Optical Alignment System for the PHENIX Muon Tracker

Hiroki Kanoh

Department of Physics
Graduate School of Science and Engineering
Tokyo Institute of Technology
Shibata Laboratory

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Abstract

PHENIX (Pioneering High Energy Nuclear Interaction eXperiment) is one of the experiments which use Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory. RHIC can accelerate both various ionized nuclei, proton to gold and polarized protons.

One of the physics goals of PHENIX experiment is the measurement of spin structure in nucleon. Proton has spin 1/2. The fraction of proton spin arising from its three valence quarks is small, about 20% to 30%. This has been studied in depth in deep inelastic scattering (DIS) experiments through 1980’s and 1990’s. Polarized proton-proton collision is a new tool to measure proton spin structure.

PHENIX detector consists of two central arms and two muon arms. The central arm is the detector to measure electrons, photons and hadrons. The muon arm is the detector to measure muons. It is composed of muon magnet, muon tracker (MuTr), and muon identifier (MuID).

The muon tracker is placed in the muon magnet. It has three layers of tracking chambers. These layers are called stations. By tracking a muon which is bent with the magnetic field, the momenta of muons are measured. The position resolution of the muon tracker is decided from chamber resolution and accuracy for the relative position of tracking chambers. Chamber resolution is about 70 $\mu$m to 100 $\mu$m for 1 station. Each chamber moves 50 to 300 $\mu$m by the magnetic field or temperature excursion during the experiment period. In order to achieve better momentum resolution, we should correct for these relative movement. The purpose of the optical alignment system (OASYS) is the real-time monitoring of the relative alignment among the stations.

The OASYS consists of a light source at the 1st station, a convex lens at the 2nd station, and a CCD camera at the 3rd station. When an individual station moves, the image on the CCD camera moves. The relation between CCD image movement and station movement is already studied by T. Watanabe. I improved his method and succeeded to express the chamber movement with six parameters, and archived the relative alignment accuracy of less than 40 $\mu$m.
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Chapter 1

Introduction

Spin is the most essential nature of the particle. The fermions which compose matter, such as quarks and leptons, have spin $\frac{1}{2}$. Proton has also spin $\frac{1}{2}$. In deep inelastic scattering (DIS) experiments through the 1960s, it became clear that proton is composed of quark, anti-quark, and gluon. We expect that proton spin should be dominantly contributed from three valence quarks. The experiment of the European Muon Collaboration(EMC) has shown that its contribution is small\[1\][2]. At present, contribution of quark spin to the proton spin is measured to be 20 % to 30 %, in SMC(Spin muon Collaboration)[3][4],SLAC[5], and HERMES[6]. We consider the rest of the proton spin should be carried by gluon and orbital angular momentum of parton. In this picture, proton spin is written as follows.

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + \Delta L \quad (1.1)$$

Here, $\Delta \Sigma$ is the sum of quark spin contributions,

$$\Delta \Sigma = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} + \Delta s + \Delta \bar{s} \quad (1.2)$$

$\Delta G$ is contribution from gluon spin, $\Delta L$ is contribution from orbital angular momentum.

RHIC(Relativistic Heavy Ion Collider) at BNL(Brookhaven National Laboratory) accelerates polarized protons. The design value of the proton polarization at RHIC is 70 % and design value of the integrated luminosity is 320 pb$^{-1}$ at $\sqrt{s} = 200$GeV. PHENIX experiment in RHIC measures $\Delta G$ using polarized proton-proton collision. $\Delta G$ is measured using direct photon production ($gg \rightarrow g\gamma$) or heavy quark production ($gg \rightarrow Q\bar{Q}$). On the other hand, it is also possible to distinguish quark flavor by using $W$ production mechanism. Muon plays an important role in these measurement because $W$ is identified by detecting muons.
This thesis is organized as follows. The second chapter describes spin physics of the PHENIX experiment. The measurement of spin structure of the proton, using muon detection is explained. The third chapter describes the construction of PHENIX muon detector. The fourth chapter describes overview of muon measurement and how momentum resolution and invariant mass resolution of dimuon are evaluated from detector specifications. The fifth chapter describes relative movement of tracking chamber in detail and how to correct for it. The result is also presented in fifth chapter. The sixth chapter describes the conclusion.
Chapter 2

Physics with the PHENIX Muon Arm

2.1 General idea of measurement

Figure 2.1 shows general idea of measurement based on QCD (Quantum Chromodynamics) parton model framework. Here, A and B mean proton beams which collide each other. Particle “a” means parton in proton A, and particle “b” is parton in proton B. Particles “c” and “d” are final states of parton after scattering $ab \rightarrow cd$. Particle c fragments into C as probe. $f$ is a parton distribution function in proton. $D$ is fragmentation function.

![Figure 2.1: General idea of measurement](image)

In this parton model framework, cross section for process $AB \rightarrow CX$ is writ-
\[ d\sigma_{AB \rightarrow CX}^{AB} = \sum_{abcd} \int dx_a dx_b dz f_{a/A}(x_a) f_{b/B}(x_b) d\sigma_{ab \rightarrow cd} D_{C/c}(z) \] (2.1)

where, \( x_a \) is momentum fraction of parton \( a \) in proton \( A \). We obtain polarized parton distribution function \( \Delta f \) by measuring asymmetry \( A \) with respect to proton spin state.

\[ A = \frac{d\Delta \sigma_{AB \rightarrow CX}}{d\sigma_{AB \rightarrow CX}} \sim \sum_{abcd} \frac{\Delta f_{a/A}}{f_{a/A}} \frac{\Delta f_{b/B}}{f_{b/B}} \alpha_{LL}(ab \rightarrow cd) \] (2.2)

where parton level spin asymmetry \( \alpha_{LL}(ab \rightarrow cd) = \frac{d\Delta \sigma_{ab \rightarrow cd}}{d\sigma_{ab \rightarrow cd}} \) is calculated from perturbative QCD theory.

### 2.2 Measurement of polarized gluon distribution

In order to measure polarized gluon distribution function, we can use heavy quarkonium (charmonium and bottomonium) production process. For example, we can obtain polarized gluon distribution function using \( J/\psi \) production process \( gg \rightarrow J/\psi X \).
2.2. MEASUREMENT OF POLARIZED GLUON DISTRIBUTION

\[ A_{LL}^{gg \rightarrow J/\psi X} \sim \frac{\Delta f(x_1) f(x_2)}{f(x_1)} a_{LL}(gg \rightarrow J/\psi X) \] (2.3)

\[ a_{LL}(gg \rightarrow J/\psi X) = \frac{d\Delta \sigma(gg \rightarrow J/\psi X)}{d\sigma(gg \rightarrow J/\psi X)} \] (2.4)

where double spin asymmetry \( A_{LL} \) is defined as follows.

\[ A_{LL} = \frac{\sigma(++) + \sigma(--) - \sigma(+-) - \sigma(-+)}{\sigma(++) + \sigma(--) + \sigma(+-) + \sigma(-+)} \] (2.5)

“++”, “−−”, “+−” and “−+” express proton beam helicity configurations. “+” means positive helicity and “−” means negative helicity.

\( J/\psi \) is the charmonium which is \( n^{2S+1} L_J = 1^3 S_1 \) state. The level scheme and mass and decay modes for charmonia are shown in figure 2.3 and table 2.1, 2.2.

\[ \text{Figure 2.3: Level scheme of charmonium states} \]

It is considered that charmonia are dominantly produced via gluon fusion process \((gg \rightarrow Q\bar{Q})\) at the PHENIX. Especially, s-wave state charmonium, such as \( J/\psi(1S) \) or \( \psi'(2S) \), is useful for \( \Delta g \) measurement because of their very clear experimental signal. These s-wave charmonia decay into positive and negative charged muon pair. It is very easy to identify it from dimuon invariant mass spectrum.
### CHAPTER 2. PHYSICS WITH THE PHENIX MUON ARM

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<tr>
<th>parent</th>
<th>decay modes</th>
<th>fraction</th>
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<td>$J/\psi (1S)$</td>
<td>$\mu^+\mu^-$</td>
<td>$(5.88 \pm 0.10)%$</td>
</tr>
<tr>
<td>$\psi (2S)$</td>
<td>$\mu^+\mu^-$</td>
<td>$(7.3 \pm 0.8) \times 10^{-3}%$</td>
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<tr>
<td>$J/\psi (1S)$</td>
<td>J/$\psi (1S)$anything</td>
<td>$(57.6 \pm 2.0)%$</td>
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<tr>
<td></td>
<td>$\gamma \chi c_1 (1P)$</td>
<td>$(8.4 \pm 0.8)%$</td>
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<tr>
<td></td>
<td>$\gamma \chi c_2 (1P)$</td>
<td>$(6.4 \pm 0.6)%$</td>
</tr>
<tr>
<td>$\chi c_1 (1P)$</td>
<td>$\gamma J\psi (1S)$</td>
<td>$(31.6 \pm 3.3)%$</td>
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<tr>
<td>$\chi c_2 (1P)$</td>
<td>$\gamma J\psi (1S)$</td>
<td>$(5.88 \pm 0.10)%$</td>
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Table 2.1: Mass of charmonium states and $D\bar{D}$ pair

Table 2.2: decay modes for several charmonium states

However, there are theoretical ambiguity on the charmonium production mechanism ($a_{LL}$ in equation 2.3), and we should resolve it first.

### 2.3 Quarkonium production

The process of quarkonium production[7] in hadron collision is separated into two stages, the creation of heavy quark pair in various color states and the subsequent non-perturbative transition from the intermediate $Q\bar{Q}$ pair to a physical quarkonium. The first stage is a short-distance process on scales of the order $1/m_Q$ or smaller and calculable in perturbation theory, where $m_Q$ is the mass of heavy quark. On the other hand, the second stage has a long-distance scale of the order of the quarkonium size $1/m_Q v$ or larger. These short-distance and long-distance processes can be separated by factorizing binding effects into universal non-perturbative parameters.

The production cross section of quarkions in p-p collision is expressed as follows.
\[
d\sigma_{pp \rightarrow HX} = \sum_{i,j} \int dx_1 dx_2 f_{i/A}(x_1) f_{j/B}(x_2) d\hat{\sigma}(ij \rightarrow HX) \tag{2.6}
\]

where, \(d\hat{\sigma}(ij \rightarrow HX)\) denotes the parton level cross section for quarkonium \(H\) production and depends on production mechanism of qurakonium.

### 2.3.1 Production mechanism

Currently, three models for quarkonium production have been proposed.

**Color Singlet Model**

The color singlet model (CSM) which can unambiguously predict production cross section, was born in the 1980’s. In this model, only \(Q\bar{Q}\) pair that has right quantum number and spin and that is in the color singlet state should form a physical quarkonium. For example, cross section for the \(\chi_J\) production is written as follows.

\[
d\sigma^{CSM}(\chi_J X) = d\hat{\sigma}(Q\bar{Q} [1,3/2] P_J) (2J + 1) \frac{3N_c}{2\pi} |R(0)|^2 \tag{2.7}
\]

where, \(d\hat{\sigma}(Q\bar{Q} [1,3/2] P_J)\) and \((2J + 1) \frac{3N_c}{2\pi} |R(0)|^2\) means the production cross section of \(Q\bar{Q} [1,3/2] P_J\) and the probability of bound state formation from \(Q\bar{Q} [1,3/2] P_J\) state. The cross section is factorised into a short-distance coefficient \(d\hat{\sigma}(Q\bar{Q} [1,3/2] P_J)\) and a single long-distance factor \((2J + 1) \frac{3N_c}{2\pi} |R(0)|^2\).

The CSM model has explained \(p_T\) distributions of \(J/\psi\) production at ISR \((\sqrt{s} = 30\) to 63 GeV) well, however failed to explain \(p_T\) differential cross section of \(J/\psi\) and \(\psi'\) at CDF experiment. The CSM prediction has large discrepancies (factor 30 to 50) from experimental data (“CDF anomaly” see Fig 2.4).

**Color Octet Model**

In order to explain large discrepancies at CDF results, the CSM model has been superseded by the color octet model which is based on non-relativistic QCD (NRQCD) framework. The CSM model considers only color singlet \(Q\bar{Q}\) state, while color octet states are taken into account in the COM model.

To separate short-distance scales from long-distance scales, the effective-field-theory framework of non-relativistic QCD(NRQCD\cite{9}) is used in COM. Within this framework of NRQCD factorisation\cite{8}, the cross section for producing a
quarkonium state $H$ is expressed using a short-distance coefficient and a long-distance matrix element.

$$d\hat{\sigma}(H X) = \sum_n d\hat{\sigma}(Q\bar{Q}[n] X)\langle O^H[n]\rangle \quad (2.8)$$

where $H$, $d\hat{\sigma}(Q\bar{Q}[n] + X)$ and $\langle O^H[n]\rangle$ denotes the created quarkonium states, a short-distance coefficient and a long-distance matrix element. The sum includes all color and angular momentum states of the $Q\bar{Q}$ pair in equation 2.8. The short-distance coefficients $d\hat{\sigma}(Q\bar{Q}[n] X)$ are proportional to the cross section for producing a $Q\bar{Q}$ pair in the state $n$. They can be calculated perturbatively in the strong coupling $\alpha_s^2$. A long-distance matrix element expresses non-perturbative transition probabilities from the $Q\bar{Q}$ state $n$ into the quarkonium $H$ and should be extracted from experiments.

For example, $\chi J$ production cross section is replaced from equation 2.7,

$$d\sigma^{\text{COM}}(\chi_J X) = d\hat{\sigma}(Q\bar{Q} [1, 3 P_J] X)\langle O^{\chi_J} [1, 3 P_J]\rangle + d\hat{\sigma}(Q\bar{Q} [8, 3 S_1] X)\langle O^{\chi_J} [8, 3 S_1]\rangle + O(v^2) \quad (2.9)$$

The first term in equation 2.9 corresponds to the expression of the color singlet and the second term represents a contribution to the cross section from a color octet mechanism.

This COM model has succeeded to explain $p_T$ distributions at CDF as shown in figure 2.4.

**Color Evaporation Model**

The color evaporation model (CEM) takes a different way from color singlet or color octet. $Q\bar{Q}$ pair is produced in various color states and quantum numbers, and a quarkonium is formed by emission of soft gluons. In this picture, a long-distance factor which expresses hadronization of $Q\bar{Q}$ pair into a quarkonium state is decided process independently.

