Double Helicity Asymmetry in Jet Production from Polarized pp Collisions at PHENIX

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Abstract

The proton consists of quarks and gluons. The spin of the proton is expressed with sum of spins and orbital angular momenta of quarks and gluons. With lepton-nucleon deep inelastic scattering experiments, the fraction of proton spin carried by the quark spins was revealed to be only \(20 \sim 30\%\). It indicates that a large amount of proton spin should be carried by the remaining components. The PHENIX experiment is studying this problem with longitudinally polarized proton-proton collisions at Relativistic Heavy Ion Collider (RHIC) of BNL. One of the goals of the PHENIX experiment is to obtain the polarized gluon distribution function in the proton, \(\Delta g\), whose integrated value is equal to the contribution of gluon spins to the proton spin.

We investigate jet production process to obtain \(\Delta g\). Jet means a group of particles fragmented from a scattered parton. Particles in a jet concentrates into the direction of the original parton momentum. For 2\(\rightarrow\)2 hard scattering (\(qq \rightarrow qq\), \(gg \rightarrow qg\) or \(gg \rightarrow gg\)) events, two jets are produced almost back-to-back. We used polarized proton-proton collision data taken in 2003 at \(\sqrt{s} = 200\text{ GeV}\) with an average beam polarization of about 26\% and an integrated luminosity of about 0.27 pb\(^{-1}\). The PHENIX has two Central Arms, each of which covers the pseudorapidity region \(|\eta| < 0.35\) and 90-degree azimuthal angle. They are positioned almost back-to-back on their azimuth.

We have measured the total transverse momentum, \(p_T^{\text{sum}} \equiv \sum_i p_T^i\), by summing up \(p_T\) of photons and charged particles in one of Central Arms which involves photons with a \(p_T\) of \(> 2\text{ GeV}/c\). The \(p_T\) of a jet is evaluated from the \(p_T^{\text{sum}}\) with simulation. We used the PYTHIA simulator with not only default parameters but also tuned parameters with Multi-Parton Interaction (MPI). The differences between the PYTHIA default and PYTHIA MPI have been discussed with \(p_T^{\text{sum}}\) distribution and transverse momentum density.

The acceptance of Central Arms is not large enough to detect all particles in a jet, therefore jet structure should be identified to precisely estimate \(p_T\) of a jet from \(p_T^{\text{sum}}\). We have evaluated jet structures with “thrust”, which represents topology of particles. We have compared thrust distribution between the real data, the PYTHIA output, and isotropic events.

The double helicity asymmetry, \(A_{LL}\), of jet production process has been measured in the \(p_T^{\text{sum}}\) range of 4 to 12 GeV/c. The \(A_{LL}\) is defined with the difference of cross sections between the two helicity patterns of protons; same helicity (++ or −−) and opposite helicity (+− or −+) . It is theoretically
connected to the $\Delta g$. Within statistical errors, the result is consistent with theoretical predictions based on representative polarized gluon distribution functions. But the errors are not yet small enough to discriminate different theoretical predictions. Using the methods developed and presented in this thesis, it will become possible to determine $\Delta g$ in the coming beamtime, PHENIX RUN5.
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Chapter 1

Introduction

The motivation of this research is to understand the spin structure of proton. We aim at evaluating the polarized gluon distribution function in proton with polarized proton-proton collisions. The longitudinally polarized proton beams are used. Double helicity asymmetry in jet production cross section is measured.

The static structure of proton can be explained with a picture that proton consists of three constituent quarks. But when a proton is probed with a large momentum transfer, $Q^2$, that is to say a short distance, many quarks and gluons are observed. They are produced via gluon radiation, quark-antiquark pair production from a gluon, or quark-antiquark pair annihilation to a gluon.

