>
<td>-0.051 $\pm$ 0.032 $\pm$ 0.006</td>
</tr>
<tr>
<td>2</td>
<td>0.048</td>
<td>1.450</td>
<td>-0.011 $\pm$ 0.033 $\pm$ 0.006</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.750</td>
<td>-0.030 $\pm$ 0.034 $\pm$ 0.006</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.140</td>
<td>-0.035 $\pm$ 0.039 $\pm$ 0.006</td>
</tr>
<tr>
<td>5</td>
<td>0.118</td>
<td>2.710</td>
<td>-0.024 $\pm$ 0.041 $\pm$ 0.007</td>
</tr>
<tr>
<td>6</td>
<td>0.165</td>
<td>3.680</td>
<td>0.006 $\pm$ 0.049 $\pm$ 0.008</td>
</tr>
<tr>
<td>7</td>
<td>0.238</td>
<td>5.180</td>
<td>-0.155 $\pm$ 0.067 $\pm$ 0.009</td>
</tr>
<tr>
<td>8</td>
<td>0.337</td>
<td>7.210</td>
<td>-0.092 $\pm$ 0.138 $\pm$ 0.013</td>
</tr>
<tr>
<td>9</td>
<td>0.447</td>
<td>9.810</td>
<td>-0.110 $\pm$ 0.289 $\pm$ 0.020</td>
</tr>
</tbody>
</table>

Table C.2: The 1995 asymmetries on the $^3$He target. These data are from [49].
### C.2.2 1996 Asymmetries on the Proton Target

<table>
<thead>
<tr>
<th>Bin</th>
<th>$x$</th>
<th>$Q^2$</th>
<th>$A_{LL}^{+}$ ± stat. ± sys.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.033</td>
<td>1.210</td>
<td>0.086 ± 0.012 ± 0.006</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.470</td>
<td>0.097 ± 0.015 ± 0.008</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.720</td>
<td>0.121 ± 0.015 ± 0.011</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.000</td>
<td>0.169 ± 0.017 ± 0.013</td>
</tr>
<tr>
<td>5</td>
<td>0.119</td>
<td>2.310</td>
<td>0.214 ± 0.019 ± 0.018</td>
</tr>
<tr>
<td>6</td>
<td>0.168</td>
<td>2.660</td>
<td>0.234 ± 0.022 ± 0.025</td>
</tr>
<tr>
<td>7</td>
<td>0.244</td>
<td>3.080</td>
<td>0.326 ± 0.028 ± 0.032</td>
</tr>
<tr>
<td>8</td>
<td>0.342</td>
<td>3.770</td>
<td>0.375 ± 0.048 ± 0.039</td>
</tr>
<tr>
<td>9</td>
<td>0.465</td>
<td>5.250</td>
<td>0.574 ± 0.068 ± 0.050</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$x$</th>
<th>$Q^2$</th>
<th>$A_{LL}^{-}$ ± stat. ± sys.</th>
</tr>
</thead>
<tbody>
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<td>1.200</td>
<td>0.083 ± 0.031 ± 0.008</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.450</td>
<td>0.100 ± 0.032 ± 0.011</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.750</td>
<td>0.149 ± 0.034 ± 0.013</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.140</td>
<td>0.172 ± 0.036 ± 0.016</td>
</tr>
<tr>
<td>5</td>
<td>0.118</td>
<td>2.720</td>
<td>0.251 ± 0.038 ± 0.020</td>
</tr>
<tr>
<td>6</td>
<td>0.166</td>
<td>3.680</td>
<td>0.227 ± 0.047 ± 0.027</td>
</tr>
<tr>
<td>7</td>
<td>0.239</td>
<td>5.180</td>
<td>0.480 ± 0.064 ± 0.032</td>
</tr>
<tr>
<td>8</td>
<td>0.338</td>
<td>7.260</td>
<td>0.234 ± 0.130 ± 0.041</td>
</tr>
<tr>
<td>9</td>
<td>0.449</td>
<td>9.810</td>
<td>0.891 ± 0.261 ± 0.052</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Bin</th>
<th>$x$</th>
<th>$Q^2$</th>
<th>$A_{LU}^{-}$ ± stat. ± sys.</th>
</tr>
</thead>
<tbody>
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<td>0.033</td>
<td>1.210</td>
<td>0.090 ± 0.039 ± 0.004</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.460</td>
<td>0.106 ± 0.042 ± 0.004</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.750</td>
<td>0.064 ± 0.043 ± 0.006</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.140</td>
<td>0.165 ± 0.049 ± 0.008</td>
</tr>
<tr>
<td>5</td>
<td>0.118</td>
<td>2.700</td>
<td>0.256 ± 0.052 ± 0.011</td>
</tr>
<tr>
<td>6</td>
<td>0.166</td>
<td>3.670</td>
<td>0.284 ± 0.068 ± 0.017</td>
</tr>
<tr>
<td>7</td>
<td>0.239</td>
<td>5.160</td>
<td>0.276 ± 0.093 ± 0.024</td>
</tr>
<tr>
<td>8</td>
<td>0.338</td>
<td>7.230</td>
<td>0.438 ± 0.197 ± 0.034</td>
</tr>
<tr>
<td>9</td>
<td>0.449</td>
<td>9.750</td>
<td>0.108 ± 0.413 ± 0.050</td>
</tr>
</tbody>
</table>

Table C.3: The 1996 asymmetries on the proton target. These data are from [49].
C.2.3 1997 Asymmetries on the Proton Target

<table>
<thead>
<tr>
<th>Bin</th>
<th>$x$</th>
<th>$Q^2$</th>
<th>$A_{11}^{c+} \pm \text{stat.} \pm \text{sys.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.033</td>
<td>1.210</td>
<td>0.075 ± 0.008 ± 0.005</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.470</td>
<td>0.107 ± 0.009 ± 0.007</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.720</td>
<td>0.116 ± 0.010 ± 0.009</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>1.990</td>
<td>0.162 ± 0.011 ± 0.012</td>
</tr>
<tr>
<td>5</td>
<td>0.119</td>
<td>2.300</td>
<td>0.191 ± 0.011 ± 0.015</td>
</tr>
<tr>
<td>6</td>
<td>0.168</td>
<td>2.660</td>
<td>0.252 ± 0.014 ± 0.021</td>
</tr>
<tr>
<td>7</td>
<td>0.245</td>
<td>3.060</td>
<td>0.325 ± 0.018 ± 0.027</td>
</tr>
<tr>
<td>8</td>
<td>0.342</td>
<td>3.740</td>
<td>0.486 ± 0.030 ± 0.034</td>
</tr>
<tr>
<td>9</td>
<td>0.465</td>
<td>5.160</td>
<td>0.640 ± 0.042 ± 0.050</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$x$</th>
<th>$Q^2$</th>
<th>$A_{11}^{c-} \pm \text{stat.} \pm \text{sys.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.033</td>
<td>1.210</td>
<td>0.089 ± 0.018 ± 0.006</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.460</td>
<td>0.116 ± 0.020 ± 0.009</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.750</td>
<td>0.133 ± 0.020 ± 0.011</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.140</td>
<td>0.185 ± 0.022 ± 0.014</td>
</tr>
<tr>
<td>5</td>
<td>0.118</td>
<td>2.700</td>
<td>0.250 ± 0.024 ± 0.017</td>
</tr>
<tr>
<td>6</td>
<td>0.165</td>
<td>3.670</td>
<td>0.258 ± 0.029 ± 0.024</td>
</tr>
<tr>
<td>7</td>
<td>0.238</td>
<td>5.160</td>
<td>0.400 ± 0.040 ± 0.028</td>
</tr>
<tr>
<td>8</td>
<td>0.339</td>
<td>7.230</td>
<td>0.490 ± 0.081 ± 0.035</td>
</tr>
<tr>
<td>9</td>
<td>0.447</td>
<td>9.750</td>
<td>0.556 ± 0.164 ± 0.047</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$x$</th>
<th>$Q^2$</th>
<th>$A_{11}^{f-} \pm \text{stat.} \pm \text{sys.}$</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>0.033</td>
<td>1.210</td>
<td>0.034 ± 0.024 ± 0.003</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.460</td>
<td>0.090 ± 0.025 ± 0.004</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.750</td>
<td>0.067 ± 0.026 ± 0.005</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.140</td>
<td>0.021 ± 0.030 ± 0.007</td>
</tr>
<tr>
<td>5</td>
<td>0.118</td>
<td>2.700</td>
<td>0.155 ± 0.033 ± 0.010</td>
</tr>
<tr>
<td>6</td>
<td>0.165</td>
<td>3.670</td>
<td>0.166 ± 0.041 ± 0.015</td>
</tr>
<tr>
<td>7</td>
<td>0.238</td>
<td>5.160</td>
<td>0.197 ± 0.059 ± 0.021</td>
</tr>
<tr>
<td>8</td>
<td>0.339</td>
<td>7.230</td>
<td>0.656 ± 0.122 ± 0.030</td>
</tr>
<tr>
<td>9</td>
<td>0.447</td>
<td>9.750</td>
<td>0.120 ± 0.249 ± 0.045</td>
</tr>
</tbody>
</table>