Cross section for a specific quarkonium state $H$ is given as follows.

$$d\sigma^{\text{CEM}}(H) = f_H \int_{2m_Q}^{2m_D} dM_{Q\bar{Q}} \frac{d\sigma(Q\bar{Q})}{dM_{Q\bar{Q}}} \quad (2.10)$$

where $f_H$ is universal long-distance factor. It is assumed that only quark pair below the $D\bar{D}$ can bind into a physical quarkonium state. The CEM describes experimental data at low $p_T$ or $p_T$ integrated results well.
2.3. QUARKONIUM PRODUCTION

2.3.2 $\psi(nS)$ polarization measurement

Polarization of $J^{PC} = 1^{--}$ quarkonium state($J/\psi, \psi', \Upsilon$) provides a sensitive probe to the quarkonium production mechanism. It is pointed out by Cho and Wise[11] that $1^{--}$ state should be transversely polarized at sufficiently large transverse momentum according to the non-relativistic QCD(NRQCD) factorisation. This NRQCD factorisation theory has well explained charmonium production cross section in the CDF experiments. For convenience of the measurement, polarization $\alpha$ is defined as

$$\alpha = \frac{\sigma_T - 2\sigma_L}{\sigma_T + 2\sigma_L}$$ (2.11)

where $\sigma_T$ and $\sigma_L$ are the transverse and longitudinal components of the cross section. The polarization $\alpha$ can be obtained by measuring the lepton angular distribution in $\psi(nS) \rightarrow l^+l^-$ decay channel. $\psi(nS)$ means $J/\psi(1S)$ and $\psi'(2S)$ in this thesis. The lepton angular distribution generally behaves according to
\[ \frac{d\sigma}{d\cos \theta} (\psi(nS) \rightarrow l^+l^-) \propto (1 + \alpha \cos^2 \theta) \quad (2.12) \]

where \( \theta \) is the angle between the lepton three-momentum in the \( \psi \) rest frame and the \( \psi \) three-momentum in the lab frame. \( \alpha \) ranges from -1 to 1. If NRQCD prediction is correct, \( \alpha \) should be equal to 1.

For the polarization measurement, we use \( J/\psi \) or \( \psi' \). Particularly, \( \psi' \) is suitable for the measurement. Prompt \( J/\psi \) signal includes \( J/\psi \) that has come from decay of higher charmonium states, \( \chi_{c1}, \chi_{c2} \) and \( \psi' \)(shown in table 2.2). The \( \chi'_{cJ} \) mesons are produced in various spin states and decay into \( J/\psi \) through radiative transitions, hence these processes make theoretical predictions of \( J/\psi \) polarization complicated. On the other hand, it is quite simple for the \( \psi' \) case. The \( \chi'_{cJ} \) states lie above the \( DD \) threshold and its branching fraction \( \text{Br}(\chi'_{cJ} \rightarrow \psi' + \gamma) \) is considered to be small. Therefore, it is useful to measure \( \psi' \).

The polarization measurement for \( J/\psi \) and \( \psi' \) has been done at CDF experiment. For the \( H/\psi \) measurement, NRQCD prediction is consistent with experiment at low \( pt \), but it has disagreement with data. \( \psi' \) measurement has not enough statistics.

### 2.4 Quark and anti-quark polarization

\( W \) production provides a powerful tool for the measurement of quark and anti-quark polarization. We obtain anti-quark polarization using \( u\bar{d} \rightarrow W^+ \) and \( d\bar{u} \rightarrow W^- \).

\[
A_{W^+} = \frac{\Delta u(x_1)\overline{d}(x_2) - \Delta \overline{d}(x_1)u(x_2)}{u(x_1)\overline{d}(x_2) + \overline{u}(x_1)d(x_2)} \\
\sim -\frac{\Delta \overline{d}(x_1)}{d(x_1)} (x_1 \ll x_2) \quad (2.13)
\]

\[
A_{W^-} = \frac{\Delta d(x_1)\overline{u}(x_2) - \Delta \overline{u}(x_1)d(x_2)}{d(x_1)\overline{u}(x_2) + \overline{d}(x_1)u(x_2)} \\
\sim -\frac{\Delta \overline{u}(x_1)}{\overline{u}(x_1)} (x_1 \ll x_2) \quad (2.14)
\]

where \( x_1(x_2) \) is momentum fraction carried by quarks (anti-quarks) and \( A_L \) is written as follows.
In the equation 2.15, $\sigma(\pm)$ means that one of the proton beams has positive helicity and another is not polarized. $W$ bosons are measured using decay modes, $W^+ \rightarrow \mu^+ \nu$ or $W^- \rightarrow \mu^- \bar{\nu}$ in the muon arm. $A_L$ is measured in PHENIX and $\bar{d}(x)$ and $\bar{u}(x)$ have been measured in DIS experiments. Therefore $\Delta \bar{u}(x)$ and $\Delta \bar{d}(x)$ can be obtained.

2.4.1 $W$ kinematics

$x_1$ and $x_2$ are determined from rapidity of $W_\gamma W$. 

$$A_L = \frac{\sigma(-) - \sigma(\pm)}{\sigma(-) + \sigma(\pm)}$$  (2.15)
Here, the transverse momentum of $W$ is assumed to be zero. $W$ is observed through semi-leptonic decay $W \rightarrow \nu l$ and only charged lepton is detected at PHENIX. And therefore, relations between lepton kinematics and $y_W$ is needed. The rapidity of $W$ boson is related to the lepton rapidity in the $W$ rest frame($y^*_{l}$) and in the lab frame($y_{l}^{lab}$).

$$y_{l}^{lab} =  y^*_{l} + y_W$$  (2.18)

$$y^*_{l} = \frac{1}{2} \ln \left[ \frac{1 + \cos \theta^*}{1 - \cos \theta^*} \right]$$  (2.19)
where $\theta^*$ is the decay angle of the lepton in the $W$ rest frame and $\theta^*$ can be determined from the transverse momentum ($p_T$) of the lepton with an irreducible uncertainty of the sign.

\[
p_T^{\text{lepton}} = p_T^* = \frac{M_W}{2} \sin \theta^* \tag{2.20}
\]

The transverse momentum of $W$ is neglected in this reconstruction.
2.4.2 $W$ identification

The cross section for $W$ production in PHENIX is $1.2 \times 10^{-6}$ mb at $\sqrt{s} = 500$ GeV. Considering integrated luminosity $800pb^{-1}$ and acceptance for muons, 8000 events for $W$ is expected. $W$ boson is identified by a high transverse momentum muon. Figure 2.8 shows fractions of $W$, $Z$ and heavy flavor at high momentum muon, which is calculated by PYTHIA. As shown in this figure, $W$ production is dominant at $p > 25$GeV/c, and heavy flavor is negligible in this region. The fraction of $Z$ can be rejected by identifying $Z$ in dimuon event.

![Inclusive $\mu$ Production, 500 GeV/c](image)

Figure 2.8: $W$ production at $\sqrt{q} = 500$ GeV(simulation)

$p_T$ distribution of muons from $W$, $Z$ and heavy quark.(simulation in PHENIX)
Chapter 3

PHENIX Muon Arm

Figure 3.1: PHENIX detector components

are placed at mid-rapidities and muon arms are placed at forward rapidities. The muon arm includes muon magnet[16], muon tracker(MuTr), muon identifier(MuID). The muon tracker is located inside of the muon magnet. Table 3.1 shows $\eta$ and $\phi$ acceptance of these muon subsystems. $\eta$ is the pseudo-rapidity defined as

$$\eta = -\ln \tan \frac{\theta}{2}$$

(3.1)

$\phi$ is the azimuthal angle around the proton beam axis.

Figure 3.2: PHENIX detector (cross section)
Figure 3.3: PHENIX global coordinate system

<table>
<thead>
<tr>
<th>detector</th>
<th>$\eta$ acceptance</th>
<th>$\phi$ acceptance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central Magnet</td>
<td>±0.35</td>
<td>360 degree</td>
</tr>
<tr>
<td>South Muon Magnet</td>
<td>-1.1 to -2.2</td>
<td>360 degree</td>
</tr>
<tr>
<td>North Muon Magnet</td>
<td>1.2 to 2.4</td>
<td>360 degree</td>
</tr>
<tr>
<td>Multiplicity-Vertex Detector</td>
<td>±2.6</td>
<td>360 degree</td>
</tr>
<tr>
<td>Beam-Beam Counter</td>
<td>±(3.1 to 3.9)</td>
<td>360 degree</td>
</tr>
<tr>
<td>Normalization Trigger Counter</td>
<td>±(1 to 2)</td>
<td>360 degree</td>
</tr>
<tr>
<td>Zero-Degree Calorimeter</td>
<td>±2 mrad</td>
<td>360 degree</td>
</tr>
<tr>
<td>Drift Chamber</td>
<td>±0.35</td>
<td>90 degree $\times 2$</td>
</tr>
<tr>
<td>Pad Chamber</td>
<td>±0.35</td>
<td>90 degree $\times 2$</td>
</tr>
<tr>
<td>Time Expansion Chamber</td>
<td>±0.35</td>
<td>90 degree</td>
</tr>
<tr>
<td>Ring Imaging CHERENKOV Counter</td>
<td>±0.35</td>
<td>90 degree $\times 2$</td>
</tr>
<tr>
<td>Time-of-Flight</td>
<td>±0.35</td>
<td>45 degree</td>
</tr>
<tr>
<td>T0</td>
<td>±0.35</td>
<td>45 degree</td>
</tr>
<tr>
<td>PbSc Calorimeter</td>
<td>±0.38</td>
<td>90 + 45 degree</td>
</tr>
<tr>
<td>PbGl Calorimeter</td>
<td>±0.35</td>
<td>45 degree</td>
</tr>
<tr>
<td>Muon Tracker (South)</td>
<td>-1.15 to -2.25</td>
<td>360 degree</td>
</tr>
<tr>
<td>Muon Tracker (North)</td>
<td>1.15 to 2.44</td>
<td>360 degree</td>
</tr>
<tr>
<td>Muon Identifier (South)</td>
<td>-1.15 to 2.25</td>
<td>360 degree</td>
</tr>
<tr>
<td>Muon Identifier (North)</td>
<td>1.15 to 2.44</td>
<td>360 degree</td>
</tr>
</tbody>
</table>

Table 3.1: PHENIX subsystems
CHAPTER 3. PHENIX MUON ARM

3.1 hadron absorber

The central magnet made of iron and the nosecone made of copper play the role of hadron absorber for the muon arm. These material’s width and radiation length are shown in table 3.2.

![Figure 3.4: absorbing materials](image)

<table>
<thead>
<tr>
<th></th>
<th>width</th>
<th>radiation length</th>
</tr>
</thead>
<tbody>
<tr>
<td>nosecone (Cu)</td>
<td>20 cm</td>
<td>1.43 cm</td>
</tr>
<tr>
<td>central magnet (Fe)</td>
<td>60 cm</td>
<td>1.76 cm</td>
</tr>
</tbody>
</table>

Table 3.2: hadron absorber

3.2 Muon Magnet

The north muon magnet is designed with integrated magnetic field $\int B \, dl = 0.72 \text{Tm at } \theta = 15^\circ$. The south muon magnet was added to the PHENIX detector after the original design. It is a little smaller than north muon arm. It is
designed with $\int Bdl = 0.76 \text{Tm}$ at $\theta = 15$. Figure 3.5 shows field line in the magnet. The magnetic field is toward radial direction in the muon magnet, and therefore injected muon is kicked toward azimuthal direction.

![Figure 3.5: fields in the magnet](image)

Magnetic field is radial in the muon magnet.

<table>
<thead>
<tr>
<th>polar angle</th>
<th>15 degree</th>
<th>20 degree</th>
<th>25 degree</th>
<th>30 degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>North</td>
<td>0.731</td>
<td>0.492</td>
<td>0.359</td>
<td>0.276</td>
</tr>
<tr>
<td>South</td>
<td>0.774</td>
<td>0.492</td>
<td>0.344</td>
<td>0.255</td>
</tr>
</tbody>
</table>

Table 3.3: Integrated field ($\int Bdl/$Tm)

Integrated field in four different polar angles in tesla meter.

### 3.3 Muon Tracker

The PHENIX muon arms are designed to detect dimuon from decay of heavy quarkonium (such as $J/\Psi$, $\Psi'$, $\Upsilon$), vector meson (such as $\rho/\omega,\phi$) and $Z$. It is also required to detect single muons from decay of $W$ mesons.

The muon tracker consists of three tracking chambers called stations. The tracking chamber which is nearest to the collision point is station1, the one most far from collision point is station3, and the one among them is station2. The station1 is separated into four individual cathode strip chambers called “quadrant”
and the station2 and the station3 are separated into eight individual chambers called “octant”.

![Muon Tracker](image)

**Figure 3.6: Muon Tracker**

### 3.3.1 Cathode strip chamber

The cathode strip chamber has two(station3) or three(station1,3) structure called “gap”. One gap is composed of one anode wires and two cathode strip planes. Anode wires are sandwiched between two cathode strip planes. Figure 3.7 shows schematic view of gap construction.

The cathode plane has cathode strips of 5 mm width with accuracy of 25 $\mu$m and strips run in radial directions. The anode plane has alternating structure of sense wire for high voltage and field wire for ground. These wires run in azimuthal direction. One of the cathode planes for each gap is perpendicular to the anode wire(labeled non-stereo). Another cathode plane has diagonal direction, 3.75 to 11.25 degrees, with respect to non-stereo strips (labeled stereo). Relative angles among stereo and non-stereo strips are shown in table 3.4.
3.3. **MUON TRACKER**

![Cathode strip chamber structure](image)

**Figure 3.7: Cathode strip chamber structure**

<table>
<thead>
<tr>
<th>station</th>
<th>gap</th>
<th>angle (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>-11.25</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>+6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>+11.25</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>+7.5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>+3.75</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>11.25</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>-11.25</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>-11.25</td>
</tr>
</tbody>
</table>

**Table 3.4: Stereo strip angle with respect to non-stereo cathode strip**
Figure 3.8: Structure of the station

Figure 3.9: Octant number
3.4 Muon Identifier

The muon identifier is designed to reject charged pion from muon. Pions produced in the collision are absorbed in the central magnet steel, nosecone, and the 30 cm thick steel plate which is backplate of the muon magnet. Roughly, $1.0 \times 10^{-2}$ of pions compared to produced pions reach to MuID. In order to further decrease this rejection factor, MuID has totally 60 cm steel itself. This steel is separated into four plates of thickness 10 cm, 10cm, 20cm, and 20cm. These four steel plates are sandwiched among the five detectors.