The total linear momentum carried by the quarks in the proton has been measured by lepton-nucleon Deep Inelastic Scattering (DIS) experiments and was revealed to be $\sim 0.5$ at $Q^2 = 10 \sim 100$ GeV$^2$. The remaining linear momentum of the proton is carried by gluons. The mechanism of sharing the momentum among the quarks and gluons is well understood with the DGLAP (Dokshitzer, Gribov, Lipatov, Altarelli and Parisi) $Q^2$-evolution equation motivated by Quantum ChromoDynamics (QCD).

Also the fraction of the proton spin carried by the quark spin was measured by several polarized lepton-nucleon DIS experiments. It was revealed that the contribution of quark spin to the proton spin is only $20 \sim 30\%$ [1] [2].

$$\sum_f \Delta q_f = 0.2 \sim 0.3$$  \hspace{1cm} (1.1)

It indicates that the large amount of proton spin should be carried by the remaining components; the spin of gluons, $\Delta g$, and/or the orbital angular
momenta of quarks and gluons, $L_{q,g}$.

$$\frac{1}{2_{proton}} = \frac{1}{2} \sum_f \Delta q_f + \Delta g + L_q + L_g$$  \hspace{1cm} (1.2)

The PHENIX experiment is running with longitudinally polarized proton-proton collisions at Relativistic Heavy Ion Collider. One of the goals of the PHENIX experiment is to obtain the polarized gluon distribution function in proton, $\Delta g$. Until recently the PHENIX experiment mainly focused on single-particle productions, and the first result on $\pi^0$ production measurement has been published \[12\]. Now we have investigated jet production process, which is the multi-particle production. The jet production measurement gives us higher statistics in high-$p_T$ region than the single particle measurements. But the acceptance of Central Arms is not large enough to detect all particles in a jet. Therefore jet structure should be identified to precisely estimate $p_T$ of a jet from measured $p_T$.

In Chapter 2 we will explain the main topic of the proton spin structure and jet production. The former includes the definition of double helicity asymmetry. In Chapter 3 we will describe the RHIC-PHENIX experiment; goal of the PHENIX experiment, RHIC and PHENIX detector, and experimental and analyses conditions. In Chapter 4 we will show the results and discussions: First we will show results on transverse momentum distributions. Second we will show results on jet structure; transverse momentum density and thrust distribution. Finally we will show the double helicity asymmetry of jet production.
Chapter 2

Physics of proton structure and jet production

2.1 Proton structure

2.1.1 Parton model and parton distribution function

The structure of proton has been investigated with lepton-proton deep inelastic scattering (DIS),

\[ e(k)p(P) \to e(k')X \tag{2.1} \]

Figure 2.1 is a schematic drawing of DIS. The kinematic variables listed in Tab. 2.1 are commonly used.

It is well investigated that the structure of proton in high-energy (large \( Q^2 \)) interactions can be described with the parton model, in which all partons are assumed to be independent of each other. In the parton model, the time

![Figure 2.1: Schematic drawing of DIS.](image-url)
Table 2.1: Kinematic variables in DIS.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = (E, k)$, $k' = (E', k')$</td>
<td>4-momenta of the initial- and final-state leptons</td>
<td></td>
</tr>
<tr>
<td>$P_{\text{lab}} = (M, 0)$</td>
<td>4-momentum of the initial target proton</td>
<td></td>
</tr>
<tr>
<td>$q = k - k'$</td>
<td>4-momentum of the virtual photon</td>
<td></td>
</tr>
<tr>
<td>$Q^2 \equiv -q^2$</td>
<td>Negative squared 4-momentum transfer</td>
<td></td>
</tr>
<tr>
<td>$\nu \equiv \frac{P_{q, \text{lab}}}{E} = E - E'$</td>
<td>Energy of the virtual photon</td>
<td></td>
</tr>
<tr>
<td>$x = \frac{Q^2}{2P_{q, \text{lab}}} = \frac{Q^2}{2M\nu}$</td>
<td>Bjorken scaling variable</td>
<td></td>
</tr>
<tr>
<td>$y \equiv \frac{P_{q, \text{lab}}}{E}$</td>
<td>Fractional energy of the virtual photon</td>
<td></td>
</tr>
<tr>
<td>$W^2 = (P + q)^2$</td>
<td>the mass squared of the proton-virtual photon system</td>
<td></td>
</tr>
<tr>
<td>$s = (k + P)^2$</td>
<td>the center-of-mass energy squared of the lepton-proton system</td>
<td></td>
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</tbody>
</table>