Table C.4: The 1997 asymmetries on the proton target. These data are from [49].
### C.2.4 1998 Asymmetries on the D Target

<table>
<thead>
<tr>
<th>Bin</th>
<th>( x )</th>
<th>( Q^2 )</th>
<th>( A_t^{++} ) ± stat. ± sys.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.033</td>
<td>1.216</td>
<td>0.010 ± 0.010 ± 0.001</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.474</td>
<td>0.014 ± 0.011 ± 0.002</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.727</td>
<td>0.022 ± 0.012 ± 0.004</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.002</td>
<td>0.037 ± 0.014 ± 0.007</td>
</tr>
<tr>
<td>5</td>
<td>0.119</td>
<td>2.319</td>
<td>0.084 ± 0.015 ± 0.012</td>
</tr>
<tr>
<td>6</td>
<td>0.168</td>
<td>2.682</td>
<td>0.136 ± 0.018 ± 0.021</td>
</tr>
<tr>
<td>7</td>
<td>0.244</td>
<td>3.085</td>
<td>0.186 ± 0.023 ± 0.037</td>
</tr>
<tr>
<td>8</td>
<td>0.342</td>
<td>3.770</td>
<td>0.309 ± 0.038 ± 0.062</td>
</tr>
<tr>
<td>9</td>
<td>0.464</td>
<td>5.215</td>
<td>0.269 ± 0.052 ± 0.092</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>( x )</th>
<th>( Q^2 )</th>
<th>( A_t^{+-} ) ± stat. ± sys.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.033</td>
<td>1.209</td>
<td>-0.009 ± 0.026 ± 0.001</td>
</tr>
<tr>
<td>2</td>
<td>0.048</td>
<td>1.459</td>
<td>0.053 ± 0.027 ± 0.005</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.772</td>
<td>0.011 ± 0.028 ± 0.003</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.169</td>
<td>0.036 ± 0.032 ± 0.006</td>
</tr>
<tr>
<td>5</td>
<td>0.118</td>
<td>2.744</td>
<td>0.124 ± 0.035 ± 0.013</td>
</tr>
<tr>
<td>6</td>
<td>0.165</td>
<td>3.728</td>
<td>0.091 ± 0.042 ± 0.013</td>
</tr>
<tr>
<td>7</td>
<td>0.238</td>
<td>5.216</td>
<td>0.269 ± 0.058 ± 0.028</td>
</tr>
<tr>
<td>8</td>
<td>0.337</td>
<td>7.298</td>
<td>0.041 ± 0.120 ± 0.022</td>
</tr>
<tr>
<td>9</td>
<td>0.449</td>
<td>9.974</td>
<td>0.562 ± 0.237 ± 0.063</td>
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</table>

<table>
<thead>
<tr>
<th>Bin</th>
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<th>( Q^2 )</th>
<th>( A_t^{-+} ) ± stat. ± sys.</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>0.033</td>
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<td>0.008 ± 0.030 ± 0.001</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.467</td>
<td>-0.008 ± 0.032 ± 0.002</td>
</tr>
<tr>
<td>3</td>
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<td>1.779</td>
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<tr>
<td>4</td>
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<td>2.186</td>
<td>-0.042 ± 0.040 ± 0.006</td>
</tr>
<tr>
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<td>-0.007 ± 0.043 ± 0.007</td>
</tr>
<tr>
<td>6</td>
<td>0.165</td>
<td>3.742</td>
<td>-0.011 ± 0.054 ± 0.009</td>
</tr>
<tr>
<td>7</td>
<td>0.238</td>
<td>5.226</td>
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</tr>
<tr>
<td>8</td>
<td>0.337</td>
<td>7.241</td>
<td>0.029 ± 0.160 ± 0.021</td>
</tr>
<tr>
<td>9</td>
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<td>10.002</td>
<td>0.302 ± 0.314 ± 0.044</td>
</tr>
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Table C.5: The 1998 asymmetries on the D target. These data are from [50].
Table C.6: The 1998 asymmetries on the D target. These data are from [50].

<table>
<thead>
<tr>
<th>Bin</th>
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<th>$A_T^{7+}$ ± stat. ± sys.</th>
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</thead>
<tbody>
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<td>1</td>
<td>0.033</td>
<td>1.209</td>
<td>-0.024 ± 0.030 ± 0.002</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.460</td>
<td>0.048 ± 0.032 ± 0.005</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.775</td>
<td>0.005 ± 0.033 ± 0.003</td>
</tr>
<tr>
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<td>-0.016 ± 0.038 ± 0.005</td>
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<td>0.118</td>
<td>2.753</td>
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</tr>
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<td>0.165</td>
<td>3.746</td>
<td>0.086 ± 0.050 ± 0.012</td>
</tr>
<tr>
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<td>5.215</td>
<td>0.206 ± 0.070 ± 0.024</td>
</tr>
<tr>
<td>8</td>
<td>0.337</td>
<td>7.301</td>
<td>0.032 ± 0.145 ± 0.021</td>
</tr>
<tr>
<td>9</td>
<td>0.449</td>
<td>10.062</td>
<td>0.535 ± 0.291 ± 0.060</td>
</tr>
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</table>

<table>
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<th>$A_T^{7-}$ ± stat. ± sys.</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>0.033</td>
<td>1.206</td>
<td>0.020 ± 0.032 ± 0.002</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.466</td>
<td>-0.001 ± 0.035 ± 0.002</td>
</tr>
<tr>
<td>3</td>
<td>0.064</td>
<td>1.774</td>
<td>-0.038 ± 0.037 ± 0.005</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.173</td>
<td>-0.045 ± 0.043 ± 0.006</td>
</tr>
<tr>
<td>5</td>
<td>0.118</td>
<td>2.773</td>
<td>-0.009 ± 0.047 ± 0.007</td>
</tr>
<tr>
<td>6</td>
<td>0.165</td>
<td>3.712</td>
<td>-0.016 ± 0.059 ± 0.010</td>
</tr>
<tr>
<td>7</td>
<td>0.237</td>
<td>5.189</td>
<td>0.093 ± 0.082 ± 0.016</td>
</tr>
<tr>
<td>8</td>
<td>0.337</td>
<td>7.210</td>
<td>-0.027 ± 0.172 ± 0.021</td>
</tr>
<tr>
<td>9</td>
<td>0.451</td>
<td>10.032</td>
<td>0.095 ± 0.337 ± 0.037</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$x$</th>
<th>$Q^2$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.034</td>
<td>1.209</td>
<td>0.050 ± 0.071 ± 0.005</td>
</tr>
<tr>
<td>2</td>
<td>0.048</td>
<td>1.457</td>
<td>0.113 ± 0.072 ± 0.007</td>
</tr>
<tr>
<td>3</td>
<td>0.065</td>
<td>1.760</td>
<td>0.066 ± 0.075 ± 0.006</td>
</tr>
<tr>
<td>4</td>
<td>0.087</td>
<td>2.185</td>
<td>0.179 ± 0.085 ± 0.015</td>
</tr>
<tr>
<td>5</td>
<td>0.118</td>
<td>2.856</td>
<td>0.012 ± 0.088 ± 0.006</td>
</tr>
<tr>
<td>6</td>
<td>0.165</td>
<td>3.904</td>
<td>0.111 ± 0.105 ± 0.014</td>
</tr>
<tr>
<td>7</td>
<td>0.239</td>
<td>5.455</td>
<td>0.390 ± 0.146 ± 0.034</td>
</tr>
<tr>
<td>8</td>
<td>0.336</td>
<td>7.480</td>
<td>0.029 ± 0.305 ± 0.024</td>
</tr>
<tr>
<td>9</td>
<td>0.450</td>
<td>9.704</td>
<td>0.530 ± 0.617 ± 0.070</td>
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</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$x$</th>
<th>$Q^2$</th>
<th>$A_T^{K^{-}}$ ± stat. ± sys.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.034</td>
<td>1.212</td>
<td>-0.173 ± 0.102 ± 0.011</td>
</tr>
<tr>
<td>2</td>
<td>0.047</td>
<td>1.463</td>
<td>-0.012 ± 0.104 ± 0.002</td>
</tr>
<tr>
<td>3</td>
<td>0.064</td>
<td>1.776</td>
<td>-0.103 ± 0.110 ± 0.008</td>
</tr>
<tr>
<td>4</td>
<td>0.086</td>
<td>2.242</td>
<td>-0.056 ± 0.126 ± 0.008</td>
</tr>
<tr>
<td>5</td>
<td>0.117</td>
<td>2.881</td>
<td>-0.025 ± 0.137 ± 0.006</td>
</tr>
<tr>
<td>6</td>
<td>0.165</td>
<td>3.942</td>
<td>-0.065 ± 0.174 ± 0.008</td>
</tr>
<tr>
<td>7</td>
<td>0.237</td>
<td>5.450</td>
<td>0.183 ± 0.254 ± 0.019</td>
</tr>
<tr>
<td>8</td>
<td>0.341</td>
<td>7.590</td>
<td>-0.081 ± 0.555 ± 0.033</td>
</tr>
<tr>
<td>9</td>
<td>0.432</td>
<td>9.461</td>
<td>2.757 ± 0.960 ± 0.217</td>
</tr>
</tbody>
</table>
C.3 Correlation Coefficients for Particle Count Rates