![Figure 3.10: Front of Muon Identifier](image)

The muon identifier consists of four large panels and two small panels. The inscribed circle corresponds to $\theta = 37^\circ$. The Gap1 is the first layer looking from the collision point. The Gap5 is the fifth layer from the collision point.
Figure 3.11: cross section of Muon Identifier

This figure shows the side view of the muon identifier. The red line at the center shows the beam line. The muon identifier is shown in red and pink, and the absorbing materials are shown in green.
3.5 Optical Alignment System

It is important to align relative positions among the three stations, because it affects the momentum measurement (it is discussed in chapter 4). We align position among the three stations using field off run at the beginning of the experiment [24]. However, each station moved 100-300 $\mu$m during the experiment period. In order to monitor this real-time movement, an optical alignment system (OASYS) has been installed into muon tracking chamber.

The OASYS consists of a light source at station 1, a convex lens at station 2, and a CCD camera at station 3. When an individual station moves, the image on the CCD camera moves reflecting the station movement. Similar alignment systems are used in L3 [18], GEM SSC Muon System [19], and ATLAS [20] muon spectrometer. There are 7 sets of optics on each octant. There are $7(\text{octics/octant}) \times 8(\text{octant/arm}) \times 2(\text{arm}) = 112$ optics in total. By observing the position of the light spot on the image of the CCD camera, we can monitor each station’s movement.

Positions where CCD camera is installed are shown in figure 3.13. This is a view from proton-proton collision vertex. Each CCD camera is tagged by its number 1 to 7. This id number is used in this thesis.
3.5.1 Optics specifications

We use a halogen lamp and optical fiber as a light source for the OASYS. Optical fibers guide light from the halogen lamp It is attached on the edge of station1. The radius of fiber core is $31.25 \, \mu m$. Lifetime of the halogen lamp is about four thousand hours.

CCD cameras used in OASYS have $8.8 \, mm(768 \text{ (horizontal pixel)}) \times 6.6 \, mm \text{ (498 (vertical pixel))}$ acceptance. Size of 1 pixel segment is $11.0 \, \mu m \text{ (horizontal)} \times 13.0 \, \mu m \text{ (vertical)}$. However, when the video signal is read in a PCI capture board, only 640 pixel (horizontal) $\times$ 480 pixel (vertical) range is selected.

![Figure 3.13: CCD camera position and their number](image)

![Figure 3.14: Image of light source. Diameter is about 15\(\mu m\)](image)
3.5. OPTICAL ALIGNMENT SYSTEM

3.5.2 Image position determination

Figure 3.14 shows obtained image from CCD camera. We determine the center position of the image by method as follows.

First, we make two histograms as shown in figure 3.15 which are projected image on the horizontal (upper one) or vertical (lower one) axis. In these histograms, horizontal axis means pixel number and vertical axis means integrated brightness of image. We fit these histograms using a function expressed in below.

\[
f(x) = p[0] + p[3] \exp\left(-\frac{(x - p[1])^2}{2p[2]^2}\right)
\]  

(3.2)

This function is a combination of a Gaussian and a constant term. Here, \( p[i] \) \((i = 0, 1, 2, 3)\) means fitting parameters. \( p[0] \) is offset, \( p[1] \) is the peak position, \( p[2] \) is the width of distribution, and \( p[3] \) is the peak height. We define \( p[2] \) as the image position. Furthermore, we define the fitting chi square \( \chi^2_{fit} \) which evaluates the fitting quality.
CHAPTER 3. PHENIX MUON ARM

\[ \chi^2_{\text{fit}} = \sum_{i=0}^{N_{\text{pixel}}} (f(x_{\text{pixel}}) - y_{\text{pixel}})^2 \]  \hspace{1cm} (3.3)

where \( x_{\text{pixel}} \) is horizontal position of histogram and \( y_{\text{pixel}} \) is brightness of each \( x_{\text{pixel}} \). \( f(x_{\text{pixel}}) \) means the obtained brightness from fitting. We call this fitting method “gauss fit” in this thesis.

There are some unfocused image as shown in figure 3.16, because the focal point of the lens is not perfectly on the CCD camera. When we make histograms from these unfocused images. The images are not Gaussian like shape(figure 3.17). Therefore, we use specified fitting method called “window fit”. The window fit procedure is as follows. At first, we fit histograms using equation 3.2 and determine the peak position. Then, we reject \( 0.7 \times p[2] \) range around the peak position, and fit histograms again using equation 3.2. Figure 3.17 is the result of window fit.

![Image of light source (broad peak)](image)

Figure 3.16: Image of light source (broad peak)

3.5.3 Focal position resolution for the CCD camera

The focal position resolution is measured by J. Murata et al.[17] The results of measurement are shown in figure 3.18. In this figure, peak position distributions for 1000 samples obtained within 30 minutes for the typical sharp image(such as figure 3.14) and for the typical broad image(such as figure 3.16) are displayed. The measured resolution is 1.4 \( \mu \text{m} \) for sharp image, and 3.1 \( \mu \text{m} \) for the broad image.
Figure 3.17: Histogram made from CCD image (broad peak)
Figure 3.18: Resolution for peak position determination
Chapter 4

Resolution study on the PHENIX Muon Tracker

4.1 Overview of momentum measurement

Muons which enter into the muon tracker are bent toward azimuthal direction by the magnetic field. We measure the momentum of muons by using sagitta which is the deviation of hit in station2 from the straight line between the hit...
CHAPTER 4. RESOLUTION STUDY ON THE PHENIX MUON TRACKER

in station1 and the hit in station 3. Figure 4.2 shows trajectory of muon track, where hits in station 1, 2 and 3 are \( A(x_1), B(x_2) \) and \( C(x_3) \). Here, three vectors \( x_i (i = 1, 2, 3) \) are positions of hits on each station. Sagitta is defined as length of line \( BS \) and expressed using hits on each stations \( x_i \) as follows, because S is dividing point between A and C.

\[
sagitta = |BS| = \left| \frac{1}{l_1 + l_2} \{l_2(x_1 - x_2) + l_1(x_3 - x_2)\} \right| \quad (4.1)
\]

Figure 4.2: muon trajectory in MuTr

A : hit on station1.
B : hit on station2.
C : hit on station3.
O : the center of arc AC.
P : the intersection point of line AC and line OB.
S : the intersection point of line AC and station2.
r : curvature radius.
where \( l_1 \) is distance between station1 and station2, \( l_2 \) is distance between station2 and station3 as shown in figure 4.1. The vector \( \mathbf{BS} \) is toward azimuthal direction, because muons are bent toward that direction. Therefore, we consider only azimuthal component of three vector \( \mathbf{BS} \) in the equation 4.1. Defining azimuthal components of hits for each station as \( x_i, (i = 1, 2, 3) \) and then, sagitta is re-defined as scalar type quantity.

\[
\text{sagitta} \equiv |\mathbf{BS}|
\]

\[
= \frac{1}{l_1 + l_2} \left\{ l_2(x_1 - x_2) + l_1(x_3 - x_2) \right\}
\]

\[
\equiv s 
\]

(4.2)

This sagitta corresponds to the momentum of the muon one to one through the curvature radius and strength of magnetic field. First, we will calculate their relationship. In figure 4.2, \( \Delta APQ \sim \Delta CPR \) and similar ratio is,

\[
\frac{AP}{CP} = \frac{AS - SP}{CS + SP}
\]

\[
= \frac{AS}{CS} \left( 1 - \frac{SP}{AS} \right)
\]

\[
\simeq \frac{l_1}{l_2} \quad (\because SP \ll AS, CS, \Delta ASE \sim \Delta ACD)
\]

(4.3)

In equation 4.3, accuracy of approximation \( SP \ll AS, CS \) is about \( 10^{-4} \). Considering \( \Delta APQ \) and \( \Delta CPR \),

\[
AQ = r \sin \theta_1
\]

\[
\sim r \theta_1
\]

(4.4)

\[
CR = r \sin \theta_2
\]

\[
\sim r \theta_2
\]

(4.5)

(4.6)

From equation 4.5, 4.7, 4.3 and \( \Delta APQ \sim \Delta CPR \),

\[
AQ : CR = \frac{AP}{CP} \quad (4.8)
\]

\[
= \frac{l_1}{l_2} \quad (4.9)
\]

\[
= r \theta_1 : r \theta_2 \quad (4.10)
\]

\[
\therefore \theta_1 : \theta_2 = l_1 : l_2 \quad (4.11)
\]

Because point P is dividing point between Q and R,
\[ OP = \frac{l_2 OQ + l_1 OR}{l_1 + l_2} \]
\[ = r \left\{ \frac{l_2}{l_1 + l_2} \cos \theta_1 + \frac{l_1}{l_1 + l_2} \cos \theta_2 \right\} \] (4.12)

We can approximate that \( \cos \theta_i \simeq 1 - \frac{1}{2} \theta_i^2 \) because \( \theta_i \ll 1 \)

\[ OP \simeq \frac{r}{l_1 + l_2} \left\{ l_2 \left( 1 - \frac{1}{2} \theta_1^2 \right) + l_1 \left( 1 - \frac{1}{2} \theta_2^2 \right) \right\} \]
\[ = \frac{r}{l_1 + l_2} \left\{ l_1 + l_2 - l_2 \frac{\theta_1^2}{2} - l_1 \frac{\theta_2^2}{2} \right\} \]
\[ = \frac{r}{l_1 + l_2} \left\{ l_1 + l_2 - l_2 \frac{\theta_1^2}{2} - l_1 \left( l_2 \frac{\theta_1^2}{2} \right) \right\} \left( \because eq. 4.11 \right) \]
\[ = \frac{r}{l_1 + l_2} \left\{ (l_1 + l_2) - (l_1 + l_2) \frac{l_1 \theta_1^2}{l_2} \right\} \]
\[ = r \left\{ 1 - \frac{l_1 \theta_1^2}{l_2} \right\} \] (4.13)

From equation 4.13 and \( OB = r \),

\[ PB = OB - OP \]
\[ = r - r \left\{ 1 - \frac{l_1 \theta_1^2}{l_2} \right\} \]
\[ = r \frac{l_2 \theta_1^2}{l_1} \] (4.14)

\( \triangle OAP \sim \triangle BSP \) and then,

\[ BS = PB \frac{OA}{OP} \] (4.15)
\[ = \frac{r \frac{\theta_1^2}{2} l_2}{1 - \frac{\theta_1^2}{2} l_1} \]
\[ \simeq \frac{\theta_1^2}{2} \frac{l_2}{l_1} \] (4.16)
4.1. OVERVIEW OF MOMENTUM MEASUREMENT

Here, define $\theta \equiv \theta_1 + \theta_2$ and $L \equiv l_1 + l_2$ and because $OA = OC = r$,

$$\angle OAC = \frac{\pi - \theta}{2}, \quad (4.17)$$

$$\angle CAD = \frac{\pi}{2} - \angle OAC = \frac{\theta}{2} \quad (4.18)$$

Therefore relation between $L$ and $\theta$ is,

$$L \equiv l_1 + l_2 = AC \cos \left( \frac{\pi - \theta}{2} \right) \simeq AC \quad (4.19)$$

$$\Delta d \equiv CD = AC \sin \left( \frac{\theta}{2} \right) \simeq AC \frac{\theta_1 + \theta_2}{2} \quad (4.20)$$

$$AC = 2r \sin \left( \frac{\theta}{2} \right) \simeq r (\theta_1 + \theta_2)$$

$$= \frac{l_1 + l_2}{l_1} r \theta_1 = L \quad (4.21)$$

$$\therefore l_1 = r \theta_1 \quad (4.22)$$

$$\therefore \Delta d = L \frac{\theta_1 + \theta_2}{2} \quad (4.23)$$

$$= \frac{L^2 \theta_1}{2 l_1} \quad (4.24)$$

$$= \frac{L^2}{2r} \left( \therefore eq\text{4.22} \right) \quad (4.25)$$

From equation 4.16 and 4.22,

$$s = BS \quad (4.26)$$

$$= r \frac{l_1}{2} \left( \frac{l_1}{r} \right)^2 \quad (4.27)$$

$$= \frac{l_1 l_2}{2r} \quad (4.28)$$

There is relationship between muon’s momentum (defined as $p_{tr}$) and curvature radius $r$ as follows.

$$p_{tr} = 0.3 Br \quad (4.29)$$

where $B$ is strength of magnetic field.
Finally, relationship between muon’s momentum $p_{tr}$ and sagitta $s$ is written as follows using equation 4.28 and 4.29

$$s = \frac{l_1l_2}{2} \left( \frac{0.3B}{p_{tr}} \right)$$ \hspace{1cm} (4.30)

or,

$$p_{tr} = \frac{l_1l_2}{2} \left( \frac{0.3B}{s} \right)$$ \hspace{1cm} (4.31)

$\Delta d$ is calculated from equation 4.25 and 4.29,

$$\Delta d = \frac{0.3BL^2}{2p_{tr}}$$ \hspace{1cm} (4.32)

$p_{tr}$ in equation 4.31 is muon’s momentum in the muon tracker. Muons lose their energy in hadron absorber. Considering energy loss in the nosecone($\Delta E_1$) and the central magnet($\Delta E_2$), the original energy of muon $E$ is,

$$E = E_{tr} + \Delta E_1 + \Delta E_2$$ \hspace{1cm} (4.33)

or using the momentum of muons $p$ and $p_{tr}$,

$$\sqrt{p^2 + m_\mu^2} = \sqrt{p_{tr}^2 + m_\mu^2 + \Delta E_1 + \Delta E_2}$$ \hspace{1cm} (4.34)

In the muon tracker, muons have more than 2GeV of momentum due to thickness of absorber, and $\Delta E \equiv \Delta E_1 + \Delta E_2 \sim 1$GeV typically. Therefore, we can approximate $\sqrt{p^2 + m_\mu^2} \sim p$ ($\because m_\mu = 0.106$GeV $\ll p$) and vice versa.

$$p = p_{tr} + \Delta E$$ \hspace{1cm} (4.35)

From equation 4.31 and 4.35, momentum of muons is expressed as,

$$p_i = \frac{l_1l_2}{2} \left( \frac{0.3B}{s} \right) + \Delta E$$ \hspace{1cm} (4.36)

where sagitta $s$ is defined in equation 4.2.
4.2 Single muon momentum resolution

From equation 4.35, the momentum resolution of Muon Tracker is estimated as follows,

\[
\left( \frac{\sigma_p}{p} \right)^2 = \left( \frac{\partial p}{\partial p_\text{tr}} \right)^2 \left( \frac{\sigma_{m_\mu}}{p} \right)^2 + \sum_i \left( \frac{\partial p}{\partial \Delta E_i} \right)^2 \left( \frac{\sigma_{\Delta E_i}}{p} \right)^2
\]

\[(4.37)\]

\[
= \left( \frac{\sigma_{p_\text{tr}}}{p} \right)^2 + \sum_i \left( \frac{\sigma_{\Delta E_i}}{p} \right)^2
\]

\[
= \left( \frac{p_\text{tr}}{p} \right)^2 \left( \frac{\sigma_{p_\text{tr}}}{p_\text{tr}} \right)^2 + \sum_i \left( \frac{\sigma_{\Delta E_i}}{p} \right)^2
\]

\[
= \left( \frac{p - \Delta E}{p} \right)^2 \left( \frac{\sigma_{p_\text{tr}}}{p_\text{tr}} \right)^2 + \sum_i \frac{\sigma_{\Delta E_i}^2}{p^2}
\]

\[(4.38)\]

where \(\sigma_{p_\text{tr}}\) is the momentum resolution of the muon tracker, and \(\sigma_{\Delta E_i}\) is fluctuations of energy loss due to energy straggling.