of interaction should be much smaller than the time during which partons are free from each other. This condition is satisfied by

$$E \gg M, \quad Q^2 \gg M^2, \quad 2M\nu \gg M^2, \quad 0 < x < 1 \quad (2.2)$$

The distribution of partons in the proton and the cross section of lepton-parton scattering can be considered separately to obtain the total cross section of lepton-proton DIS.

The characteristics of partons, which include gluons as well as quarks, are represented with parton distribution function (PDF), $f(x)$. The Bjorken scaling variable $x$ represents the fraction of momentum of a parton to that of the proton,

$$x = \frac{p_{\text{parton}}}{P_{\text{proton}}} \quad (2.3)$$

$f(x)$ means the probability of finding partons with a certain $x$. Often $q(x)$ and $g(x)$ are used instead of the $f(x)$ of quarks and gluons, respectively, and also $u(x)$, $\bar{u}(x)$, $d(x)$, etc. instead of $f(x)$ of each quark flavor.

The PDFs have been measured by many lepton-nucleon DIS experiments. Figure 2.2 summarizes the data from both electromagnetic (BCDMS, BFP and EMC) and neutrino (CCFRR, CDHSW and CHARM) experiments [4]. It shows the PDF of whole quarks, $F_2(x) = q(x) + \bar{q}(x)$, that of valence quarks, $xF_2(x) = q(x) - \bar{q}(x)$, and that of sea anti-quarks, $\bar{q}(x)$. The valence quarks dominate at $x > 0.3$. The sea quarks become dominant as $x \to 0$. This tendency grows with the $Q^2$ increasing. Also $g(x)$ can be indirectly measured.
with DIS via virtual photon-gluon fusion process, $g\gamma^* \rightarrow q\bar{q}$, illustrated in Fig. 2.3. Figure 2.4 shows $xg(x)$ from the H1 and ZEUS NLO-QCD fits. 

### 2.1.2 Polarized parton distribution function

The polarized distribution functions of partons, $\Delta q$ and $\Delta g$, are defined as

$$
\Delta q(x) \equiv q^+(x) - q^-(x)
$$

(2.4)

$$
\Delta g(x) \equiv g^+(x) - g^-(x)
$$

(2.5)

Here, $q^+(x)$ ($q^-(x)$) is the probability of finding quarks which have a spin parallel (anti-parallel) to that of proton. The total contributions of parton spin to the proton spin, $\frac{1}{2}\Delta q$ and $\Delta g$, are written as

$$
\frac{1}{2}\Delta q \equiv \frac{1}{2} \int_0^1 dx \Delta q(x) = \frac{1}{2} \int_0^1 dx \left(q^+(x) - q^-(x)\right)
$$

(2.6)

$$
\Delta g \equiv \int_0^1 dx \Delta g(x) = \int_0^1 dx \left(g^+(x) - g^-(x)\right)
$$

(2.7)

The factor $\frac{1}{2}$ comes from the magnitude of quark spin.
Figure 2.3: Diagram illustrating how the photon couples to the gluon in the proton.

Figure 2.4: Gluon distribution function $xg(x)$ from the H1 and ZEUS NLO-QCD fits. (from [6])
The proton spin is composed of spins and orbital angular momenta of quarks and gluons:

\[
\frac{1}{2} = \frac{1}{2} \sum_f \Delta q_f + \Delta g + L_q + L_g
\]  

(2.8)