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\rho(N^{e^+}, N^{h^+})$</th>
<th>$\rho(N^{e^+}, N^{h^-})$</th>
<th>$\rho(N^{h^+}, N^{h^-})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.446</td>
<td>0.395</td>
<td>0.128</td>
</tr>
<tr>
<td>2</td>
<td>0.492</td>
<td>0.417</td>
<td>0.137</td>
</tr>
<tr>
<td>3</td>
<td>0.507</td>
<td>0.411</td>
<td>0.130</td>
</tr>
<tr>
<td>4</td>
<td>0.497</td>
<td>0.386</td>
<td>0.118</td>
</tr>
<tr>
<td>5</td>
<td>0.451</td>
<td>0.336</td>
<td>0.105</td>
</tr>
<tr>
<td>6</td>
<td>0.364</td>
<td>0.260</td>
<td>0.096</td>
</tr>
<tr>
<td>7</td>
<td>0.260</td>
<td>0.177</td>
<td>0.083</td>
</tr>
<tr>
<td>8</td>
<td>0.183</td>
<td>0.120</td>
<td>0.067</td>
</tr>
<tr>
<td>9</td>
<td>0.127</td>
<td>0.081</td>
<td>0.054</td>
</tr>
</tbody>
</table>

Note: The correlation coefficients of count rates for the $^3$He target in 1995. These data are from [49].

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\rho(N^{e^+}, N^{h^+})$</th>
<th>$\rho(N^{e^+}, N^{h^-})$</th>
<th>$\rho(N^{h^+}, N^{h^-})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.452</td>
<td>0.394</td>
<td>0.130</td>
</tr>
<tr>
<td>2</td>
<td>0.500</td>
<td>0.414</td>
<td>0.140</td>
</tr>
<tr>
<td>3</td>
<td>0.517</td>
<td>0.406</td>
<td>0.134</td>
</tr>
<tr>
<td>4</td>
<td>0.509</td>
<td>0.379</td>
<td>0.120</td>
</tr>
<tr>
<td>5</td>
<td>0.464</td>
<td>0.328</td>
<td>0.107</td>
</tr>
<tr>
<td>6</td>
<td>0.375</td>
<td>0.253</td>
<td>0.098</td>
</tr>
<tr>
<td>7</td>
<td>0.267</td>
<td>0.171</td>
<td>0.083</td>
</tr>
<tr>
<td>8</td>
<td>0.188</td>
<td>0.115</td>
<td>0.067</td>
</tr>
<tr>
<td>9</td>
<td>0.130</td>
<td>0.076</td>
<td>0.051</td>
</tr>
</tbody>
</table>

Note: The correlation coefficients of count rates for the proton target in 1997. These data are from [49].
<table>
<thead>
<tr>
<th>Bin</th>
<th>$\rho(N_{\nu}^{e+}, N_{\nu}^{h^{+}})$</th>
<th>$\rho(N_{\nu}^{e+}, N_{\nu}^{h^{-}})$</th>
<th>$\rho(N_{\nu}^{e+}, N_{\nu}^{\pi^{+}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.374</td>
<td>0.325</td>
<td>0.324</td>
</tr>
<tr>
<td>2</td>
<td>0.402</td>
<td>0.344</td>
<td>0.345</td>
</tr>
<tr>
<td>3</td>
<td>0.401</td>
<td>0.337</td>
<td>0.343</td>
</tr>
<tr>
<td>4</td>
<td>0.385</td>
<td>0.313</td>
<td>0.326</td>
</tr>
<tr>
<td>5</td>
<td>0.353</td>
<td>0.281</td>
<td>0.298</td>
</tr>
<tr>
<td>6</td>
<td>0.292</td>
<td>0.226</td>
<td>0.244</td>
</tr>
<tr>
<td>7</td>
<td>0.215</td>
<td>0.166</td>
<td>0.179</td>
</tr>
<tr>
<td>8</td>
<td>0.155</td>
<td>0.119</td>
<td>0.129</td>
</tr>
<tr>
<td>9</td>
<td>0.106</td>
<td>0.081</td>
<td>0.085</td>
</tr>
</tbody>
</table>

**Table C.9**: The correlation coefficients of count rates of inclusive events for the D target in 1998. These data are from [67].

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\rho(N_{\nu}^{e+}, N_{\nu}^{\pi^{-}})$</th>
<th>$\rho(N_{\nu}^{e+}, N_{\nu}^{K^{+}})$</th>
<th>$\rho(N_{\nu}^{e+}, N_{\nu}^{K^{-}})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.299</td>
<td>0.140</td>
<td>0.097</td>
</tr>
<tr>
<td>2</td>
<td>0.316</td>
<td>0.155</td>
<td>0.107</td>
</tr>
<tr>
<td>3</td>
<td>0.310</td>
<td>0.157</td>
<td>0.106</td>
</tr>
<tr>
<td>4</td>
<td>0.290</td>
<td>0.149</td>
<td>0.097</td>
</tr>
<tr>
<td>5</td>
<td>0.261</td>
<td>0.135</td>
<td>0.085</td>
</tr>
<tr>
<td>6</td>
<td>0.210</td>
<td>0.113</td>
<td>0.067</td>
</tr>
<tr>
<td>7</td>
<td>0.155</td>
<td>0.082</td>
<td>0.047</td>
</tr>
<tr>
<td>8</td>
<td>0.110</td>
<td>0.060</td>
<td>0.033</td>
</tr>
<tr>
<td>9</td>
<td>0.076</td>
<td>0.043</td>
<td>0.024</td>
</tr>
</tbody>
</table>

**Table C.10**: The correlation coefficients of count rates of semi-inclusive $h^{+}$ for the D target in 1998. These data are from [67].
### Table C.11: The correlation coefficients of count rates for semi-inclusive $h^-$ for the D target in 1998. These data are from [67].

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\rho(N_h^-, N_{\pi^+})$</th>
<th>$\rho(N_h^-, N_{\pi^-})$</th>
<th>$\rho(N_h^-, N_{K^+})$</th>
<th>$\rho(N_h^-, N_{K^-})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.115</td>
<td>0.924</td>
<td>0.048</td>
<td>0.297</td>
</tr>
<tr>
<td>2</td>
<td>0.112</td>
<td>0.919</td>
<td>0.053</td>
<td>0.313</td>
</tr>
<tr>
<td>3</td>
<td>0.111</td>
<td>0.922</td>
<td>0.051</td>
<td>0.313</td>
</tr>
<tr>
<td>4</td>
<td>0.102</td>
<td>0.926</td>
<td>0.045</td>
<td>0.309</td>
</tr>
<tr>
<td>5</td>
<td>0.090</td>
<td>0.927</td>
<td>0.042</td>
<td>0.304</td>
</tr>
<tr>
<td>6</td>
<td>0.077</td>
<td>0.928</td>
<td>0.037</td>
<td>0.296</td>
</tr>
<tr>
<td>7</td>
<td>0.062</td>
<td>0.931</td>
<td>0.029</td>
<td>0.287</td>
</tr>
<tr>
<td>8</td>
<td>0.048</td>
<td>0.928</td>
<td>0.024</td>
<td>0.279</td>
</tr>
<tr>
<td>9</td>
<td>0.041</td>
<td>0.936</td>
<td>0.017</td>
<td>0.282</td>
</tr>
</tbody>
</table>

### Table C.12: The correlation coefficients of count rates for semi-inclusive $\pi$ and $K$ for the D target in 1998. These data are from [67].