<table>
<thead>
<tr>
<th>absorber</th>
<th>nosecone (Cu)</th>
<th>central magnet (Fe)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x(\text{thickness})[\text{cm}])</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>(\rho(\text{density})[\text{g/cm}^3])</td>
<td>8.96</td>
<td>7.87</td>
</tr>
<tr>
<td>(Z(\text{atomic number}))</td>
<td>29</td>
<td>26</td>
</tr>
<tr>
<td>(A(\text{atomic weight}))</td>
<td>55.845</td>
<td>63.546</td>
</tr>
<tr>
<td>(\frac{dE}{dx}(\text{energy loss})[\text{MeV/g/cm}^2])</td>
<td>1.403</td>
<td>1.451</td>
</tr>
<tr>
<td>(X_0(\text{radiation length})[\text{g/cm}^2])</td>
<td>12.86</td>
<td>13.84</td>
</tr>
</tbody>
</table>

Table 4.1: property of absorber(referred from [22])

4.2.1 Energy straggling

For a thick absorber, the energy loss distribution can be shown in Gaussian in form. The mean value of energy loss distribution \(\Delta E\) is calculated from MIP
CHAPTER 4. RESOLUTION STUDY ON THE PHENIX MUON TRACKER

(Minimum Ionization Particle),

\[
\Delta E_1 = \left( \frac{dE}{dx} \right)_{Cu} \times \rho_{Cu} \times \frac{x(\text{nosecone})}{\cos \theta} \quad (4.39)
\]
\[
= (1.403 \times 20 \times 8.96) / \cos \theta \quad (4.40)
\]
\[
\Delta E_2 = \left( \frac{dE}{dx} \right)_{Fe} \times \rho_{Fe} \times \frac{x(\text{central magnet})}{\cos \theta} \quad (4.41)
\]
\[
= (1.451 \times 60 \times 7.87) / \cos \theta \quad (4.42)
\]

where \( \theta \) is the polar angle of the muon. The spread of Gaussian \( \sigma_{\Delta E} \) is calculated as follows for the relativistic particles[21],

\[
\sigma_{\Delta E}^2 = \frac{(1 - \frac{1}{2} \beta^2)}{1 - \beta^2} \sigma_0^2 \quad (4.43)
\]
\[
\sigma_0^2 = 4\pi N_a r_e^2 (m_e c^2)^2 \rho \frac{Z}{A} x \quad (4.44)
\]
\[
= 0.1569 \rho \frac{Z}{A} x [\text{MeV}^2] \quad (4.45)
\]

where,

- \( r_e \): classical electron radius = \( 2.817 \times 10^{-13} \text{cm} \)
- \( m_e \): electron mass = 511 keV
- \( N_a \): Avogadro’s number = \( 6.022 \times 10^{23} \text{mol}^{-1} \)
- \( Z \): atomic number of absorbing material
- \( A \): atomic weight of absorbing material
- \( \rho \): density of absorbing material
- \( \beta \): \( v/c \) of the incident particle
- \( x \): thickness of absorbing material

In the equation 4.43, coefficient is calculated as follows.
4.2. SINGLE MUON MOMENTUM RESOLUTION

\[
\frac{(1 - \frac{1}{2} \beta^2)}{1 - \beta^2} = \frac{1}{1 - \beta^2} - \frac{1}{2} \frac{\beta^2}{1 - \beta^2} \\
= \gamma^2 - \frac{1}{2} \gamma^2 \beta^2 \\
= \left( \frac{E}{m} \right)^2 - \frac{1}{2} \left( \frac{p}{m} \right)^2 \\
\sim \frac{1}{2} \left( \frac{p}{m} \right)^2
\]  

(4.46)

we use next three equations in this calculation.

\[
\gamma \equiv \frac{1}{\sqrt{1 - \beta^2}} \quad (4.47) \\
\gamma = \frac{E}{m} \quad (4.48) \\
\gamma \beta = \frac{p}{m} \quad (4.49)
\]

From equations 4.43, 4.45, and 4.46, \( \sigma_{\Delta E} \) is written as follows, using values shown in table 4.1.

\[
\sigma^2_{\Delta E_i} = 0.1569 \times \frac{1}{2} \left( \frac{p}{m_\mu} \right)^2 \times \rho_{Cu} \times \frac{Z_{Cu}}{A_{Cu}} \times \frac{x(nosecone)}{\cos \theta} \\
= 0.1569 \times \frac{1}{2} \left( \frac{p}{m_\mu} \right)^2 \times 8.96 \times \frac{29}{55.845} \times 20 \times \frac{1}{\cos \theta} \\
= 0.1569 \times \frac{1}{2} \left( \frac{p}{m_\mu} \right)^2 \times 93.04 \cos \theta \\
= \frac{7.299}{\cos \theta} \times \left( \frac{p}{m_\mu} \right)^2 [\text{MeV}^2] \\
= \frac{7.299}{106^2} \times \frac{p^2}{\cos \theta} \\
= (6.50 \times 10^{-4}) \times \frac{p^2}{\cos \theta}
\]

(4.50)
CHAPTER 4. RESOLUTION STUDY ON THE PHENIX MUON TRACKER

$$\sigma_{\Delta E_2}^2 = 0.1569 \times \frac{1}{2} \left( \frac{p - \Delta E_1}{m_\mu} \right)^2 \times \rho_{Fe} \times \frac{Z_{Fe}}{A_{Fe}} \times \frac{x(\text{central magnet})}{\cos \theta}$$

$$= 0.1569 \times \frac{1}{2} \left( \frac{p - \Delta E_1}{m_\mu} \right)^2 \left( 7.87 \times \frac{26}{63.546 \times 60} \right) \frac{1}{\cos \theta}$$

$$= 0.1569 \times \frac{1}{2} \left( \frac{p - \Delta E_1}{m_\mu} \right)^2 \times 193.2 \frac{\cos \theta}{\cos \theta}$$

$$= \frac{15.16}{\cos \theta} \times \left( \frac{p - \Delta E_1}{m_\mu} \right)^2 \text{[MeV}^2\text{]}$$

$$= \frac{15.16 \text{[MeV}^2\text{]}}{106^2\text{[MeV}^2\text{]}} \times \frac{(p - \Delta E_1)^2}{\cos \theta}$$

$$= (1.35 \times 10^{-3}) \times \frac{(p - \Delta E_1)^2}{\cos \theta} \quad (4.51)$$

where $\theta$ is the polar angle of the muon. Total fluctuation via energy straggling is calculated from equations 4.50 and 4.51,

$$\sum_i \left( \frac{\sigma_{\Delta E_i}}{p} \right)^2 = \left( \frac{\sigma_{\Delta E_1}}{p} \right)^2 + \left( \frac{\sigma_{\Delta E_2}}{p} \right)^2$$

$$= \frac{1}{p^2} \left\{ (6.50 \times 10^{-4}) \times \frac{p^2}{\cos \theta} + (1.35 \times 10^{-3}) \frac{(p - \Delta E_1)^2}{\cos \theta} \right\}$$

$$= \frac{(6.50 \times 10^{-4})}{\cos \theta} + \frac{(1.35 \times 10^{-3}) (p - \Delta E_1)^2}{p^2} \quad (4.52)$$

$$= \frac{(6.50 \times 10^{-4})}{\cos \theta} + \frac{(1.35 \times 10^{-3})}{\cos \theta} \left( 1 - \frac{0.2514 \text{[GeV]}}{pcos \theta} \right)^2 \quad (4.53)$$

4.2.2 Momentum resolution of the muon tracker

Resolution of the muon tracker ($\sigma_{p_{tr}}$ in equation 4.38) is calculated as follows from equation 4.31.

$$\left( \frac{\sigma_{p_{tr}}}{p_{tr}} \right)^2 = \sum_i \left( \frac{\partial p_{tr}}{\partial l_i} \right)^2 \left( \frac{\sigma_{l_i}}{p_{tr}} \right)^2 + \left( \frac{\partial p_{tr}}{\partial s} \right)^2 \left( \frac{\sigma_{s}}{p_{tr}} \right)^2 + \left( \frac{\partial p_{tr}}{\partial B} \right)^2 \left( \frac{\sigma_{B}}{p_{tr}} \right)^2$$

$$= \sum_i \left( \frac{\sigma_{l_i}}{l_i} \right)^2 + \left( \frac{\sigma_{s}}{s} \right)^2 + \left( \frac{\sigma_{B}}{B} \right)^2 \quad (4.54)$$

Accuracy of sagitta $\sigma_{s/s}$ is calculated from equation 4.2,
4.2. SINGLE MUON MOMENTUM RESOLUTION

\[
\left( \frac{\sigma_s}{s} \right)^2 = \sum_{i=1}^{3} \left( \frac{\partial s}{\partial x_i} \right)^2 \left( \frac{\sigma_{x_i}}{s} \right)^2 + \sum_{i=1}^{2} \left( \frac{\partial s}{\partial l_i} \right)^2 \left( \frac{\sigma_{l_i}}{s} \right)^2 \\
= \left( \frac{l_2}{L} \right)^2 \left( \frac{\sigma_{x_1}}{s} \right)^2 + \left( \frac{\sigma_{x_2}}{s} \right)^2 + \left( \frac{l_1}{L} \right)^2 \left( \frac{\sigma_{x_3}}{s} \right)^2 \\
+ \left( \frac{l_2}{L^2} (x_3 - x_2) \right)^2 \left( \frac{\sigma_{l_1}}{s} \right)^2 + \left( \frac{l_1}{L^2} (x_1 - x_2) \right)^2 \left( \frac{\sigma_{l_2}}{s} \right)^2 \tag{4.55}
\]

where \( \sigma_{x_i} (i = 1, 2, 3) \) means the resolution of each station along the azimuthal direction. Last two terms are negligible. \( l_i \) and \( L \) are of order of meter and \( (x_3 - x_2) \) and \( (x_1 - x_2) \) are of order of centimeter and therefore, \( \frac{l_i}{L^2} \) is of order of \( 10^{-2} \).

Hits on station 3 fluctuate due to multiple scattering in station 2, and therefore resolution of station 3 is separated into two terms: multiple scattering fluctuations and resolution of cathode strip chambers.

\[
\sigma_{x_3}^2 = \sigma_{\text{strip}3}^2 + \sigma_{\text{ms}}^2 \tag{4.56}
\]

where \( \sigma_{\text{strip}3} \) means the resolution of cathode strip chambers in station 3, and \( \sigma_{\text{ms}} \) means multiple scattering fluctuations. \( \sigma_{\text{ms}} \) is decided from the deflection of muon angle \( \theta_{\text{st}2} \) and the distance between station 2 and station 3, \( l_2 \).

\[
\theta_{\text{st}2} = \frac{13.6[\text{MeV}]}{p_{tr}} \sqrt{\frac{X}{X_0}} \left( 1 + 0.038 \ln \frac{X}{X_0} \right) \tag{4.57}
\]

where \( X/X_0 \) is the radiation length of station 2 and \( X/X_0 = 8.5 \times 10^{-4} \). Then, \( \sigma_{\text{ms}} \) is

\[
\sigma_{\text{ms}} = \frac{l_2}{\cos \theta} \theta_{\text{st}2} \tag{4.58}
\]

\[
= \frac{l_2}{\cos \theta} \frac{13.6[\text{MeV}]}{p_{tr}} \sqrt{\frac{X}{X_0}} \left( 1 + 0.038 \ln \frac{X}{X_0} \right) \tag{4.59}
\]

\[
\sim \frac{1.23}{p_{tr} \cos \theta} [\text{mm}] \tag{4.60}
\]

Similarly, the relative movement of three stations affects resolution. This effect can be included in the resolution of station 2.

\[
\sigma_{x_2}^2 = \sigma_{\text{strip}2}^2 + \sigma_{\text{relative}}^2 \tag{4.61}
\]
where \( \sigma_{\text{strip2}} \) means the resolution of cathode strip chambers in station2, and \( \sigma_{\text{relative}} \) means the effect of relative movement. Resolution of station 1 just come from cathode strip chamber’s resolution.

\[
\sigma_{x1} = \sigma_{\text{strip1}} \quad (4.62)
\]

From equations 4.55, 4.56, 4.61, 4.62

\[
\left( \frac{\sigma_s}{s} \right)^2 = \left( \frac{l_2}{L} \right)^2 \left( \frac{\sigma_{\text{strip1}}}{s} \right)^2 + \left( \frac{\sigma_{\text{strip2}}}{s} \right)^2 + \left( \frac{l_1}{L} \right)^2 \left( \frac{\sigma_{\text{strip3}}}{s} \right)^2 + \left( \frac{\sigma_{\text{relative}}}{s} \right)^2 + \left( \frac{l_1}{L} \right)^2 \left( \frac{\sigma_{\text{ms}}}{s} \right)^2 \quad (4.63)
\]

The resolutions of cathode strip chamber is 100 [\( \mu \text{m} \)] for non-stereo cathode plane. Station1 and station2 have three gaps and station3 has two gaps,

\[
\sigma_{\text{strip1}} = \sigma_{\text{strip2}} = \frac{100}{\sqrt{3}} = 57.7 [\mu \text{m}] \quad (4.64)
\]

\[
\sigma_{\text{strip3}} = \frac{100}{\sqrt{2}} = 70.7 [\mu \text{m}] \quad (4.65)
\]