Here, \( f \) is a flavor of quarks (\( u, d, s, \bar{u}, \bar{d}, \) and \( \bar{s} \)). The fraction of proton spin carried by the quark spin, \( \frac{1}{2} \sum_f \Delta q_f \), was primarily measured by the EMC experiment at CERN in 1988, and it was revealed that the fraction was only \( \sim 15\% \) \([2]\). After this discovery, many experiments have measured \( \Delta q \) precisely. Figure 2.5 shows \( x\Delta q(x) \) measured by EMC, E142, E143, SMC, HERMES, E154 and E155 experiments, and from these results, now we derive

\[
\sum_f \Delta q_f = 0.2 \sim 0.3
\]  

(2.9)

Also \( \Delta g \) can be indirectly measured with DIS via virtual photon-gluon fusion process, \( g\gamma^* \rightarrow q\bar{q} \). Figure 2.6 shows \( \Delta g \) determined from the pQCD analysis at \( Q^2 = 1 \text{ GeV}^2 \) by SMC experiment \([3]\). The theoretical uncertainty as well as experimental ones were still large due to various input parameters to the pQCD analysis.

### 2.1.3 Measurement of gluon distribution function

Polarized proton-proton collision is a powerful tool to access to \( \Delta g \). We use longitudinally polarized proton beams, and represent the spin direction of the proton with helicity. Helicity “+” (“–”) means that the spin of proton is parallel (anti-parallel) to its momentum.

First, for example, the unpolarized cross section for inclusive pion production (\( pp \rightarrow \pi X \)), illustrated in Fig. 2.7, can be written as

\[
\sigma^{pp \rightarrow \pi X} = \sum_{f_1,f_2,f'} \int dx_1 dx_2 dz f_1(x_1, \mu^2) f_2(x_1, \mu^2) \,
\]

\[
\times \hat{\sigma}^{f_1,f_2 \rightarrow fX'}(x_1p_1; x_2p_2; p_\pi/z, \mu) D_f^\pi(z, \mu^2)
\]

(2.10)

Here, \( f_1 \) and \( f_2 \) are each \( q_f \) or \( g \). The equation is factorized into three elements; 1) parton distribution function \( (f_1, f_2) \), 2) parton-parton cross section \( (\hat{\sigma}^{f_1,f_2 \rightarrow fX'}) \), and 3) fragmentation function \( (D_f^\pi) \). The fragmentation function represents the finding probability of a particle (\( \pi \) in the case) with a momentum fraction of \( z \) in the parton \( f \).

Then, in the polarized case, using a polarized parton-parton cross section, \( \Delta \hat{\sigma}^{f_1,f_2 \rightarrow fX'} \), the polarized cross section for inclusive pion production is
Figure 2.5: Polarized PDF of quarks multiplied by the Bjorken scaling variable, $xg_1(x) = x\Delta q(x)$, in the proton, deuteron and neutron. (from [7])

defined as

$$
\Delta \sigma^{pp\to\pi X} \equiv \frac{1}{4} \left[ \sigma_{++}^{pp\to\pi X} - \sigma_{+-}^{pp\to\pi X} - \sigma_{-+}^{pp\to\pi X} + \sigma_{--}^{pp\to\pi X} \right] 
$$

$$
= \sum_{f_1,f_2,f} \int dx_1 dx_2 dz \Delta f_1(x_1,\mu^2) \Delta f_2(x_1,\mu^2) \times \Delta \hat{f}_{12}\to fX'(x_1p_1, x_2p_2, p_\pi/z, \mu) D_f^p(z, \mu^2) 
$$

(2.11)

$$
\Delta \hat{f}_{12}\to fX' \equiv \frac{1}{4} \left[ \hat{\sigma}_{++}^{f_{12}\to fX'} - \hat{\sigma}_{+-}^{f_{12}\to fX'} - \hat{\sigma}_{-+}^{f_{12}\to fX'} + \hat{\sigma}_{--}^{f_{12}\to fX'} \right] 
$$

(2.12)