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\rho(N_{\pi^+}, N_{\pi^-})$</th>
<th>$\rho(N_{\pi^+}, N_{K^+})$</th>
<th>$\rho(N_{\pi^+}, N_{K^-})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.116</td>
<td>0.016</td>
<td>0.020</td>
</tr>
<tr>
<td>2</td>
<td>0.111</td>
<td>0.016</td>
<td>0.023</td>
</tr>
<tr>
<td>3</td>
<td>0.110</td>
<td>0.017</td>
<td>0.024</td>
</tr>
<tr>
<td>4</td>
<td>0.101</td>
<td>0.016</td>
<td>0.023</td>
</tr>
<tr>
<td>5</td>
<td>0.088</td>
<td>0.017</td>
<td>0.019</td>
</tr>
<tr>
<td>6</td>
<td>0.076</td>
<td>0.018</td>
<td>0.016</td>
</tr>
<tr>
<td>7</td>
<td>0.061</td>
<td>0.012</td>
<td>0.013</td>
</tr>
<tr>
<td>8</td>
<td>0.045</td>
<td>0.011</td>
<td>0.013</td>
</tr>
<tr>
<td>9</td>
<td>0.040</td>
<td>0.016</td>
<td>0.015</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\rho(N_{\pi^-}, N_{K^+})$</th>
<th>$\rho(N_{\pi^-}, N_{K^-})$</th>
<th>$\rho(N_{K^+}, N_{K^-})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.027</td>
<td>0.010</td>
<td>0.063</td>
</tr>
<tr>
<td>2</td>
<td>0.030</td>
<td>0.011</td>
<td>0.067</td>
</tr>
<tr>
<td>3</td>
<td>0.028</td>
<td>0.008</td>
<td>0.072</td>
</tr>
<tr>
<td>4</td>
<td>0.025</td>
<td>0.010</td>
<td>0.059</td>
</tr>
<tr>
<td>5</td>
<td>0.023</td>
<td>0.011</td>
<td>0.057</td>
</tr>
<tr>
<td>6</td>
<td>0.023</td>
<td>0.010</td>
<td>0.043</td>
</tr>
<tr>
<td>7</td>
<td>0.016</td>
<td>0.009</td>
<td>0.044</td>
</tr>
<tr>
<td>8</td>
<td>0.009</td>
<td>0.000</td>
<td>0.051</td>
</tr>
<tr>
<td>9</td>
<td>0.009</td>
<td>0.000</td>
<td>0.030</td>
</tr>
</tbody>
</table>
C.4 Results of Quark Polarizations and Distributions

C.4.1 Valence Decomposition

Results with the Mixing Matrix

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\Delta u_{e_u}$ $u_{e_u}$</th>
<th>$\Delta d_{e_u}$ $d_{e_u}$</th>
<th>$\Delta u_{q_u}$ $q_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.184 ± 0.104</td>
<td>-0.437 ± 0.220</td>
<td>0.029 ± 0.082</td>
</tr>
<tr>
<td>2</td>
<td>0.201 ± 0.075</td>
<td>-0.341 ± 0.177</td>
<td>0.024 ± 0.081</td>
</tr>
<tr>
<td>3</td>
<td>0.292 ± 0.060</td>
<td>-0.158 ± 0.157</td>
<td>-0.098 ± 0.087</td>
</tr>
<tr>
<td>4</td>
<td>0.376 ± 0.056</td>
<td>-0.106 ± 0.166</td>
<td>-0.208 ± 0.114</td>
</tr>
<tr>
<td>5</td>
<td>0.309 ± 0.050</td>
<td>-0.119 ± 0.174</td>
<td>-0.116 ± 0.153</td>
</tr>
<tr>
<td>6</td>
<td>0.345 ± 0.048</td>
<td>0.037 ± 0.206</td>
<td>-0.311 ± 0.259</td>
</tr>
<tr>
<td>7</td>
<td>0.367 ± 0.038</td>
<td>-0.211 ± 0.209</td>
<td>-0.107 ± 0.444</td>
</tr>
<tr>
<td>8</td>
<td>0.472 ± 0.050</td>
<td>0.289 ± 0.369</td>
<td>—</td>
</tr>
<tr>
<td>9</td>
<td>0.694 ± 0.066</td>
<td>-0.996 ± 0.665</td>
<td>—</td>
</tr>
</tbody>
</table>

Table C.13: Results of valence decomposition with the mixing matrix. Only statistic errors are shown.

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\Delta u_{e_u}$ $u_{e_u}$</th>
<th>$\Delta d_{u_{e_u}}$ $d_{u_u}$</th>
<th>$\Delta u_{q_u}$ $q_u$</th>
<th>$\Delta d_{u_{q_u}}$ $d_{q_u}$</th>
<th>$\Delta u_{q_u}$ $q_u$</th>
<th>$\Delta d_{u_{q_u}}$ $d_{q_u}$</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.863</td>
<td>-0.981</td>
<td>-0.935</td>
<td>0.091</td>
<td>0.082</td>
<td>0.925</td>
<td>0.701</td>
</tr>
<tr>
<td>2</td>
<td>0.797</td>
<td>-0.966</td>
<td>-0.911</td>
<td>0.067</td>
<td>0.500</td>
<td>0.911</td>
<td>0.675</td>
</tr>
<tr>
<td>3</td>
<td>0.748</td>
<td>-0.952</td>
<td>-0.896</td>
<td>0.500</td>
<td>0.525</td>
<td>0.896</td>
<td>0.500</td>
</tr>
<tr>
<td>4</td>
<td>0.745</td>
<td>-0.948</td>
<td>-0.899</td>
<td>1.525</td>
<td>1.564</td>
<td>0.899</td>
<td>1.525</td>
</tr>
<tr>
<td>5</td>
<td>0.743</td>
<td>-0.941</td>
<td>-0.906</td>
<td>1.564</td>
<td>1.583</td>
<td>0.906</td>
<td>1.564</td>
</tr>
<tr>
<td>6</td>
<td>0.723</td>
<td>-0.926</td>
<td>-0.910</td>
<td>1.583</td>
<td>1.805</td>
<td>0.910</td>
<td>1.583</td>
</tr>
<tr>
<td>7</td>
<td>0.510</td>
<td>-0.839</td>
<td>-0.859</td>
<td>1.805</td>
<td>2.421</td>
<td>0.859</td>
<td>1.805</td>
</tr>
<tr>
<td>8</td>
<td>0.370</td>
<td>—</td>
<td>—</td>
<td>2.421</td>
<td>—</td>
<td>—</td>
<td>2.421</td>
</tr>
<tr>
<td>9</td>
<td>0.353</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.748</td>
<td>—</td>
<td>0.748</td>
</tr>
</tbody>
</table>

Table C.14: Correlations among Results of the valence decomposition with the mixing matrix. The last column shows the minimum value of $\chi^2$ in Eq.5.9.
Results with the New Definition of Purity

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\Delta u_v$</th>
<th>$\Delta d_v$</th>
<th>$\Delta g_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.230 ± 0.111</td>
<td>-0.332 ± 0.233</td>
<td>-0.010 ± 0.088</td>
</tr>
<tr>
<td>2</td>
<td>0.193 ± 0.078</td>
<td>-0.358 ± 0.182</td>
<td>0.033 ± 0.085</td>
</tr>
<tr>
<td>3</td>
<td>0.304 ± 0.060</td>
<td>-0.133 ± 0.156</td>
<td>-0.114 ± 0.088</td>
</tr>
<tr>
<td>4</td>
<td>0.371 ± 0.052</td>
<td>-0.134 ± 0.159</td>
<td>-0.193 ± 0.110</td>
</tr>
<tr>
<td>5</td>
<td>0.318 ± 0.048</td>
<td>-0.070 ± 0.160</td>
<td>-0.153 ± 0.143</td>
</tr>
<tr>
<td>6</td>
<td>0.344 ± 0.045</td>
<td>0.029 ± 0.184</td>
<td>-0.301 ± 0.233</td>
</tr>
<tr>
<td>7</td>
<td>0.373 ± 0.037</td>
<td>-0.175 ± 0.196</td>
<td>-0.194 ± 0.422</td>
</tr>
<tr>
<td>8</td>
<td>0.470 ± 0.049</td>
<td>0.168 ± 0.347</td>
<td>—</td>
</tr>
<tr>
<td>9</td>
<td>0.630 ± 0.061</td>
<td>-1.517 ± 0.579</td>
<td>—</td>
</tr>
</tbody>
</table>

Table C.15: Results of valence decomposition with the new definition of purity. Only statistic errors are shown.