Therefore,

\[
\sum \alpha_i^2 \left( \frac{\sigma_{\text{stripi}}}{s} \right)^2 = \left( \frac{l_2}{L} \right)^2 \left( \frac{\sigma_{\text{strip1}}}{s} \right)^2 + \left( \frac{\sigma_{\text{strip2}}}{s} \right)^2 + \left( \frac{l_1}{L} \right)^2 \left( \frac{\sigma_{\text{strip3}}}{s} \right)^2 \]

\[
= \frac{1}{s^2} \left\{ \left( \frac{2655}{4325} \right)^2 \left( \frac{100^2}{3} \right) + \left( \frac{100^2}{3} \right) + \left( \frac{1670}{4325} \right)^2 \left( \frac{100^2}{2} \right) \right\} \]

\[
= \frac{5.34 \times 10^{-3} [\text{mm}^2]}{s^2} \quad (4.66)
\]

For the north muon arm, sagitta \( s \) is calculated from equation 4.28.

\[
s = \frac{l_1 l_2}{2} \left( \frac{0.3B}{p_{\text{tr}}} \right) = \frac{l_1}{2} \left( 1 - \frac{l_1}{L} \right) \frac{0.3BL}{p_{\text{tr}}} = \frac{154BL}{p_{\text{tr}}} [\text{mm}] \quad (4.67)
\]
Finally, the momentum resolution of the muon tracker is written in the equation below, using equations 4.54, 4.60, 4.63, 4.66 and 4.67

\[
\left( \frac{\sigma_{p_{tr}}}{p_{tr}} \right)^2 = \left( \frac{p_{tr}}{154BL} \right)^2 \left\{ 5.34 \times 10^{-3} + \sigma_{relative}^2 [\text{mm}^2] + \frac{0.255}{p_{tr} \cos^2 \theta} \right\} \quad (4.68)
\]

### 4.2.3 Total momentum resolution

Total momentum resolution of the muon arm is shown below, from equations 4.38, 4.53, 4.68.

\[
\left( \frac{\sigma_p}{p} \right)^2 = \left( 1 - \frac{\Delta E}{p} \right)^2 \left( \frac{p - \Delta E}{154BL(\theta)} \right)^2 \left\{ 5.34 \times 10^{-3} + \sigma_{relative}^2 [\text{mm}^2] + \frac{0.255}{(p - \Delta E)^2 \cos^2 \theta} \right\} \\
\quad + \left( 6.50 \times 10^{-4} \right) \left( \frac{6.35 \times 10^{-3}}{\cos \theta} \right) \frac{(p - \Delta E_1)^2}{p^2} \quad (4.69)
\]

Figure 4.3 and figure 4.4 show the momentum resolution for North Arm which is calculated using this equation. Here \( \theta = 20 \text{[degree]} \), \( \sigma_{oasys} = 0 \) to 300 [\( \mu \text{m} \)]. In the large momentum region, chamber resolution and relative alignment accuracy are dominant. Therefore, it is very important to align relative position via OASYS. On the other hand, relative alignment is not essential in the small momentum region because its term is suppressed by \( \left( 1 - \frac{\Delta E}{p} \right)^2 \). In this region, energy straggling is dominant.
Figure 4.3: Momentum resolution vs. momentum.
Momentum resolution for $3 \sigma_{\text{OASYS}}$ value, 0$\mu$m (green), 100$\mu$m (yellow), 200$\mu$m (pink) and 300$\mu$m (red).

Figure 4.4: Momentum resolution vs. momentum ($p < 20\text{GeV}$)
Momentum resolution ($p < 20\text{GeV}$) for $3 \sigma_{\text{OASYS}}$ value, 0$\mu$m (green), 100$\mu$m (yellow) 200$\mu$m (pink) and 300$\mu$m (red).
4.3 Direction measurement basic

A Charged muon through a material is deflected by multiple scattering. Projected angular distribution is approximated via Gaussian and its width is given as,

\[ \theta_0 = \frac{13.6\text{MeV}}{\beta c p} \sqrt{\frac{x}{X_0}} \left[ 1 + 0.038 \ln \left( \frac{x}{X_0} \right) \right] \]  \hspace{1cm} (4.70)

\[ \Psi_{rms_{plane}} = \frac{1}{\sqrt{3}} \theta_0 \]  \hspace{1cm} (4.71)

Here \( p, \beta c \) are the momentum, velocity of the incident particle, \( x/X_0 \) is the thickness of the scattering medium in radiation lengths.

Actually, muon passes through two materials, the nosecone made of copper and the central magnet made of iron. In this case, smearing of the muon direction in rms plane is estimated as follows.

\[ \theta_{ms} = \sqrt{\theta_1^2 + \theta_2^2} \]  \hspace{1cm} (4.72)

\[ \Psi_{ms} = \frac{1}{d} \sqrt{d_1^2 \Psi_1^2 + d_2^2 \theta_1^2 + d_2^2 \Psi_2^2} \]

\[ = \frac{1}{\sqrt{3d}} \sqrt{d_1^2 \theta_1^2 + 3d_2^2 \theta_1^2 + d_2^2 \theta_2^2} \]  \hspace{1cm} (4.73)

\[ \begin{array}{|c|c|c|}
\hline
\text{d}_1 \text{ (nosecone)} & \text{d}_2 \text{ (central magnet)} & \text{d} \text{ (nosecone + central magnet)} \\
200 \text{ [mm]} & 600 \text{ [mm]} & 800 \text{ [mm]} \\
\hline
\end{array} \]
CHAPTER 4. RESOLUTION STUDY ON THE PHENIX MUON TRACKER

where $\theta_1$ and $\Psi_1$ are deflection angles in the nosecone, $\theta_2$ and $\Psi_2$ are in the central magnet. $d_1$, $d_2$ are width of nosecone and central magnet respectively, and $d = d_1 + d_2$.

![Figure 4.6: multiple scattering in nosecone and central magnet](image)

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>800</td>
<td>600</td>
<td>1800</td>
</tr>
</tbody>
</table>

When we reconstruct a muon track, we obtain track direction from collision vertex point measured by BBC and hit on station1. The angle deflection is expressed with $\Psi_{ms}$ and $\theta_{ms}$.

$$\sigma_{\theta} = \frac{1}{x} \sqrt{x_2^2 \Psi_{ms}^2 + x_3^2 \theta_{ms}^2}$$  \hspace{1cm} (4.74)

From equations 4.72,4.73,4.74,

$$\sigma_{\theta} = \frac{1}{x^2} \left\{ x_2^2 \Psi_{ms}^2 + x_3^2 \theta_{ms}^2 \right\}$$

$$= \frac{1}{x^2} \left[ x_2^2 \left( \frac{d_1}{d} \right)^2 \Psi_1^2 + \left( \frac{d_2}{d} \right)^2 \theta_1^2 + \left( \frac{d_2}{d} \right)^2 \Psi_2^2 \right] + x_3^2 (\theta_1^2 + \theta_2^2)$$

$$= (0.255 \times \theta_1^2 + 0.123 \times \theta_2^2)$$  \hspace{1cm} (4.75)

In North Arm, $\theta_1$ and $\theta_2$ are calculated from radiation length and density of copper (nosecone) or iron (central magnet). Path length of muon is increase, when azimuthal angle $\theta$ increases. Therefore, $\frac{x}{X_0}$ is written as,

$$\frac{x}{X_0} = \frac{(path) \times (density)}{X_0 \cos(\theta)}$$  \hspace{1cm} (4.76)

$$= \frac{19.5}{\cos \theta} \text{ (nosecone) or } \frac{105.5}{\cos \theta} \text{ (central magnet)}$$  \hspace{1cm} (4.77)
4.4. \( J/\psi \) INVARIANT MASS

Then,

\[
\theta_1 = \frac{13.6 \text{MeV}}{p} \sqrt{\frac{19.5}{\cos \theta}} \left[ 1 + 0.038 \ln \left( \frac{19.5}{\cos \theta} \right) \right] \\
\simeq \frac{66.7 \text{MeV}}{p \sqrt{\cos \theta}} \tag{4.78}
\]

\[
\theta_2 = \frac{13.6 \text{MeV}}{p} \sqrt{\frac{105.5}{\cos \theta}} \left[ 1 + 0.038 \ln \left( \frac{105.5}{\cos \theta} \right) \right] \\
\simeq \frac{165 \text{MeV}}{p \sqrt{\cos \theta}} \tag{4.79}
\]

Finally,

\[
\sigma_\theta^2 \simeq \frac{4.52 \times 10^{-3}}{p^2 \cos \theta} \tag{4.80}
\]

4.4 \( J/\psi \) invariant mass

4.4.1 Mass resolution

Charmonia(\( J/\psi, \psi' \)) are measured with a positive and negative muon pair in the muon tracker.

\[
\psi \rightarrow \mu^+ + \mu^-
\]

Mass of charmonium \( \psi \) is written using the momenta of dimuon \( p_2, p_2 \) and angle between them, \( \theta_{\mu\mu} \).

\[
M^2 = (p_1^\mu + p_2^\mu)^2 \\
= (E_1 + E_2)^2 - (p_1 + p_2)^2 \\
\simeq 2p_1p_2 - 2p_1p_2 \cos \theta_{\mu\mu} \\
= 2p_1p_2(1 - \cos \theta_{\mu\mu}) \\
= 4p_1p_2 \sin^2 \frac{\theta_{\mu\mu}}{2} \tag{4.81}
\]

\[
\therefore M = \sqrt{4p_1p_2 \sin^2 \frac{\theta_{\mu\mu}}{2}} \tag{4.82}
\]

Then mass resolution \( \sigma(M) \) is,
\[ \frac{\sigma(M)}{M} = \sqrt{\left( \frac{\partial M}{\partial p_1} \right)^2 \left( \frac{\sigma_{p_1}}{M} \right)^2 + \left( \frac{\partial M}{\partial p_2} \right)^2 \left( \frac{\sigma_{p_2}}{M} \right)^2 + \left( \frac{\partial M}{\partial \theta_{\mu\mu}} \right)^2 \left( \frac{\sigma_{\mu\mu}}{M} \right)^2} \]

\[ = \frac{1}{2} \sqrt{\frac{\sigma_{p_1}^2}{p_1^2} + \frac{\sigma_{p_2}^2}{p_2^2} + \sigma_{\mu\mu}^2 \cot^2 \theta_{\mu\mu}} \]  

(4.83)

where \( \sigma_{\mu\mu} \) is the error of \( \theta_{\mu\mu} \) and is due to from multiple scattering in the nosecone and the central magnet which are located in front of muon tracker.

\( \sigma_{\mu\mu} \) arises from angle deflection of positive and negative muon track, and calculated with equation 4.80,

\[ \sigma_{\mu\mu}^2 = \sum \sigma_{\theta_i}^2 \]

\[ = \sum \frac{4.52 \times 10^{-3}}{p_i^2 \cos \theta_i} \]  

(4.84)

Finally, the mass resolution of \( J/\psi \) is expressed as follows.

\[ \left( \frac{\sigma(M)}{M} \right)^2 = \frac{1}{4} \sum \left\{ \left( \frac{\sigma_{p_i}}{p_i} \right)^2 + \frac{4.52 \times 10^{-3}}{p_i^2 \cos \theta_i} \cot^2 \theta_{\mu\mu} \right\} \]

\[ = \frac{1}{4} \sum \left\{ \left( \frac{\sigma_{p_i}}{p_i} \right)^2 + \frac{4.52 \times 10^{-3}}{p_i^2 \cos \theta_i} \left( \frac{4p_1p_2}{M^2} - 1 \right) \right\} \]  

(4.85)

\[ \therefore \cot^2 x = \frac{1 - \sin^2 x}{\sin^2 x}, \quad \sin^2 \frac{\theta_{\mu\mu}}{2} = M^2 \frac{1}{4p_1p_2} \]  

(4.86)
4.4. $J/\psi$ INVARIANT MASS

Here, figure 4.7 shows $\theta_{\mu\mu}$ distribution generated by PYTHIA event generator, and figure 4.8 shows the momentum relation between positive and negative muon. Each value is in lab frame.

4.4.2 Simulation study

We studied contributions of relative movement of MuTr to mass resolution by using Monte Carlo simulation. Dimuon event from $J/\psi$ is generated with PYTHIA event generator at $\sqrt{s} = 200$ GeV proton-proton collisions. The behavior of muons in the PHENIX detector is calculated from the equations above. How the accuracy of relative alignment affects on mass resolution is studied using this simulation.

Figure 4.9 shows invariant mass of dimuon. Figure 4.10 shows momentum distribution of muon. Both of them are calculated by simulation. The contribution of relative movement of tacking chamber to the mass resolution is shown in figure 4.11.

$\psi'$ is 588 MeV apart from $J/\psi$. If the mass resolution of $J/\psi$ ($\sigma_M$) reaches 140 MeV, $4.2\sigma$ separation becomes possible. As shown in figure 4.11, 140 MeV mass resolution will be archived at less than 50 [$\mu$m] accuracy of the relative alignment and this value is required for OASYS.
Figure 4.9: Dimuon invariant Mass (simulation)
Dimuon invariant mass distribution generated by Monte Carlo simulation. Histogram is fitted by using Gaussian.

Figure 4.10: Momentum distribution (simulation)
Momentum distribution generated by Monte Carlo.
Figure 4.11: Relative movement dependence of Mass resolution
Horizontal axis is $\sigma_{\text{relative}}[\mu m]$, position error from relative movement. Vertical axis is momentum resolution.
CHAPTER 4. RESOLUTION STUDY ON THE PHENIX MUON TRACKER
Chapter 5

Relative alignment for the PHENIX Muon Tracker

5.1 Alignment model

To establish relative alignment method using Optical Alignment System at PHENIX Muon Tracker, we study tracker chamber’s relative movement. Momentum resolution of the muon tracker is affected by relative positions among three stations as mentioned in chapter 4. Only the movement of station 2 is taken into account in our movement model. We assume that tracking chamber is a rigid body, and we model the movement of tracking chamber in this chapter.

5.1.1 Movement of optics

In this chapter, we use PHENIX global coordinate system, if there are no other notation. We define three component vector $\Delta x_{\text{parallel}}$ and matrix $R(\delta \phi)$. $\Delta x_{\text{parallel}}$ expresses the chamber’s parallel translation along $X$, $Y$, and $Z$ axes, $R(\delta \phi)$ expresses the rotation of the chamber around $X'$, $Y'$ and $Z$ axes as shown in figure 5.1. Here, $\Delta x_{\text{parallel}}$ and $\Delta x_{\text{rotate}}$ denotes the displacement from initial position of tracking chamber.

$X'$ axis is the projection of global $X$ axis on to the chamber surface plane, and $Y'$ axis is defined similarly. Here,

$$\Delta x_{\text{parallel}} = (\delta x, \delta y, \delta z) \quad (5.1)$$

and $R(\delta \phi)$ has three variables, $\delta \phi_x, \delta \phi_y, \delta \phi_z$ (see figure 5.1).