Here, "++" and "--" denote the helicity of proton or parton. The parton-parton cross section is well predicted by perturbative Quantum-Chromo Dynamics (pQCD), and the fragmentation function has been measured with electron-positron collision experiments and lepton-nucleon DIS experiments. Therefore the polarized parton distribution function can be extracted from the polarized cross section for pion production.
Figure 2.6: $\Delta g(x)$ determined from the pQCD analysis at $Q^2 = 1 \text{ GeV}^2$ by SMC experiment. The statistical uncertainty as obtained from the QCD fit is shown by a band with crossed hatch. The experimental systematic uncertainty is indicated by the vertically hatched band, and the theoretical uncertainty by the horizontally hatched band. (from [8])

Figure 2.7: Production of a large-$p_T$ pion in a hard $pp$ collision. (from [9])
2.1.4 Double helicity asymmetry

To obtain information on $\Delta g$, we measure the double helicity asymmetry instead of the absolute value of $\Delta \sigma$ itself. The double helicity asymmetry, $A_{LL}$, is defined as

$$A_{LL} \equiv \frac{\Delta \sigma}{\sigma} \tag{2.13}$$

The $A_{LL}$ of pion production is written with Eq. 2.10 and 2.11 as

$$A_{LL} = \frac{\sum f_{1,f_2,f} \int dx_1 dx_2 dz \cdot \Delta f_1 \cdot \Delta f_2 \cdot \hat{a}_{LL} f_{2} - f X' \cdot D^\pi_f}{\sum f_{1,f_2,f} \int dx_1 dx_2 dz \cdot f_1 \cdot f_2 \cdot \hat{\sigma} f_{1,2} - f X' \cdot D^\pi_f} \tag{2.14}$$

Here $\hat{a}_{LL} f_{2} - f X'$ is the spin asymmetry for the parton-parton scattering,

$$\hat{a}_{LL} f_{2} - f X' = \Delta \hat{\sigma} f_{1,2} - f X' \tag{2.15}$$

and is called “analyzing power”. Figure 2.8 shows the lowest-order analyzing powers as functions of the partonic scattering angle in center-of-mass system.

In experiments, $A_{LL}$ can be obtained by measuring the helicity-dependent cross sections

$$A_{LL} = \frac{\sigma_{++} - \sigma_{+-}}{\sigma_{++} + \sigma_{+-}} \tag{2.16}$$

Here $\sigma_{++}$ ($\sigma_{+-}$) is the cross section with both colliding particles having the same (opposite) helicity. That is to say $\sigma_{++}$ contains $\sigma_{--}$ as well as $\sigma_{++}$ itself and it is based on the parity conservation. Using experimental yield ($N$) and integrated luminosity ($L$), we can derive

$$A_{LL} = \frac{1}{|P_B||P_Y|} \frac{N_{++} - RN_{+-}}{N_{++} + RN_{+-}} \tag{2.17}$$

and

$$R = \frac{L_{++}}{L_{+-}} \tag{2.18}$$

Here $P_B$ and $P_Y$ are the beam polarizations, and $R$ is the relative luminosity. Therefore the $A_{LL}$ measurement needs

- the beam polarizations, $P_B$ and $P_Y$,
- the relative luminosity, $R$, and
- the helicity-dependent yields, $N_{++}$ and $N_{+-}$.
Figure 2.8: Lowest-order analyzing powers for various reactions relevant for RHIC, as functions of the partonic scattering angle in center-of-mass system, $\theta$. Left: longitudinal polarization, right: transverse polarization (a factor $\cos(2\phi)$ has been taken out, where $\phi$ is the azimuthal angle of one produced particle). (from [10])
2.2 Jet production in pp collisions

2.2.1 Jet cone and clustering procedure

Jet is a group of particles fragmented from a scattered parton as shown in Fig. 2.9. The fragmentation is caused by the deconfinement of a parton by strong color interaction. The phenomenon of jet production was discovered in 1975 at SLAC with $e^+e^- \rightarrow q\bar{q}$ reaction[14]. The fragmentation from the parton to the hadrons can be described with QCD at small $Q^2 \ (\ < \ 1 \ \text{GeV}^2)$, but it needs non-perturbative calculation and thus currently we should use some model descriptions.