<table>
<thead>
<tr>
<th>Bin</th>
<th>$(\Delta u_v, \Delta d_v)$</th>
<th>$(\Delta u_v, \Delta q_s)$</th>
<th>$(\Delta d_v, \Delta q_s)$</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.878</td>
<td>-0.983</td>
<td>-0.943</td>
<td>1.058</td>
</tr>
<tr>
<td>2</td>
<td>0.810</td>
<td>-0.969</td>
<td>-0.916</td>
<td>0.656</td>
</tr>
<tr>
<td>3</td>
<td>0.749</td>
<td>-0.953</td>
<td>-0.895</td>
<td>0.374</td>
</tr>
<tr>
<td>4</td>
<td>0.729</td>
<td>-0.946</td>
<td>-0.890</td>
<td>1.686</td>
</tr>
<tr>
<td>5</td>
<td>0.708</td>
<td>-0.935</td>
<td>-0.889</td>
<td>1.651</td>
</tr>
<tr>
<td>6</td>
<td>0.673</td>
<td>-0.914</td>
<td>-0.890</td>
<td>1.221</td>
</tr>
<tr>
<td>7</td>
<td>0.482</td>
<td>-0.832</td>
<td>-0.847</td>
<td>1.666</td>
</tr>
<tr>
<td>8</td>
<td>0.342</td>
<td>—</td>
<td>—</td>
<td>1.943</td>
</tr>
<tr>
<td>9</td>
<td>0.267</td>
<td>—</td>
<td>—</td>
<td>1.275</td>
</tr>
</tbody>
</table>

Table C.16: Correlations among Results of the valence decomposition with the new definition of purity. The last column shows the minimum value of $\chi^2$ in Eq.5.9.
C.4.2 Flavor Decomposition

Results with the Mixing Matrix

<table>
<thead>
<tr>
<th>Bin</th>
<th>(Δu+Δd)_{u+6}</th>
<th>(Δd+Δd)_{d+d}</th>
<th>(Δd+Δs)_{s-s}</th>
<th>χ^2/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.098 ± 0.009</td>
<td>-0.114 ± 0.025</td>
<td>0.029 ± 0.082</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.118 ± 0.010</td>
<td>-0.109 ± 0.028</td>
<td>0.024 ± 0.081</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.140 ± 0.011</td>
<td>-0.123 ± 0.030</td>
<td>-0.098 ± 0.087</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.195 ± 0.012</td>
<td>-0.160 ± 0.036</td>
<td>-0.208 ± 0.114</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.212 ± 0.013</td>
<td>-0.118 ± 0.041</td>
<td>-0.116 ± 0.153</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.252 ± 0.016</td>
<td>-0.100 ± 0.053</td>
<td>-0.311 ± 0.259</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.334 ± 0.019</td>
<td>-0.180 ± 0.075</td>
<td>-0.107 ± 0.444</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.416 ± 0.033</td>
<td>-0.080 ± 0.168</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.661 ± 0.047</td>
<td>-1.173 ± 0.341</td>
<td>—</td>
<td></td>
</tr>
</tbody>
</table>

Table C.17: Results of flavor decomposition with the mixing matrix. Only statistic errors are shown.

<table>
<thead>
<tr>
<th>Bin</th>
<th>(Δu+Δs)_{u+0}</th>
<th>(Δd+Δd)_{d+d}</th>
<th>(Δd+Δs)_{s-s}</th>
<th>(Δd+Δs)_{s-s}</th>
<th>χ^2/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.778</td>
<td>0.027</td>
<td>-0.238</td>
<td>0.074</td>
<td>0.701</td>
</tr>
<tr>
<td>2</td>
<td>-0.745</td>
<td>-0.037</td>
<td>-0.254</td>
<td>0.067</td>
<td>0.675</td>
</tr>
<tr>
<td>3</td>
<td>-0.724</td>
<td>-0.067</td>
<td>-0.267</td>
<td>0.050</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-0.703</td>
<td>-0.107</td>
<td>-0.286</td>
<td>1.252</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>-0.686</td>
<td>-0.132</td>
<td>-0.303</td>
<td>1.564</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.715</td>
<td>-0.100</td>
<td>-0.234</td>
<td>1.583</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-0.741</td>
<td>0.094</td>
<td>0.111</td>
<td>1.805</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-0.636</td>
<td>—</td>
<td>—</td>
<td>2.421</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-0.546</td>
<td>—</td>
<td>—</td>
<td>0.748</td>
<td></td>
</tr>
</tbody>
</table>

Table C.18: Correlations among Results of the flavor decomposition with the mixing matrix. The last column shows the minimum value of χ^2 in Eq.5.9.
Results with the New Definition of Purity

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\frac{\Delta u + \Delta s}{u + s}$</th>
<th>$\frac{\Delta d + \Delta \bar{d}}{d + \bar{d}}$</th>
<th>$\Delta q_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.097 ± 0.009</td>
<td>-0.109 ± 0.025</td>
<td>-0.010 ± 0.088</td>
</tr>
<tr>
<td>2</td>
<td>0.118 ± 0.010</td>
<td>-0.109 ± 0.028</td>
<td>0.033 ± 0.085</td>
</tr>
<tr>
<td>3</td>
<td>0.141 ± 0.011</td>
<td>-0.122 ± 0.030</td>
<td>-0.114 ± 0.088</td>
</tr>
<tr>
<td>4</td>
<td>0.196 ± 0.012</td>
<td>-0.165 ± 0.035</td>
<td>-0.193 ± 0.110</td>
</tr>
<tr>
<td>5</td>
<td>0.211 ± 0.013</td>
<td>-0.108 ± 0.040</td>
<td>-0.153 ± 0.143</td>
</tr>
<tr>
<td>6</td>
<td>0.252 ± 0.016</td>
<td>-0.101 ± 0.052</td>
<td>-0.301 ± 0.233</td>
</tr>
<tr>
<td>7</td>
<td>0.333 ± 0.019</td>
<td>-0.181 ± 0.074</td>
<td>-0.194 ± 0.422</td>
</tr>
<tr>
<td>8</td>
<td>0.424 ± 0.032</td>
<td>-0.102 ± 0.163</td>
<td>—</td>
</tr>
<tr>
<td>9</td>
<td>0.662 ± 0.045</td>
<td>-0.866 ± 0.318</td>
<td>—</td>
</tr>
</tbody>
</table>

Table C.19: Results of flavor decomposition with the new definition of purity. Only statistic errors are shown.

<table>
<thead>
<tr>
<th>Bin</th>
<th>$(\frac{\Delta u + \Delta s}{u + s}, \frac{\Delta d + \Delta \bar{d}}{d + \bar{d}})$</th>
<th>$(\frac{\Delta u + \Delta s}{u + s}, \frac{\Delta q_2}{q_2})$</th>
<th>$(\frac{\Delta d + \Delta \bar{d}}{d + \bar{d}}, \frac{\Delta q_2}{q_2})$</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.775</td>
<td>0.034</td>
<td>-0.262</td>
<td>1.058</td>
</tr>
<tr>
<td>2</td>
<td>-0.740</td>
<td>-0.050</td>
<td>-0.254</td>
<td>0.656</td>
</tr>
<tr>
<td>3</td>
<td>-0.722</td>
<td>-0.093</td>
<td>-0.243</td>
<td>0.374</td>
</tr>
<tr>
<td>4</td>
<td>-0.707</td>
<td>-0.136</td>
<td>-0.241</td>
<td>1.686</td>
</tr>
<tr>
<td>5</td>
<td>-0.696</td>
<td>-0.170</td>
<td>-0.230</td>
<td>1.651</td>
</tr>
<tr>
<td>6</td>
<td>-0.722</td>
<td>-0.131</td>
<td>-0.160</td>
<td>1.221</td>
</tr>
<tr>
<td>7</td>
<td>-0.734</td>
<td>0.067</td>
<td>0.147</td>
<td>1.666</td>
</tr>
<tr>
<td>8</td>
<td>-0.630</td>
<td>—</td>
<td>—</td>
<td>1.943</td>
</tr>
<tr>
<td>9</td>
<td>-0.566</td>
<td>—</td>
<td>—</td>
<td>1.275</td>
</tr>
</tbody>
</table>

Table C.20: Correlations among Results of the flavor decomposition with the new definition of purity. The last column shows the minimum value of $\chi^2$ in Eq.5.9.
C.4.3 Flavor Asymmetry of Light Sea Quarks

Results with the Mixing Matrix

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\Delta u + \Delta \bar{u}$</th>
<th>$\Delta d + \Delta \bar{d}$</th>
<th>$\Delta \bar{u} + \Delta \bar{d}$</th>
<th>$\Delta \bar{u} - \Delta \bar{d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.982 ± 0.327</td>
<td>-2.102 ± 0.476</td>
<td>0.149 ± 1.264</td>
<td>0.245 ± 1.409</td>
</tr>
<tr>
<td>2</td>
<td>1.941 ± 0.235</td>
<td>-1.337 ± 0.369</td>
<td>-0.565 ± 0.756</td>
<td>1.307 ± 0.909</td>
</tr>
<tr>
<td>3</td>
<td>1.675 ± 0.185</td>
<td>-1.068 ± 0.292</td>
<td>-0.748 ± 0.538</td>
<td>0.582 ± 0.659</td>
</tr>
<tr>
<td>4</td>
<td>1.677 ± 0.158</td>
<td>-1.025 ± 0.255</td>
<td>-0.553 ± 0.455</td>
<td>-0.005 ± 0.539</td>
</tr>
<tr>
<td>5</td>
<td>1.503 ± 0.124</td>
<td>-0.493 ± 0.204</td>
<td>-0.557 ± 0.349</td>
<td>0.680 ± 0.401</td>
</tr>
<tr>
<td>6</td>
<td>1.289 ± 0.096</td>
<td>-0.256 ± 0.160</td>
<td>-0.605 ± 0.270</td>
<td>0.706 ± 0.335</td>
</tr>
<tr>
<td>7</td>
<td>0.931 ± 0.073</td>
<td>-0.196 ± 0.124</td>
<td>0.125 ± 0.149</td>
<td>-0.377 ± 0.242</td>
</tr>
<tr>
<td>8</td>
<td>0.957 ± 0.085</td>
<td>-0.343 ± 0.141</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0.497 ± 0.055</td>
<td>-0.138 ± 0.105</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table C.21: Results of flavor asymmetry extraction with the mixing matrix. Only statistic errors are shown.