The rotation matrix $R(\delta \phi)$ is written in the form below,
Figure 5.1: Free parameters for station movement

\[ R(\delta \phi) \equiv R(\delta \phi_x)R(\delta \phi_y)R(\delta \phi_z) \]  

\[ R(\delta \phi_x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \delta \phi_x & -\sin \delta \phi_x \\ 0 & \sin \delta \phi_x & \cos \delta \phi_x \end{pmatrix} \]  

\[ R(\delta \phi_y) = \begin{pmatrix} \cos \delta \phi_y & 0 & \sin \delta \phi_y \\ 0 & 1 & 0 \\ -\sin \delta \phi_y & 0 & \cos \delta \phi_y \end{pmatrix} \]  

\[ R(\delta \phi_z) = \begin{pmatrix} \cos \delta \phi_z & -\sin \delta \phi_z & 0 \\ \sin \delta \phi_z & \cos \delta \phi_z & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  

Actually, \( \delta \phi_i \) (i = x, y, z) is of order of \( 10^{-3} \) to \( 10^{-4} \) [rad] and much smaller than 1. Therefore, we can approximate as follows.
5.1. ALIGNMENT MODEL

\[
\cos \delta \phi_i \sim 1 \quad (5.6) \\
\sin \delta \phi_i \sim \delta \phi_i \quad (5.7)
\]

From equations 5.2-5.7, we obtain \( R(\delta \phi) \) in a convenient form.

\[
R(\delta \phi) \sim 1 + \epsilon(\delta \phi) \quad (5.8)
\]

\[
\epsilon(\delta \phi) = \begin{pmatrix}
0 & -\delta \phi_z & \delta \phi_y \\
\delta \phi_z & 0 & -\delta \phi_x \\
-\delta \phi_y & \delta \phi_x & 0
\end{pmatrix} \quad (5.9)
\]

Here, we define the position of optics on the station 2 as a three component vector on \( X' - Y' \) plane.

\[
r = (x, y, 0) \quad (5.10)
\]

Using equations 5.8, 5.9, and 5.10, the movement due to the rotation of a station is expressed as follows,

\[
\Delta x_{\text{rotate}} = R(\delta \phi)r - r = (1 + \epsilon(\delta \phi)r) - r = \epsilon(\delta \phi)r \quad (5.11)
\]

\[
= \begin{pmatrix}
-y\delta \phi_z \\
x\delta \phi_z \\
-x\delta \phi_y + y\delta \phi_x
\end{pmatrix} \quad (5.12)
\]

From equations 5.1 and 5.12, total optics’ movement is written as,

\[
\Delta x_{\text{total}} = \Delta x_{\text{parallel}} + \Delta x_{\text{rotate}} \quad (5.13)
\]

\[
= \begin{pmatrix}
\delta x - y\delta \phi_z \\
\delta y + x\delta \phi_z \\
\delta z - x\delta \phi_y + y\delta \phi_x
\end{pmatrix} \quad (5.14)
\]

It is convenient to convert \( X \) and \( Y \) components of vector \( \Delta x_{\text{total}} \) into radial and azimuthal components.
CHAPTER 5. RELATIVE ALIGNMENT FOR THE PHENIX MUON TRACKER

\[
\Delta x_{\text{total}} = \begin{pmatrix}
\cos \phi \delta x - \sin \phi \delta y \\
\sin \phi \delta x + \cos \phi \delta y + \delta x r \\
\delta z - \delta \phi y x + \delta \phi y y
\end{pmatrix}
\]

(5.15)

\[
\equiv \begin{pmatrix}
\delta x_r \\
\delta x_\phi \\
\delta x_z
\end{pmatrix}
\]

(5.16)

5.1.2 Movement of image on the CCD camera

![Image of camera block angle]

Figure 5.2: Camera block angle

We will discuss CCD image movement. As shown in figure 5.2 the image plane of CCD (that is the light accept plane of camera) makes angle \( \theta_{cb} \) from octant surface plane. Therefore, we should convert optics’ movement to the movement along the surface of the CCD camera. Then we define local image frame \( r' - z' \), as shown in figure 5.2.

Optics’ movement on the local image frame \( \Delta x'_{\text{total}} \) is written in an equation below,
\[
\begin{pmatrix}
\delta x' \\
\delta x'_\phi \\
\delta x'_z
\end{pmatrix}
= \begin{pmatrix}
\cos \theta_{eb} & 0 & \sin \theta_{eb} \\
0 & 1 & 0 \\
-\sin \theta_{eb} & 0 & \cos \theta_{eb}
\end{pmatrix}
\begin{pmatrix}
\delta x_r \\
\delta x_\phi \\
\delta x_z
\end{pmatrix}
\] (5.17)

\[
= \begin{pmatrix}
\delta x_r \cos \theta_{eb} + \delta x_z \sin \theta_{eb} \\
\delta x_z \cos \theta_{eb} - \delta x_r \sin \theta_{eb}
\end{pmatrix}
\] (5.18)

The movement of optics (lens) in the local image frame is expressed as follows.

\[
\delta x'_r = \cos \theta_{eb} (\cos \phi \delta x - \sin \phi \delta y) + \sin \theta_{eb}(\delta z + \delta \phi_x y - \delta \phi_y x) \quad (5.19)
\]

\[
\delta x'_\phi = r \delta \phi_z + \sin \phi \delta x + \cos \phi \delta y \quad (5.20)
\]

\[
\delta x'_z = - \sin \theta_{eb}(\sin \phi \delta x - \cos \phi \delta x) + \cos \theta_{eb}(\delta z + \delta \phi_x y - \delta \phi_y x) \quad (5.21)
\]

where \(\delta x'_r\) and \(\delta x'_\phi\) mean the movement of image on the CCD camera.

The movement of CCD image is decided from movement of lens on station2. When a lens on the station2 moves \(dx_2\) and CCD image moves \(dx_{img}\), relation between \(dx_2\) and \(dx_{img}\) is written as bellow.

\[
dx_{img} \frac{l_1}{l_1 + l_2} = dx_2 \quad (5.22)
\]

define \(k\) as \(k \equiv \frac{l_1}{l_1 + l_2}\),

\[
\begin{pmatrix}
\delta x'_r \\
\delta x'_\phi
\end{pmatrix}
= k \begin{pmatrix}
\delta x_{img,r} \\
\delta x_{img,\phi}
\end{pmatrix}
\] (5.23)

Finally, relation with image movement and station2 movement is calculated as follows.

\[
k \delta x_{img,r} = \cos \theta_{eb}(\cos \phi \delta x - \sin \phi \delta y) + \sin \theta_{eb}(\delta z + \delta \phi_x y - \delta \phi_y x) \quad (5.24)
\]

\[
k \delta x_{img,\phi} = r \delta \phi_z + \sin \phi \delta x + \cos \phi \delta y \quad (5.25)
\]

5.1.3 Thermal expansion of tracking chamber

Relative movement of tracking chamber is treated under the assumption that tracking chamber has rigid body. However, temperature in the experiment hall fluctuates and tracking chamber expands due to it. Here, it is assumed that expansion and contraction of the tracking chamber is isotropic.
The expansion and contraction of chamber are expressed by coefficient of linear expansion which is defined as the change in length of a substance per change in temperature.

$$\alpha \equiv \frac{1}{l_0} \frac{dl}{dT}$$ (5.26)

where \(dl\) is change in length, \(l_0\) is the initial length and \(dT\) is the change in temperature. The tracking chamber consists of glass epoxy resin called G10 and duralumin. The coefficient of linear expansion for these material is shown in table 5.1.

<table>
<thead>
<tr>
<th></th>
<th>G10</th>
<th>duralumin</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>(9 \times 10^{-6})</td>
<td>(21.6 \times 10^{-6})</td>
</tr>
</tbody>
</table>

Table 5.1: \(\alpha\) for G10 and duralumin

Temperature in the experimental hall fluctuates by about \(2^\circ\). The length of octant in station2 is about 2 m. Therefore, order of \(\Delta x_{\text{expansion}}\) is estimated as follows.

$$2 \times 21.6 \times 10^{-6} = 43.2 \mu \text{m}$$ (5.27)

The displacement of the position on the tracking chamber \(\delta x_{\text{expansion}}\) is written as follows.

$$\Delta x_{\text{expansion}} = \alpha r$$

$$\Delta x_{\text{expansion}} = \begin{pmatrix} \alpha x \\ \alpha y \end{pmatrix}$$ (5.28)

Total displacement of optics is described in equation below.

$$\Delta x_{\text{total}} = \Delta x_{\text{parallel}} + \Delta x_{\text{rotate}} + \Delta x_{\text{expansion}}$$ (5.29)

$$\Delta x_{\text{total}} = \begin{pmatrix} \delta x - y\delta \phi_z + \alpha x \\ \delta y + x\delta \phi_z + \alpha y \\ \delta z - x\delta \phi_y + y\delta \phi_z \end{pmatrix}$$ (5.30)

or, separating into radial and azimuthal components,
5.2. ALIGNMENT METHOD

\[ \Delta x_{\text{total}} = \begin{pmatrix} \cos \phi \delta x - \sin \phi \delta y + \alpha r \\ \sin \phi \delta x + \cos \phi \delta y + \delta \phi z r \\ \delta z - \delta \phi y x + \delta \phi x y \end{pmatrix} \]  

(5.31)

\[ \equiv \begin{pmatrix} \delta x_r \\ \delta x_{\phi} \\ \delta x_z \end{pmatrix} \]  

(5.32)

The equation 5.24 is modified as

\[ k \delta x_{\text{img}, r} = \cos \theta_{cb} (\cos \phi \delta x - \sin \phi \delta y) + \sin \theta_{cb} (\delta z + \delta \phi x y - \delta \phi y x) + \alpha r \]

\[ = \cos \theta_{cb} (\cos \phi \delta x - \sin \phi \delta y) + \sin \theta_{cb} (\delta z + \alpha (l_0 + l_1) + \delta \phi x y - \delta \phi y x) \]  

(5.33)

Here, we used the following expression of relations,

\[ r = (l_0 + l_1) \tan \theta_{cb} \]  

(5.34)

\[ r \cos \theta_{cb} = (l_0 + l_1) \tan \theta_{cb} \cos \theta_{cb} \]

\[ = (l_0 + l_1) \sin \theta_{cb} \]  

(5.35)

where, \( l_0 \) is distance between the collision point and station1, \( l_1 \) is the distance between station1 and station2.

In the equation 5.33, it is impossible to distinguish \( \delta z \) from \( \alpha (l_0 + l_1) \). Therefore, \( \delta z \) includes displacement due to expansion in our model.

But it is not a critical problem when considering the momentum resolution. As discussed in chapter 4, the momentum resolution is affected by the resolution of azimuthal direction. Seeing equation 5.32, movement of tracking chamber along azimuthal direction \( \delta x_{\phi} \) is expressed by \( \delta x, \delta y \) and \( \delta \phi_z \). Both \( \delta z \) and \( \alpha \) are not in \( \delta x_{\phi} \). On the other hand, the movement along radial direction and z direction is affected by expansion of tracking chamber. However, as estimated in equation 5.27, it is about 50\( \mu \)m, less than cathode strip resolution of radial direction which is about 500\( \mu \)m. Therefore, The thermal expansion of tracking chamber is negligible in our model.

5.2 Alignment method

In order to calculate the chamber movement, we define \( \chi^2 \) from equation 5.25,

\[ \chi^2_1 = \sum_{s}^{N} (r^s \delta \phi_z + \sin \phi^s \delta x + \cos \phi^s \delta y - k \delta x_{\text{img}, \phi}^s)^2 \]  

(5.36)
and $\chi_2^2$ from 5.24

$$\chi_2^2 = \sum_s (\beta^s + \sin \theta_s (\delta z + \delta \phi_x y^s - \delta \phi_y x^s) - k \delta x_{img,r}^s)^2$$

(5.37)

$$\beta^s = \cos \theta_s (\cos \phi^s \delta x - \sin \phi^s \delta y)$$

(5.38)

where $s$ is identical number of CCD cameras (1 to 7), and $N = \sum_s$ is the total number of CCD camera which is used for analysis. Minimizing $\chi_1^2$, we obtain $\delta x$, $\delta y$ and $\delta \phi_z$ and we can calculate $\beta^s$. Minimizing $\chi_2^2$, we obtain $\delta z$, $\delta \phi_x$ and $\delta \phi_y$. This method is used in general case.

When we use just three CCD cameras for alignment, we obtain the movement of chamber by using algebraic calculation.

Using three CCD cameras(indexed as $s$, $t$, $u$), we obtain transfer matrix $T$, from equation 5.25.

$$T \begin{pmatrix} \delta \phi_z \\ \delta x \\ \delta y \end{pmatrix} = k \begin{pmatrix} \delta x_{img,\phi}^s \\ \delta x_{img,\phi}^t \\ \delta x_{img,\phi}^u \end{pmatrix}$$

(5.39)

$$T = \begin{pmatrix} r^s \sin \phi^s \cos \phi^s \\ r^t \sin \phi^t \cos \phi^t \\ r^u \sin \phi^u \cos \phi^u \end{pmatrix}$$

(5.40)

By using this transfer matrix $T$, the movement of station $\delta x$ is calculated as below,

$$T \begin{pmatrix} \delta \phi_z \\ \delta x \\ \delta y \end{pmatrix} = kT^{-1} \begin{pmatrix} \delta x_{img,\phi}^s \\ \delta x_{img,\phi}^t \\ \delta x_{img,\phi}^u \end{pmatrix}$$

(5.41)

Similarly, we can calculate $\delta z$, $\delta \phi_x$ and $\delta \phi_y$ from equation 5.24.

$$T_2 \begin{pmatrix} \delta z \\ \delta \phi_x \\ \delta \phi_y \end{pmatrix} + \beta = k \begin{pmatrix} \delta x_{img,r}^s \\ \delta x_{img,r}^t \\ \delta x_{img,r}^u \end{pmatrix}$$

(5.42)

$$T_2 = \begin{pmatrix} \sin \theta_{cb}^s \sin \theta_{cb}^s y^s - \sin \theta_{cb}^s x^s \\ \sin \theta_{cb}^t \sin \theta_{cb}^t y^t - \sin \theta_{cb}^t x^t \\ \sin \theta_{cb}^u \sin \theta_{cb}^u y^u - \sin \theta_{cb}^u x^u \end{pmatrix}$$

(5.43)

$$\beta = \begin{pmatrix} \cos \phi^s \cos \theta_{cb}^s \delta x - \sin \phi^s \cos \theta_{cb}^s \delta y \\ \cos \phi^t \cos \theta_{cb}^t \delta x - \sin \phi^t \cos \theta_{cb}^t \delta y \\ \cos \phi^u \cos \theta_{cb}^u \delta x - \sin \phi^u \cos \theta_{cb}^u \delta y \end{pmatrix}$$

(5.44)
\[
\begin{pmatrix}
\delta z \\
\delta \phi_x \\
\delta \phi_y
\end{pmatrix} = T_2^{-1} \begin{pmatrix}
k \left( \frac{\delta x^i_{\text{img},\phi}}{\delta x^u_{\text{img},\phi}} \right) - \beta
\end{pmatrix}
\] (5.45)

Here, \( \beta \) is calculated by \( \delta x, \delta y \) and \( \delta \phi_z \) which are obtained from equation 5.41.