The momenta of particles in a jet have almost the same direction as that of the original parton, but spread a little by obeying the principle of uncertainty. The transverse component of the momentum of each particle against the jet direction is independent of the jet momentum itself and the mean value is almost constant;

$$\langle j_T \rangle \sim 300 \ \text{MeV}/c \quad (2.19)$$

Figure 2.10 shows energy flow in $\phi$-$\eta$ (transverse) plane measured in proton-antiproton collisions by CDF Collaboration [15]. $\phi$ is the azimuthal angle around the proton beam axis. $\eta$ is the pseudorapidity, which is defined with the polar angle from the proton beam axis, $\theta$, as

$$\eta \equiv -\ln \left( \tan \frac{\theta}{2} \right) \quad (2.20)$$
Figure 2.10: Energy flow in the transverse plane for the three events containing the highest total transverse energies [(a)-(c)] observed in the CDF high-$\sum E_T$ data sample, and (d) an example of an event with a complicated jet topology. (from [15])

Figure 2.11 shows transverse energy flow etc. in jets measured in proton-antiproton collisions by UA1 Collaboration [16].

To find jets in one event, we first pick up an initial particle with certain conditions and then collect particles around the initial particles within a threshold distance. The distance between the initial particle and each target particle is defined as

$$R \equiv \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2}$$

(2.21)

Here $\Delta \phi$ and $\Delta \eta$ are the distance in $\phi$ direction and $\eta$ direction, respectively. The major value of the threshold distance of $R$ is experimentally 0.7, which collects round 90% of $p_T$ of a jet.

2.2.2 Cross section and double helicity asymmetry

In a jet measurement, $z = 1$ and $D_f^\text{jet} = 1$ are satisfied in principle, therefore analogously to Eq. 2.10, the cross section of jet production can be written as

$$\sigma^{pp\rightarrow \text{jet}X} = \sum_{f_1, f_2, f} \int dx_1 dx_2 f_1(x_1, \mu^2) f_2(x_1, \mu^2) \hat{s} f_1 f_2 f X'(x_1 p_1, x_2 p_2, p_{\text{jet}}, \mu)$$

(2.22)
Figure 2.11: (a)-(c): Transverse energy flow as function of $\Delta \eta$, i.e. pseudo-rapidity distance from the jet axis, for 3 slices of jet $E_T$. The bin width is $d\Delta \eta = 0.05$, azimuthal integration is over $\Delta \phi = \pm 90^\circ$. (d)-(f): Charged transverse momentum flow as function of $\Delta \eta$. (g)-(i): Charged multiplicity flow as function of $\Delta \eta$. (from [16])
Figure 2.12: Relative contribution from $gg$, $qg$, and $qq$ scatterings to the NLO polarized cross section at mid pseudo-rapidities. (from [13])

Also, the double helicity asymmetry of the jet production can be written as

$$A_{LL} = \frac{\sum_{f_1,f_2,f} \int dx_1dx_2 \cdot \Delta f_1 \cdot \Delta f_2 \cdot \hat{\sigma}^{f_1f_2\rightarrow gX'} \cdot \hat{a}^{f_1f_2\rightarrow gX'}_{LL}}{\sum_{f_1,f_2,f} \int dx_1dx_2 \cdot f_1 \cdot f_2 \cdot \hat{\sigma}^{f_1f_2\rightarrow gX'}_{LL}} \quad (2.23)$$

Figure 2.12 shows the relative contribution from $gg$, $qg$, and $qq$ scatterings in the NLO polarized cross section at mid pseudo-rapidities. The processes which include gluon ($gg$ and $qg$) are dominant at the mid pseudo-rapidities in the polarized cross section and thus in the double helicity asymmetry, too. For example, the double helicity asymmetry of jet production via gluon-gluon scattering is written as

$$A_{LL}^{gg} = \frac{\int dx_1dx_2 \cdot \Delta g(x_1) \cdot \Delta g(x_2) \cdot \hat{\sigma}^{gg\rightarrow gX'} \cdot \hat{a}^{gg\rightarrow gX'}_{LL}}{\int dx_1dx_2 \cdot g(x_1) \cdot g(x_2) \cdot \hat{\sigma}^{gg\rightarrow gX'}_{LL}} \quad (2.24)$$