<table>
<thead>
<tr>
<th>Bin</th>
<th>$(\Delta u + \Delta \bar{u}, \Delta d + \Delta \bar{d})$</th>
<th>$(\Delta u + \Delta \bar{u}, \Delta \bar{u} + \Delta \bar{d})$</th>
<th>$(\Delta u + \Delta \bar{u}, \Delta \bar{u} - \Delta \bar{d})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.219</td>
<td>-0.693</td>
<td>0.820</td>
</tr>
<tr>
<td>2</td>
<td>-0.284</td>
<td>-0.657</td>
<td>0.757</td>
</tr>
<tr>
<td>3</td>
<td>-0.296</td>
<td>-0.659</td>
<td>0.733</td>
</tr>
<tr>
<td>4</td>
<td>-0.260</td>
<td>-0.681</td>
<td>0.733</td>
</tr>
<tr>
<td>5</td>
<td>-0.256</td>
<td>-0.670</td>
<td>0.711</td>
</tr>
<tr>
<td>6</td>
<td>-0.434</td>
<td>-0.589</td>
<td>0.652</td>
</tr>
<tr>
<td>7</td>
<td>-0.791</td>
<td>-0.376</td>
<td>0.619</td>
</tr>
<tr>
<td>8</td>
<td>-0.842</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-0.815</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$(\Delta d + \Delta \bar{d}, \Delta \bar{u} + \Delta \bar{d})$</th>
<th>$(\Delta d + \Delta \bar{d}, \Delta \bar{u} - \Delta \bar{d})$</th>
<th>$(\Delta \bar{u} + \Delta \bar{d}, \Delta \bar{u} - \Delta \bar{d})$</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.367</td>
<td>0.156</td>
<td>-0.714</td>
<td>0.786</td>
</tr>
<tr>
<td>2</td>
<td>-0.355</td>
<td>0.143</td>
<td>-0.709</td>
<td>0.493</td>
</tr>
<tr>
<td>3</td>
<td>-0.353</td>
<td>0.151</td>
<td>-0.743</td>
<td>0.472</td>
</tr>
<tr>
<td>4</td>
<td>-0.376</td>
<td>0.176</td>
<td>-0.764</td>
<td>1.685</td>
</tr>
<tr>
<td>5</td>
<td>-0.396</td>
<td>0.195</td>
<td>-0.767</td>
<td>1.446</td>
</tr>
<tr>
<td>6</td>
<td>-0.272</td>
<td>0.090</td>
<td>-0.763</td>
<td>1.287</td>
</tr>
<tr>
<td>7</td>
<td>0.230</td>
<td>-0.389</td>
<td>-0.690</td>
<td>1.723</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td>1.282</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td>0.391</td>
</tr>
</tbody>
</table>

Table C.22: Correlations among Results of the flavor asymmetry extraction with the mixing matrix. The last column shows the minimum value of $\chi^2$ in Eq.5.9.
### Results with the New Definition of Purity

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\Delta u + \Delta \bar{u}$</th>
<th>$\Delta d + \Delta \bar{d}$</th>
<th>$\Delta \bar{u} + \Delta \bar{d}$</th>
<th>$\Delta \bar{u} - \Delta \bar{d}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.178 \pm 0.349$</td>
<td>$-1.897 \pm 0.484$</td>
<td>$-0.979 \pm 1.405$</td>
<td>$1.202 \pm 1.520$</td>
</tr>
<tr>
<td>2</td>
<td>$1.911 \pm 0.237$</td>
<td>$-1.360 \pm 0.366$</td>
<td>$-0.435 \pm 0.760$</td>
<td>$1.173 \pm 0.907$</td>
</tr>
<tr>
<td>3</td>
<td>$1.655 \pm 0.181$</td>
<td>$-1.088 \pm 0.286$</td>
<td>$-0.676 \pm 0.496$</td>
<td>$0.451 \pm 0.618$</td>
</tr>
<tr>
<td>4</td>
<td>$1.604 \pm 0.148$</td>
<td>$-1.096 \pm 0.245$</td>
<td>$-0.257 \pm 0.381$</td>
<td>$-0.342 \pm 0.475$</td>
</tr>
<tr>
<td>5</td>
<td>$1.487 \pm 0.114$</td>
<td>$-0.495 \pm 0.193$</td>
<td>$-0.504 \pm 0.272$</td>
<td>$0.635 \pm 0.340$</td>
</tr>
<tr>
<td>6</td>
<td>$1.234 \pm 0.089$</td>
<td>$-0.300 \pm 0.154$</td>
<td>$-0.350 \pm 0.203$</td>
<td>$0.412 \pm 0.278$</td>
</tr>
<tr>
<td>7</td>
<td>$0.929 \pm 0.072$</td>
<td>$-0.189 \pm 0.123$</td>
<td>$0.091 \pm 0.133$</td>
<td>$-0.357 \pm 0.230$</td>
</tr>
<tr>
<td>8</td>
<td>$0.914 \pm 0.081$</td>
<td>$-0.273 \pm 0.132$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>9</td>
<td>$0.465 \pm 0.052$</td>
<td>$-0.045 \pm 0.091$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table C.23: Results of flavor asymmetry extraction with the new definition of purity. Only statistic errors are shown.

<table>
<thead>
<tr>
<th>Bin</th>
<th>$(\Delta u + \Delta \bar{u}, \Delta d + \Delta \bar{d})$</th>
<th>$(\Delta u + \Delta \bar{u}, \Delta \bar{u} + \Delta \bar{d})$</th>
<th>$(\Delta u + \Delta \bar{u}, \Delta \bar{u} - \Delta \bar{d})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-0.137$</td>
<td>$-0.736$</td>
<td>$0.843$</td>
</tr>
<tr>
<td>2</td>
<td>$-0.291$</td>
<td>$-0.664$</td>
<td>$0.760$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.369$</td>
<td>$-0.636$</td>
<td>$0.713$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.401$</td>
<td>$-0.623$</td>
<td>$0.686$</td>
</tr>
<tr>
<td>5</td>
<td>$-0.444$</td>
<td>$-0.593$</td>
<td>$0.644$</td>
</tr>
<tr>
<td>6</td>
<td>$-0.578$</td>
<td>$-0.493$</td>
<td>$0.576$</td>
</tr>
<tr>
<td>7</td>
<td>$-0.795$</td>
<td>$-0.359$</td>
<td>$0.616$</td>
</tr>
<tr>
<td>8</td>
<td>$-0.827$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>9</td>
<td>$-0.793$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$(\Delta d + \Delta \bar{d}, \Delta \bar{u} + \Delta \bar{d})$</th>
<th>$(\Delta d + \Delta \bar{d}, \Delta \bar{u} - \Delta \bar{d})$</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-0.405$</td>
<td>$0.211$</td>
<td>$-0.760$</td>
</tr>
<tr>
<td>2</td>
<td>$-0.340$</td>
<td>$0.129$</td>
<td>$-0.708$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.294$</td>
<td>$0.081$</td>
<td>$-0.704$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.276$</td>
<td>$0.052$</td>
<td>$-0.685$</td>
</tr>
<tr>
<td>5</td>
<td>$-0.255$</td>
<td>$0.024$</td>
<td>$-0.660$</td>
</tr>
<tr>
<td>6</td>
<td>$-0.158$</td>
<td>$-0.055$</td>
<td>$-0.635$</td>
</tr>
<tr>
<td>7</td>
<td>$0.251$</td>
<td>$-0.414$</td>
<td>$-0.649$</td>
</tr>
<tr>
<td>8</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>9</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table C.24: Correlations among Results of the flavor asymmetry extraction with the new definition of purity. The last column shows the minimum value of $\chi^2$ in Eq.5.9.
C.4.4 Single Quark Distributions