5.3 Data analysis

The data analysis and its result are presented in this section. Data of OASYS obtained during run3 experiment period (2003-2004) is used in this analysis. We use image data which are obtained by fifty six CCD cameras on the north muon arm, and analyze relative movement of the tracking chamber in the north muon arm.

5.3.1 Image processing

In order to obtain histograms for X or Y axis in CCD local frame, the method below is used.

1. Make \( 640 \times 480 \) sized 2-dimension array from CCD image. An element means brightness on the pixel.

2. Search for a peak position of the image array, and make projection around peak with \( 8\sigma \) range.

3. The method of “projection” is defined as the sum of pixels in the effective range.

\[
f_x(i) = \sum_{j=c_x-5\sigma_x}^{c_x+5\sigma_x} b(i, j)
\] (5.46)

\[
f_y(j) = \sum_{i=c_y-5\sigma_y}^{c_y+5\sigma_y} b(i, j)
\] (5.47)

\[(0 \leq i < 640, 0 \leq j < 480)\] (5.48)

where \( (i, j), b(i, j), (c_x, c_y) \), \( f_x(i) \) and \( f_y(j) \) denote the position of pixel in image array, brightness at \( (i, j) \), position of image peak, brightness distribution for X and Y direction.

As a result of this image processing, image movements \( \delta x_{\text{image}} \) such as shown in figure 5.3,5.4 are obtained.
Figure 5.3: Movement of image on the CCD camera (X direction)

Figure 5.4: Movement of image on the CCD camera (Y direction)
5.3. DATA ANALYSIS

5.3.2 Data selection

Before analyzing data, events are selected by using method as follows.

1. Make the distribution of the peak positions which are determined by Gaussian fitting and select events that are within $5\sigma$ from the mean value.

2. Make the distribution of fitting $\chi^2$ and select events that are within $5\sigma$ from the mean value.

3. Make the distribution of peak determination by Gaussian fitting and select events that are less than $16\mu m$.

Table 5.2 shows the event selection results.

5.3.3 Consistency check

Code check

We checked the alignment code using OASYS event generator which calculates the movement of image on the CCD cameras from input chamber’s movement. Figure 5.5 shows the deviation of the result from the input value.

Here, $\delta x, \delta y$ and $\delta z$ are determined with accuracy of $4.3 \times 10^{-8}$ $\sim$ $1.3 \times 10^{-6}$[mm] and $\delta \phi_x, \delta \phi_y$ and $\delta \phi_z$ are determined with accuracy of $1.2 \times 10^{-11}$ $\sim$ $2.8 \times 10^{-10}$[rad]. These values are small enough to be ignored.

Model check

We checked our alignment model by using a method as follows. We select three CCD cameras which is on the octant 3 as shown in table 5.3, and calculate the movement of tracking chamber. Here, $\langle stu \rangle$ means that we select CCD camera which has number s, t, and u (s, t, u = 1 to 7 shown in figure 3.13).

We define a difference between the CCD camera combinations $\langle stu \rangle$ as consistency of our relative movement model. Figures C.1 to C.6 in C show histogram of consistency defined above, and mean and rms for each parameter are listed in table 5.4,5.5.

These results prove that our chamber movement model is correct. The mean is 1.7 to 39.5 $\mu m$ for parallel transition and 0.81 to 11.0 $\mu rad$ for rotation.

5.3.4 Results

We calculate time dependent movement of tracking chamber for octant 3 in the north muon arm as shown in figure 5.7. The result for all octants in the north
### Table 5.2: Event selection results (North arm, run3)

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<th>Octant</th>
<th>CCD Number</th>
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<th>Total</th>
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</tr>
<tr>
<td></td>
<td>CCD2</td>
<td>2191</td>
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<tr>
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<td>CCD3</td>
<td>16</td>
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<td>1211</td>
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<td></td>
<td>CCD7</td>
<td>10</td>
<td>4389</td>
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<tr>
<td>8</td>
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<td>CCD6</td>
<td>4273</td>
<td>4389</td>
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<tr>
<td></td>
<td>CCD7</td>
<td>1958</td>
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</table>
5.3. **DATA ANALYSIS**

<table>
<thead>
<tr>
<th>label</th>
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<tbody>
<tr>
<td>(135)</td>
<td>1,3,5</td>
</tr>
<tr>
<td>(136)</td>
<td>1,3,6</td>
</tr>
<tr>
<td>(146)</td>
<td>1,4,6</td>
</tr>
<tr>
<td>(356)</td>
<td>3,5,6</td>
</tr>
</tbody>
</table>

Table 5.3: CCD camera selection

<table>
<thead>
<tr>
<th>parameter</th>
<th>mean</th>
<th>parameter</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta x)</td>
<td>2.2 to 9.3 [(\mu m)]</td>
<td>(\delta \phi_x)</td>
<td>1.2 to 9.9 [(\mu rad)]</td>
</tr>
<tr>
<td>(\delta y)</td>
<td>1.7 to 4.8 [(\mu m)]</td>
<td>(\delta \phi_y)</td>
<td>1.3 to 11.0 [(\mu rad)]</td>
</tr>
<tr>
<td>(\delta z)</td>
<td>9.6 to 39.5 [(\mu m)]</td>
<td>(\delta \phi_z)</td>
<td>0.81 to 2.7 [(\mu rad)]</td>
</tr>
</tbody>
</table>

Table 5.4: The mean for each parameter

<table>
<thead>
<tr>
<th>parameter</th>
<th>rms</th>
<th>parameter</th>
<th>rms</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta x)</td>
<td>9.1 to 37 [(\mu m)]</td>
<td>(\delta \phi_x)</td>
<td>1.3 to 24 [(\mu rad)]</td>
</tr>
<tr>
<td>(\delta y)</td>
<td>0.57 to 23 [(\mu m)]</td>
<td>(\delta \phi_y)</td>
<td>11 to 34 [(\mu rad)]</td>
</tr>
<tr>
<td>(\delta z)</td>
<td>53 to (1.4 \times 10^2) [(\mu m)]</td>
<td>(\delta \phi_z)</td>
<td>5.8 to 8.2 [(\mu rad)]</td>
</tr>
</tbody>
</table>

Table 5.5: The rms for each parameter

Muon arm is shown in appendix B. Tracking chambers turned out to have moved by about 50 to 300 \(\mu m\) during the experiment period.

5.3.5 **Error estimation**

We consider that accuracy of the relative alignment is dominantly determined from accuracy of optics position and peak position resolution on the CCD cameras. The accuracy of optics position is of the order of \(1.0[\text{\(\mu m\)}]\) from survey.

The peak position resolution is measured as mentioned in chapter 3. The measured resolution is 1.4[\(\mu m\)] for the sharp one and 3.1[\(\mu m\)] for the broad one.

The accuracy of relative alignment is calculated using Monte Carlo simulation for sharp image case and broad image case. The procedure of simulation is as follows. At first, the movement of image on the CCD camera is generated from inputed six parameters by using our model. Accuracy of optics position and peak position resolution are taken into account via Gaussian distribution on this process. Then, chamber movement is calculated from generated image movement.

Results of simulation are shown in figure 5.8 (sharp image case) and 5.9 (broad image case). Horizontal axis of each histogram is difference between true value (input) and obtained value (output). Each histogram is fitted by Gaussian. Results of fit-
CHAPTER 5. RELATIVE ALIGNMENT FOR THE PHENIX MUON TRACKER

Table 5.6: Estimated accuracy by simulation study

<table>
<thead>
<tr>
<th></th>
<th>sharp image</th>
<th>broad image</th>
<th></th>
<th>sharp image</th>
<th>broad image</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta x)</td>
<td>1.17 [\mu m]</td>
<td>2.54 [\mu m]</td>
<td>(\delta \phi_x)</td>
<td>1.32 [\mu rad]</td>
<td>2.84 [\mu rad]</td>
</tr>
<tr>
<td>(\delta y)</td>
<td>1.19 [\mu m]</td>
<td>2.58 [\mu m]</td>
<td>(\delta \phi_y)</td>
<td>1.18 [\mu rad]</td>
<td>2.54 [\mu rad]</td>
</tr>
<tr>
<td>(\delta z)</td>
<td>7.19 [\mu m]</td>
<td>15.7 [\mu m]</td>
<td>(\delta \phi_z)</td>
<td>0.32 [\mu rad]</td>
<td>0.67 [\mu rad]</td>
</tr>
</tbody>
</table>

Here, the calculated accuracy has the same order compared to the real data. But it is smaller than real data because tracking chamber is not rigid body.

5.4 Discussion

As mentioned in chapter 4, the position resolution along with azimuthal direction is important for the momentum resolution. Figure 5.10 shows chamber’s movement along with azimuthal direction at polar angle \(\theta = 256^\circ\).

The fraction of position resolution from relative alignment is calculated from the equation 5.16.

\[
\sigma_{\text{relative}}^2 = \left( \frac{\partial \delta x \phi}{\partial x} \right)^2 \sigma(\delta x)^2 + \left( \frac{\partial \delta x \phi}{\partial y} \right)^2 \sigma(\delta y)^2 + \left( \frac{\partial \delta x \phi}{\partial \phi} \right)^2 \sigma(\delta \phi z)^2 \\
\quad = \sin^2 \phi \sigma(\delta x)^2 + \cos^2 \phi \sigma(\delta y)^2 + r^2 \sigma(\delta \phi z)^2
\]

(5.49)

using result of data analysis,

\[
\sigma_{\text{relative}}^2 = (9.32 \times 10^{-3}) \sin^2 \phi + (4.8 \times 10^{-3}) \cos^2 \phi + (2.66 \times 10^{-6})^2 [\text{mm}^2] \\
\quad = 8.65 \times 10^{-5} \sin^2 \phi + 2.34 \times 10^{-5} \cos^2 \phi + 7.08 \times 10^{-12} r^2 [\text{mm}^2]
\]

(5.50)

For example, \(\sigma_{\text{relative}}\) at the position \(\theta = 25^\circ\) and \(\phi = 90^\circ\) is calculated.

\[
\sigma_{\text{relative}}^2 = 8.65 \times 10^{-5} + 7.08 \times 10^{-12} \times 1617^2 \\
\quad = 8.65 \times 10^{-5} + 1.97 \times 10^{-5} \\
\quad = 10.6 \times 10^{-5} [\text{mm}^2]
\]

\[
\therefore \sigma_{\text{relative}} = 1.03 \times 10^{-2} [\text{mm}]
\]

(5.51)

Therefore, the accuracy of relative alignment is about 10 \(\mu \text{m}\) and it is smaller than the chamber resolution and required resolution 50\(\mu \text{m}\).
### Figure 5.5: $\delta x$ error

The difference between input and output value.
Figure 5.6: $\langle 135 \rangle - \langle 136 \rangle$
Figure 5.7: Time dependent movement (octant 3)
**Figure 5.8: Accuracy of alignment (from simulation) sharp image**
Figure 5.9: Accuracy of alignment (from simulation) broad image
Figure 5.10: Chamber’s movement at $\theta = 25$
Chapter 6

Conclusion

RHIC is the first polarized proton-proton collider. PHENIX studies the spin structure of proton. $W$ production and heavy quark production measurement in PHENIX provide a new tool to study spin structure of proton.

Detection of muons is important for $W$ production and heavy quark production measurement, because these processes are identified by using muons. To separate $\psi'$ from $J/\psi$ is important in charmonium measurement and high mass resolution is required. For $W$ measurement, transverse momentum $p_T$ measurement is important because it is associated with momentum fraction of gluon and $W$ is identified by $p_T$ distributions in PHENIX. High momentum resolution is required for these measurements. Momentum resolution of muon arm is determined by energy stragglng in the hadron absorber and multiple scattering at station 2, in the low momentum region. On the other, in the middle to high momentum region, the position resolution of tracking chamber and the relative position accuracy among the three stations are dominant. This means that relative alignment among the station is important for $W$ measurement. How relative position accuracy affects $J/\psi$ mass resolution is studied by simulation. It turned out that less than 50 $\mu$m of accuracy is required to separate $\psi'$ with $4.2 \sigma$ separation.

Relative position among the stations is aligned by OASYS. We assume that tracking chamber has rigid body and we model its movement using six parameters which express parallel transition and rotation.

As a result of analysis with data obtained in run 3 experiment period, we calculate the movement of tracking chamber in the north muon arm. Each tracking chamber has moved by 50 $\mu$m to 300 $\mu$m during the experiment period. We can correct the relative movement of tracking chamber by using method developed and presented in this thesis, and the accuracy of alignment has reached less than 40 $\mu$m. This value is smaller than required accuracy.
Acknowledgment

I would like to thank Prof. Toshi-Aki Shibata, my thesis supervisor, for providing me with the opportunity to work on the PHENIX muon tracker and for providing me many advices.

I would like to thank Dr. Atsushi Takefumi of RIKEN. Helpful discussions with him are gratefully acknowledged. Without his advice and encouragement, my study would have not been completed. I also thank Dr. Hideto Enyo, the chief of radiation laboratory of RIKEN.

I wish to express my thanks to Dr. Yoshiyuki Miyachi of Tokyo Institute of Technology. He provided many helpful suggestions. I wish also to express my thanks to Mr. Nobuyuki Kamihara and Mr. Takuma Horaguchi who are graduate students. The discussions and talks with them gave me encouragement and relaxation. I would like to thank all of them.
Appendix A

OASYS DAQ system upgrade

A.1 DAQ system

DAQ (Data Acquisition) system for the OASYS consists of fifty six CCD cameras, multiplexor for selecting video signal, PCI capture board to read video signal, and two PCs for image processing and analysis. Figure A.1 shows a block diagram of the current DAQ system. Multiplexor is controlled with a windows PC through
APPENDIX A. OASYS DAQ SYSTEM UPGRADE

GPIB-Ether net device, because multiplexor is far from the PC in control room. Fifty six signals from the CCD cameras are fed into a multiplexor and one signal is fed into a PCI capture board on the Windows PC. Electrical video signal from multiplexor is converted into optical signal in the fiber transmitter at once, and converted into electrical signal again in the fiber receiver. In the Windows PC, histograms are generated, which are projected images along the vertical or horizontal axis, and then stored on the hard disk of the Windows PC and Linux server through SAMBA. Then, on the Linux server, we obtain the position of the light spot via a Gaussian fit, using ROOT. We replaced the old GUI base DAQ system with the LabVIEW base system summer of 2004. LabVIEW is commercially available software, which is widely used in the monitor and the control industry. The stability of the system has been improved and data taking speed increased from 1 image/30min/camera to 1 image/10min/camera.

![Figure A.2: Multiplexor](image)

A.2 Image processing

As another upgrade, the image processing procedure is improved as described in details below. In the old method, data from all vertical pixels in the CCD image are used for generating horizontally projected histograms and vice versa. Only the vertical pixels that contribute to the peak image are selected. In this way, the sharpness of the peak images is improved. Second, we superimpose images that are sequentially taken over a short period (16 at least and 128 maximum), and
the background noise is averaged and statistical fluctuations are minimized. Thus, the peak position accuracy is improved. Figure A.5 shows the effect of these improvements. The two raw sets of the date on the top are histograms produced by the old system, and the two raw sets of date on the bottom are histograms produced by the new one. The same CCD cameras are used for each histogram. It is difficult to find the peak in the histograms produced by the old system, while it is very easy in the histograms produced by the new one.

A.3 result

As a result of this upgrade, the number of cameras that can be used for relative alignment increases from 75 to 96. In addition, the S/N ratio increases from 1.4 to 12 at maximum ($S/N = 0.2$ is taken as a cut threshold for image analysis).
Figure A.4: Graphical user interface of DAQ software

Figure A.5: Comparison of old DAQ and upgraded DAQ.
Appendix B

Octant movement during run3

The movement of tracking chambers during run3 experimental period is shown in figure B.1 to B.8. Table B.1 to B.8 shows mean value and rms for distribution of movement paramters, $\delta x, \delta y, \delta z, \delta \phi_x, \delta \phi_y$ and $\delta \phi_z$.

<table>
<thead>
<tr>
<th>$\delta x$</th>
<th>mean [$\mu$m]</th>
<th>rms [$\mu$m]</th>
</tr>
</thead>
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<tr>
<td>21</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>51</td>
<td>$1.2 \times 10^2$</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta \phi_x$</th>
<th>mean [$\mu$rad]</th>
<th>rms [$\mu$rad]</th>
</tr>
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<tbody>
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<td>9.0</td>
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<td>44</td>
<td></td>
</tr>
<tr>
<td>-1.9</td>
<td>4.0</td>
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</tbody>
</table>

Table B.1: $\delta x$ distributions (octant1)

<table>
<thead>
<tr>
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<th>rms [$\mu$m]</th>
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<td>26</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>$1.5 \times 10^2$</td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>$\delta \phi_x$</th>
<th>mean [$\mu$rad]</th>
<th>rms [$\mu$rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>4.3</td>
<td>5.7</td>
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Table B.2: $\delta x$ distributions (octant2)

<table>
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<th>mean [$\mu$m]</th>
<th>rms [$\mu$m]</th>
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<td>7.3</td>
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</tr>
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<td>1.6</td>
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<td></td>
</tr>
<tr>
<td>$1.1 \times 10^4$</td>
<td>$1.6 \times 10^2$</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>$\delta \phi_x$</th>
<th>mean [$\mu$rad]</th>
<th>rms [$\mu$rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>37</td>
<td></td>
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<tr>
<td>-19</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>-1.0</td>
<td>1.7</td>
<td></td>
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</table>

Table B.3: $\delta x$ distributions (octant3)
## APPENDIX B. OCTANT MOVEMENT DURING RUN3

### Table B.4: \( \delta x \) distributions (octant4)

<table>
<thead>
<tr>
<th>( \delta x )</th>
<th>mean [( \mu m )]</th>
<th>rms [( \mu m )]</th>
<th>( \delta \phi_x )</th>
<th>mean [( \mu rad )]</th>
<th>rms [( \mu rad )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta x )</td>
<td>0.36</td>
<td>13</td>
<td>( \delta \phi_x )</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>( \delta y )</td>
<td>0.77</td>
<td>21</td>
<td>( \delta \phi_y )</td>
<td>15</td>
<td>22</td>
</tr>
<tr>
<td>( \delta z )</td>
<td>54</td>
<td>( 1.5 \times 10^2 )</td>
<td>( \delta \phi_z )</td>
<td>2.3</td>
<td>4.2</td>
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</table>

### Table B.5: \( \delta x \) distributions (octant5)

<table>
<thead>
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<th>( \delta x )</th>
<th>mean [( \mu m )]</th>
<th>rms [( \mu m )]</th>
<th>( \delta \phi_x )</th>
<th>mean [( \mu rad )]</th>
<th>rms [( \mu rad )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta x )</td>
<td>11</td>
<td>13</td>
<td>( \delta \phi_x )</td>
<td>20</td>
<td>86</td>
</tr>
<tr>
<td>( \delta y )</td>
<td>22</td>
<td>26</td>
<td>( \delta \phi_y )</td>
<td>17</td>
<td>38</td>
</tr>
<tr>
<td>( \delta z )</td>
<td>( 1.3 \times 10^2 )</td>
<td>( 1.6 \times 10^2 )</td>
<td>( \delta \phi_z )</td>
<td>-2.6</td>
<td>2.9</td>
</tr>
</tbody>
</table>

### Table B.6: \( \delta x \) distributions (octant6)

<table>
<thead>
<tr>
<th>( \delta x )</th>
<th>mean [( \mu m )]</th>
<th>rms [( \mu m )]</th>
<th>( \delta \phi_x )</th>
<th>mean [( \mu rad )]</th>
<th>rms [( \mu rad )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta x )</td>
<td>20</td>
<td>38</td>
<td>( \delta \phi_x )</td>
<td>8.3 \times 10^2</td>
<td>7.5 \times 10^2</td>
</tr>
<tr>
<td>( \delta y )</td>
<td>25</td>
<td>50</td>
<td>( \delta \phi_y )</td>
<td>1.7 \times 10^2</td>
<td>74</td>
</tr>
<tr>
<td>( \delta z )</td>
<td>( 3.5 \times 10^2 )</td>
<td>( 8.6 \times 10^2 )</td>
<td>( \delta \phi_z )</td>
<td>-9.9</td>
<td>12</td>
</tr>
</tbody>
</table>

### Table B.7: \( \delta x \) distributions (octant7)

<table>
<thead>
<tr>
<th>( \delta x )</th>
<th>mean [( \mu m )]</th>
<th>rms [( \mu m )]</th>
<th>( \delta \phi_x )</th>
<th>mean [( \mu rad )]</th>
<th>rms [( \mu rad )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta x )</td>
<td>19</td>
<td>23</td>
<td>( \delta \phi_x )</td>
<td>81</td>
<td>60</td>
</tr>
<tr>
<td>( \delta y )</td>
<td>18</td>
<td>23</td>
<td>( \delta \phi_y )</td>
<td>62</td>
<td>44</td>
</tr>
<tr>
<td>( \delta z )</td>
<td>( 3.0 \times 10^2 )</td>
<td>( 2.3 \times 10^2 )</td>
<td>( \delta \phi_z )</td>
<td>7.1</td>
<td>7.9</td>
</tr>
</tbody>
</table>

### Table B.8: \( \delta x \) distributions (octant8)

<table>
<thead>
<tr>
<th>( \delta x )</th>
<th>mean [( \mu m )]</th>
<th>rms [( \mu m )]</th>
<th>( \delta \phi_x )</th>
<th>mean [( \mu rad )]</th>
<th>rms [( \mu rad )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta x )</td>
<td>24</td>
<td>28</td>
<td>( \delta \phi_x )</td>
<td>92</td>
<td>56</td>
</tr>
<tr>
<td>( \delta y )</td>
<td>40</td>
<td>44</td>
<td>( \delta \phi_y )</td>
<td>16</td>
<td>20</td>
</tr>
<tr>
<td>( \delta z )</td>
<td>( 4.2 \times 10^2 )</td>
<td>( 2.8 \times 10^2 )</td>
<td>( \delta \phi_z )</td>
<td>3.0</td>
<td>5.2</td>
</tr>
</tbody>
</table>
Figure B.1: Time dependent movement (octant 1)
Figure B.2: Time dependent movement (octant 2)
Figure B.3: Time dependent movement (octant 3)
Figure B.4: Time dependent movement (octant 4)
Figure B.5: Time dependent movement (octant 5)
Figure B.6: Time dependent movement (octant 6)
Figure B.7: Time dependent movement (octant 7)
Figure B.8: Time dependent movement (octant 8)
Appendix C

Consistency of movement model

Here, result of consistency check are shown for north muon tracking chambers.

<table>
<thead>
<tr>
<th></th>
<th>mean [µm]</th>
<th>rms [µm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>δx</td>
<td>2.4</td>
<td>9.7</td>
</tr>
<tr>
<td>δy</td>
<td>4.8</td>
<td>19</td>
</tr>
<tr>
<td>δz</td>
<td>29</td>
<td>1.2 × 10^2</td>
</tr>
</tbody>
</table>

Table C.1: consistency : ⟨135⟩ - ⟨136⟩

<table>
<thead>
<tr>
<th></th>
<th>mean [µm]</th>
<th>rms [µm]</th>
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<tbody>
<tr>
<td>δx</td>
<td>3.2</td>
<td>16</td>
</tr>
<tr>
<td>δy</td>
<td>3.2</td>
<td>23</td>
</tr>
<tr>
<td>δz</td>
<td>9.5</td>
<td>1.4 × 10^2</td>
</tr>
</tbody>
</table>

Table C.2: consistency : ⟨135⟩ - ⟨146⟩

<table>
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<tr>
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<th>rms [µm]</th>
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<tbody>
<tr>
<td>δx</td>
<td>9.3</td>
<td>37</td>
</tr>
<tr>
<td>δy</td>
<td>-1.7 × 10^{-2}</td>
<td>0.57</td>
</tr>
<tr>
<td>δz</td>
<td>11</td>
<td>57</td>
</tr>
</tbody>
</table>

Table C.3: consistency : ⟨135⟩ - ⟨356⟩

<table>
<thead>
<tr>
<th></th>
<th>mean [µrad]</th>
<th>rms [µrad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>δφx</td>
<td>-6.2</td>
<td>22</td>
</tr>
<tr>
<td>δφy</td>
<td>5.3</td>
<td>16</td>
</tr>
<tr>
<td>δφz</td>
<td>2.4</td>
<td>7.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>mean [µrad]</th>
<th>rms [µrad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>δφx</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>δφy</td>
<td>14</td>
<td>34</td>
</tr>
<tr>
<td>δφz</td>
<td>0.81</td>
<td>7.1</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>mean [µrad]</th>
<th>rms [µrad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>δφx</td>
<td>2.5</td>
<td>1.3</td>
</tr>
<tr>
<td>δφy</td>
<td>2.7</td>
<td>11</td>
</tr>
<tr>
<td>δφz</td>
<td>-2.0</td>
<td>8.2</td>
</tr>
</tbody>
</table>
### APPENDIX C. CONSISTENCY OF MOVEMENT MODEL

#### Table C.4: consistency : $\langle 136 \rangle$ - $\langle 146 \rangle$

<table>
<thead>
<tr>
<th></th>
<th>mean [$\mu$m]</th>
<th>rms [$\mu$m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta x$</td>
<td>2.2</td>
<td>9.1</td>
</tr>
<tr>
<td>$\delta y$</td>
<td>1.7</td>
<td>8.4</td>
</tr>
<tr>
<td>$\delta z$</td>
<td>39</td>
<td>53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>mean [$\mu$rad]</th>
<th>rms [$\mu$rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta \phi_x$</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>$\delta \phi_y$</td>
<td>8.4</td>
<td>21</td>
</tr>
<tr>
<td>$\delta \phi_z$</td>
<td>1.2</td>
<td>7.6</td>
</tr>
</tbody>
</table>

#### Table C.5: consistency : $\langle 136 \rangle$ - $\langle 356 \rangle$

<table>
<thead>
<tr>
<th></th>
<th>mean [$\mu$m]</th>
<th>rms [$\mu$m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta x$</td>
<td>6.8</td>
<td>27</td>
</tr>
<tr>
<td>$\delta y$</td>
<td>4.8</td>
<td>19</td>
</tr>
<tr>
<td>$\delta z$</td>
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<td>$1.2 \times 10^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>mean [$\mu$rad]</th>
<th>rms [$\mu$rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta \phi_x$</td>
<td>8.5</td>
<td>23</td>
</tr>
<tr>
<td>$\delta \phi_y$</td>
<td>-2.6</td>
<td>15</td>
</tr>
<tr>
<td>$\delta \phi_z$</td>
<td>1.7</td>
<td>7.0</td>
</tr>
</tbody>
</table>

#### Table C.6: consistency : $\langle 146 \rangle$ - $\langle 356 \rangle$

<table>
<thead>
<tr>
<th></th>
<th>mean [$\mu$m]</th>
<th>rms [$\mu$m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta x$</td>
<td>8.9</td>
<td>24</td>
</tr>
<tr>
<td>$\delta y$</td>
<td>3.2</td>
<td>23</td>
</tr>
<tr>
<td>$\delta z$</td>
<td>1.0</td>
<td>$1.3 \times 10^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>mean [$\mu$rad]</th>
<th>rms [$\mu$rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta \phi_x$</td>
<td>-9.9</td>
<td>19</td>
</tr>
<tr>
<td>$\delta \phi_y$</td>
<td>-11</td>
<td>29</td>
</tr>
<tr>
<td>$\delta \phi_z$</td>
<td>-2.7</td>
<td>5.8</td>
</tr>
</tbody>
</table>
Figure C.1: ⟨135⟩ - ⟨136⟩
Figure C.2: \((135)-(146)\)
Figure C.3: $\langle 135 \rangle - \langle 356 \rangle$
Figure C.4: ⟨136⟩-⟨146⟩
Figure C.5: $\langle 136 \rangle - \langle 356 \rangle$
Figure C.6: ⟨146⟩-⟨356⟩
Bibliography