Results with the Mixing Matrix

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\Delta u$</th>
<th>$\Delta d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.785 \pm 1.002$</td>
<td>$-2.299 \pm 1.060$</td>
</tr>
<tr>
<td>2</td>
<td>$1.570 \pm 0.636$</td>
<td>$-1.709 \pm 0.697$</td>
</tr>
<tr>
<td>3</td>
<td>$1.758 \pm 0.464$</td>
<td>$-0.986 \pm 0.516$</td>
</tr>
<tr>
<td>4</td>
<td>$1.956 \pm 0.387$</td>
<td>$-0.746 \pm 0.435$</td>
</tr>
<tr>
<td>5</td>
<td>$1.442 \pm 0.293$</td>
<td>$-0.555 \pm 0.335$</td>
</tr>
<tr>
<td>6</td>
<td>$1.239 \pm 0.236$</td>
<td>$-0.306 \pm 0.268$</td>
</tr>
<tr>
<td>7</td>
<td>$1.057 \pm 0.160$</td>
<td>$-0.070 \pm 0.189$</td>
</tr>
<tr>
<td>8</td>
<td>$0.639 \pm 0.184$</td>
<td>$-0.661 \pm 0.215$</td>
</tr>
<tr>
<td>9</td>
<td>$0.623 \pm 0.123$</td>
<td>$-0.012 \pm 0.151$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\Delta \bar{u}$</th>
<th>$\Delta \bar{d}$</th>
<th>$\Delta \bar{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0.197 \pm 0.947$</td>
<td>$-0.048 \pm 0.947$</td>
<td>$0.035 \pm 0.671$</td>
</tr>
<tr>
<td>2</td>
<td>$0.371 \pm 0.591$</td>
<td>$-0.936 \pm 0.591$</td>
<td>$-0.491 \pm 0.405$</td>
</tr>
<tr>
<td>3</td>
<td>$0.083 \pm 0.425$</td>
<td>$-0.665 \pm 0.425$</td>
<td>$-0.467 \pm 0.289$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.279 \pm 0.353$</td>
<td>$-0.274 \pm 0.353$</td>
<td>$-0.276 \pm 0.243$</td>
</tr>
<tr>
<td>5</td>
<td>$0.061 \pm 0.266$</td>
<td>$-0.618 \pm 0.266$</td>
<td>$-0.387 \pm 0.186$</td>
</tr>
<tr>
<td>6</td>
<td>$0.051 \pm 0.215$</td>
<td>$-0.655 \pm 0.215$</td>
<td>$-0.415 \pm 0.145$</td>
</tr>
<tr>
<td>7</td>
<td>$-0.126 \pm 0.142$</td>
<td>$0.251 \pm 0.142$</td>
<td>$0.123 \pm 0.084$</td>
</tr>
<tr>
<td>8</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>9</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table C.25: Results of single quark distribution with the mixing matrix. Only statistic errors are shown.
Results with the New Definition of Purity

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\Delta u$</th>
<th>$\Delta d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2.067 \pm 1.092$</td>
<td>$-2.008 \pm 1.142$</td>
</tr>
<tr>
<td>2</td>
<td>$1.541 \pm 0.637$</td>
<td>$-1.729 \pm 0.696$</td>
</tr>
<tr>
<td>3</td>
<td>$1.768 \pm 0.435$</td>
<td>$-0.975 \pm 0.489$</td>
</tr>
<tr>
<td>4</td>
<td>$1.903 \pm 0.339$</td>
<td>$-0.797 \pm 0.391$</td>
</tr>
<tr>
<td>5</td>
<td>$1.421 \pm 0.246$</td>
<td>$-0.561 \pm 0.291$</td>
</tr>
<tr>
<td>6</td>
<td>$1.203 \pm 0.194$</td>
<td>$-0.331 \pm 0.231$</td>
</tr>
<tr>
<td>7</td>
<td>$1.062 \pm 0.151$</td>
<td>$-0.056 \pm 0.181$</td>
</tr>
<tr>
<td>8</td>
<td>$0.668 \pm 0.164$</td>
<td>$-0.519 \pm 0.195$</td>
</tr>
<tr>
<td>9</td>
<td>$0.620 \pm 0.103$</td>
<td>$0.109 \pm 0.127$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bin</th>
<th>$\Delta \bar{u}$</th>
<th>$\Delta \bar{d}$</th>
<th>$\Delta \bar{s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0.112 \pm 1.035$</td>
<td>$-1.091 \pm 1.035$</td>
<td>$-0.681 \pm 0.743$</td>
</tr>
<tr>
<td>2</td>
<td>$0.369 \pm 0.592$</td>
<td>$-0.804 \pm 0.592$</td>
<td>$-0.405 \pm 0.406$</td>
</tr>
<tr>
<td>3</td>
<td>$-0.112 \pm 0.396$</td>
<td>$-0.564 \pm 0.396$</td>
<td>$-0.410 \pm 0.267$</td>
</tr>
<tr>
<td>4</td>
<td>$-0.300 \pm 0.305$</td>
<td>$0.042 \pm 0.305$</td>
<td>$-0.074 \pm 0.205$</td>
</tr>
<tr>
<td>5</td>
<td>$0.066 \pm 0.218$</td>
<td>$-0.570 \pm 0.218$</td>
<td>$-0.353 \pm 0.147$</td>
</tr>
<tr>
<td>6</td>
<td>$0.031 \pm 0.172$</td>
<td>$-0.381 \pm 0.172$</td>
<td>$-0.241 \pm 0.111$</td>
</tr>
<tr>
<td>7</td>
<td>$-0.133 \pm 0.133$</td>
<td>$0.224 \pm 0.133$</td>
<td>$0.103 \pm 0.076$</td>
</tr>
<tr>
<td>8</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>9</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table C.26: Results of single quark distribution with the new definition of purity. Only statistic errors are shown.
C.5 Moments of Polarized Quark Distributions

C.5.1 Valence Decomposition

Results with the Mixing Matrix

<table>
<thead>
<tr>
<th></th>
<th>(N_q)</th>
<th>(\alpha_q)</th>
<th>(\chi^2/\text{ndf})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta u_v)</td>
<td>0.77 (\pm) 0.10</td>
<td>0.42 (\pm) 0.08</td>
<td>1.53/9</td>
</tr>
<tr>
<td>(\Delta d_v)</td>
<td>-0.01 (\pm) 0.01</td>
<td>-1.24 (\pm) 0.28</td>
<td>0.57/9</td>
</tr>
</tbody>
</table>

Table C.27: Fit parameters of the valence quarks in the valence decomposition with the mixing matrix.

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-(x)</th>
<th>high-(x)</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta u_v)</td>
<td>0.56 (\pm) 0.09</td>
<td>0.04</td>
<td>0.02</td>
<td>0.62 (\pm) 0.09</td>
</tr>
<tr>
<td>(\Delta d_v)</td>
<td>-0.14 (\pm) 0.16</td>
<td>-0.03</td>
<td>0.00</td>
<td>-0.16 (\pm) 0.16</td>
</tr>
<tr>
<td>(\Delta q_s)</td>
<td>-0.03 (\pm) 0.05</td>
<td>-0.00</td>
<td>0.00</td>
<td>-0.03 (\pm) 0.05</td>
</tr>
</tbody>
</table>

Table C.28: First moment of polarized quark distributions in the valence decomposition with the mixing matrix.

Results with the New Definition of Purity

<table>
<thead>
<tr>
<th></th>
<th>(N_q)</th>
<th>(\alpha_q)</th>
<th>(\chi^2/\text{ndf})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta u_v)</td>
<td>0.71 (\pm) 0.09</td>
<td>0.37 (\pm) 0.08</td>
<td>1.08/9</td>
</tr>
<tr>
<td>(\Delta d_v)</td>
<td>-0.01 (\pm) 0.01</td>
<td>-1.01 (\pm) 0.28</td>
<td>1.17/9</td>
</tr>
</tbody>
</table>

Table C.29: Fit parameters of the valence quarks in the valence decomposition with the new definition of purity.

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-(x)</th>
<th>high-(x)</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta u_v)</td>
<td>0.56 (\pm) 0.09</td>
<td>0.04</td>
<td>0.02</td>
<td>0.62 (\pm) 0.09</td>
</tr>
<tr>
<td>(\Delta d_v)</td>
<td>-0.14 (\pm) 0.15</td>
<td>-0.02</td>
<td>0.00</td>
<td>-0.17 (\pm) 0.15</td>
</tr>
<tr>
<td>(\Delta q_s)</td>
<td>-0.03 (\pm) 0.05</td>
<td>-0.00</td>
<td>0.00</td>
<td>-0.04 (\pm) 0.05</td>
</tr>
</tbody>
</table>

Table C.30: First moment of polarized quark distributions in the valence decomposition with the new definition of purity.
C.5.2 Flavor Decomposition

Results with the Mixing Matrix

<table>
<thead>
<tr>
<th></th>
<th>( N_q )</th>
<th>( \alpha_\eta )</th>
<th>( \chi^2/\text{ndf} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta u + \Delta \bar{u} )</td>
<td>0.90 ± 0.06</td>
<td>0.67 ± 0.03</td>
<td>1.89/9</td>
</tr>
<tr>
<td>( \Delta d + \Delta \bar{d} )</td>
<td>-0.21 ± 0.07</td>
<td>0.19 ± 0.12</td>
<td>1.53/9</td>
</tr>
</tbody>
</table>

Table C.31: Fit parameters of the valence quarks in the flavor decomposition with the mixing matrix.

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-( x )</th>
<th>high-( x )</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta u + \Delta \bar{u} )</td>
<td>0.52 ± 0.04</td>
<td>0.04</td>
<td>0.02</td>
<td>0.57 ± 0.04</td>
</tr>
<tr>
<td>( \Delta d + \Delta \bar{d} )</td>
<td>-0.22 ± 0.08</td>
<td>-0.03</td>
<td>0.00</td>
<td>-0.26 ± 0.08</td>
</tr>
<tr>
<td>( \Delta q_s )</td>
<td>-0.03 ± 0.05</td>
<td>-0.00</td>
<td>0.00</td>
<td>-0.03 ± 0.05</td>
</tr>
</tbody>
</table>

Table C.32: First moment of polarized quark distributions in the flavor decomposition with the mixing matrix.

Results with the New Definition of Purity

<table>
<thead>
<tr>
<th></th>
<th>( N_q )</th>
<th>( \alpha_\eta )</th>
<th>( \chi^2/\text{ndf} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta u + \Delta \bar{u} )</td>
<td>0.92 ± 0.06</td>
<td>0.68 ± 0.03</td>
<td>1.94/9</td>
</tr>
<tr>
<td>( \Delta d + \Delta \bar{d} )</td>
<td>-0.20 ± 0.07</td>
<td>0.19 ± 0.12</td>
<td>1.05/9</td>
</tr>
</tbody>
</table>

Table C.33: Fit parameters of the valence quarks in the flavor decomposition with the new definition of purity.

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-( x )</th>
<th>high-( x )</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta u + \Delta \bar{u} )</td>
<td>0.52 ± 0.04</td>
<td>0.04</td>
<td>0.02</td>
<td>0.57 ± 0.04</td>
</tr>
<tr>
<td>( \Delta d + \Delta \bar{d} )</td>
<td>-0.21 ± 0.07</td>
<td>-0.03</td>
<td>0.00</td>
<td>-0.24 ± 0.07</td>
</tr>
<tr>
<td>( \Delta q_s )</td>
<td>-0.03 ± 0.05</td>
<td>-0.00</td>
<td>0.00</td>
<td>-0.04 ± 0.05</td>
</tr>
</tbody>
</table>

Table C.34: First moment of polarized quark distributions in the flavor decomposition with the new definition of purity.
C.5.3 Flavor Asymmetry of Light Sea Quarks

Results with the Mixing Matrix

<table>
<thead>
<tr>
<th></th>
<th>$N_q$</th>
<th>$\alpha_\bar{q}$</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u + \Delta \bar{u}$</td>
<td>0.89 ± 0.08</td>
<td>0.63 ± 0.05</td>
<td>1.31/9</td>
</tr>
<tr>
<td>$\Delta d + \Delta \bar{d}$</td>
<td>-0.17 ± 0.06</td>
<td>0.12 ± 0.14</td>
<td>0.81/9</td>
</tr>
</tbody>
</table>

Table C.35: Fit parameters of the valence quarks in the flavor asymmetry extraction with the mixing matrix.

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-x</th>
<th>high-x</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u + \Delta \bar{u}$</td>
<td>0.54 ± 0.05</td>
<td>0.04</td>
<td>0.02</td>
<td>0.60 ± 0.05</td>
</tr>
<tr>
<td>$\Delta d + \Delta \bar{d}$</td>
<td>-0.21 ± 0.09</td>
<td>-0.03</td>
<td>0.00</td>
<td>-0.25 ± 0.09</td>
</tr>
<tr>
<td>$\Delta \bar{u} + \Delta \bar{d}$</td>
<td>-0.09 ± 0.09</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.11 ± 0.09</td>
</tr>
<tr>
<td>$\Delta \bar{u} - \Delta \bar{d}$</td>
<td>0.10 ± 0.10</td>
<td>0.02</td>
<td>0.00</td>
<td>0.12 ± 0.10</td>
</tr>
</tbody>
</table>

Table C.36: First moment of polarized quark distributions in the flavor asymmetry extraction with the mixing matrix.

Results with the New Definition of Purity

<table>
<thead>
<tr>
<th></th>
<th>$N_q$</th>
<th>$\alpha_\bar{q}$</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u + \Delta \bar{u}$</td>
<td>0.83 ± 0.08</td>
<td>0.61 ± 0.05</td>
<td>1.03/9</td>
</tr>
<tr>
<td>$\Delta d + \Delta \bar{d}$</td>
<td>-0.18 ± 0.07</td>
<td>0.16 ± 0.14</td>
<td>0.52/9</td>
</tr>
</tbody>
</table>

Table C.37: Fit parameters of the valence quarks in the flavor asymmetry extraction with the new definition of purity.

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-x</th>
<th>high-x</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta u + \Delta \bar{u}$</td>
<td>0.53 ± 0.05</td>
<td>0.04</td>
<td>0.02</td>
<td>0.59 ± 0.05</td>
</tr>
<tr>
<td>$\Delta d + \Delta \bar{d}$</td>
<td>-0.19 ± 0.08</td>
<td>-0.03</td>
<td>0.00</td>
<td>-0.22 ± 0.08</td>
</tr>
<tr>
<td>$\Delta \bar{u} + \Delta \bar{d}$</td>
<td>-0.09 ± 0.08</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.10 ± 0.08</td>
</tr>
<tr>
<td>$\Delta \bar{u} - \Delta \bar{d}$</td>
<td>0.09 ± 0.09</td>
<td>0.02</td>
<td>0.00</td>
<td>0.11 ± 0.09</td>
</tr>
</tbody>
</table>

Table C.38: First moment of polarized quark distributions in the flavor asymmetry extraction with the new definition of purity.
C.5.4 Single Quark Distributions

Results with the Mixing Matrix

<table>
<thead>
<tr>
<th></th>
<th>(N_\eta)</th>
<th>(\alpha_\eta)</th>
<th>(\chi^2/\text{ndf})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta u)</td>
<td>0.81 ± 0.18</td>
<td>0.53 ± 0.12</td>
<td>0.56/9</td>
</tr>
<tr>
<td>(\Delta d)</td>
<td>-0.13 ± 0.07</td>
<td>-0.09 ± 0.21</td>
<td>1.11/9</td>
</tr>
</tbody>
</table>

Table C.39: Fit parameters of the valence quarks in the single quark distribution with the mixing matrix.

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-(x)</th>
<th>high-(x)</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta u)</td>
<td>0.54 ± 0.13</td>
<td>0.04</td>
<td>0.02</td>
<td>0.59 ± 0.13</td>
</tr>
<tr>
<td>(\Delta d)</td>
<td>-0.22 ± 0.14</td>
<td>-0.03</td>
<td>0.00</td>
<td>-0.25 ± 0.14</td>
</tr>
<tr>
<td>(\Delta \bar{u})</td>
<td>0.01 ± 0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01 ± 0.07</td>
</tr>
<tr>
<td>(\Delta \bar{d})</td>
<td>-0.10 ± 0.07</td>
<td>-0.02</td>
<td>0.00</td>
<td>-0.11 ± 0.07</td>
</tr>
<tr>
<td>(\Delta \bar{s})</td>
<td>-0.06 ± 0.05</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.07 ± 0.05</td>
</tr>
</tbody>
</table>

Table C.40: First moment of polarized quark distributions in the single quark distribution with the mixing matrix.

Results with the New Definition of Purity

<table>
<thead>
<tr>
<th></th>
<th>(N_\eta)</th>
<th>(\alpha_\eta)</th>
<th>(\chi^2/\text{ndf})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta u)</td>
<td>0.83 ± 0.17</td>
<td>0.54 ± 0.11</td>
<td>0.68/9</td>
</tr>
<tr>
<td>(\Delta d)</td>
<td>-0.12 ± 0.07</td>
<td>-0.12 ± 0.21</td>
<td>0.98/9</td>
</tr>
</tbody>
</table>

Table C.41: Fit parameters of the valence quarks in the single quark distribution with the new definition of purity.

<table>
<thead>
<tr>
<th></th>
<th>measured region</th>
<th>low-(x)</th>
<th>high-(x)</th>
<th>total integral</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta u)</td>
<td>0.54 ± 0.12</td>
<td>0.04</td>
<td>0.02</td>
<td>0.60 ± 0.12</td>
</tr>
<tr>
<td>(\Delta d)</td>
<td>-0.18 ± 0.13</td>
<td>-0.03</td>
<td>0.00</td>
<td>-0.21 ± 0.13</td>
</tr>
<tr>
<td>(\Delta \bar{u})</td>
<td>0.00 ± 0.06</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00 ± 0.06</td>
</tr>
<tr>
<td>(\Delta \bar{d})</td>
<td>-0.09 ± 0.06</td>
<td>-0.02</td>
<td>0.00</td>
<td>-0.10 ± 0.06</td>
</tr>
<tr>
<td>(\Delta \bar{s})</td>
<td>-0.06 ± 0.04</td>
<td>-0.01</td>
<td>0.00</td>
<td>-0.07 ± 0.04</td>
</tr>
</tbody>
</table>

Table C.42: First moment of polarized quark distributions in the single quark distribution with the new definition of purity.
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