Figure 2.13 shows the theoretical prediction of the spin asymmetry at NLO in the case of four $\Delta g$ sets, and the error bars in the figures show the expected statistical accuracy for the conditions described in the figures. The figures indicate not only the typical value of the double helicity asymmetry but also the required value of the integrated luminosity and beam polarization to determine the $\Delta g$ with the required accuracy.
Figure 2.13: The spin asymmetry at NLO using the GRSV standard set as well as three other sets with very different gluon polarizations. The error bars indicate the expected statistical accuracy for 40% beam polarization and integrated luminosities of 3 pb$^{-1}$ and 20 pb$^{-1}$ for $\sqrt{s} = 200$ GeV and $\sqrt{s} = 500$ GeV, respectively. (from [13])
Chapter 3

RHIC-PHENIX experiment

3.1 Goal of PHENIX experiment

The PHENIX experiment has two goals; elucidation of proton spin structure and discovery of Quark Gluon Plazma. I refer only to the details of the proton spin physics in this thesis.

The PHENIX experiment is using longitudinally polarized proton-proton collisions at the Relativistic Heavy Ion Collider (RHIC). One of the goals of the PHENIX experiment is to obtain the polarized gluon distribution function in proton, $\Delta g$. The main reactions to probe $\Delta g$ are

- direct photon production ($p\bar{p} \rightarrow \gamma X$),
- jet production ($p\bar{p} \rightarrow jetX$), and
- heavy flavor pair production ($p\bar{p} \rightarrow c\bar{c}X, b\bar{b}X$).

Figure 3.1 shows the lowest-order Feynman diagrams for each process.

Direct photons are produced via $qg \rightarrow \gamma q$ and $q\bar{q} \rightarrow \gamma q$. The quark-gluon Compton process ($qg \rightarrow \gamma q$) is favored in proton-proton collisions as opposed to proton-antiproton collisions. Direct photons are promptly produced from parton-parton scattering and are measured without any conversion or decay, therefore the measurement of direct photons can determine $\Delta g$ precisely.

Jets are produced via $gg \rightarrow gg$ and $qg \rightarrow qg$. There are many approaches to determine $\Delta g$ with jets; measuring all produced particles as a jet or measuring only leading hadrons such as $\pi^0$, or $\pi^\pm$. The advantage of jet production is its large cross section, which is roughly 100 times larger than that of the direct photon production.

Heavy flavor pairs are produced via $gg \rightarrow q\bar{q}$ to form $D, B$, or $J/\Psi$ mesons and are detected with $\mu^+, \mu^-, e^+$, and/or $e^-$. 

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Figure 3.1: Selected lowest-order Feynman diagrams for elementary processes with gluons in the initial state in \( pp \) collisions; (a) quark-gluon Compton process for prompt-photon production, (b) gluon-gluon and gluon-quark scattering for jet production, and (c) gluon-gluon fusion for heavy quark pair production. (from \cite{9})

Table 3.1: The main specifications of RHIC in RUN 2003 for proton beam.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>beam energy</td>
<td>100 GeV (( \sqrt{s} = 200 ) GeV)</td>
</tr>
<tr>
<td>bunches/ring</td>
<td>55</td>
</tr>
<tr>
<td>protons/bunch</td>
<td>( 7 \times 10^{10} )</td>
</tr>
<tr>
<td>polarization</td>
<td>27%</td>
</tr>
<tr>
<td>luminosity</td>
<td>( 6 \mu b^{-1}s^{-1} ) (max), ( 3 \mu b^{-1}s^{-1} ) (average)*</td>
</tr>
<tr>
<td>integrated luminosity</td>
<td>( 0.27 ) pb(^{-1})</td>
</tr>
</tbody>
</table>

* \( 1 \mu b^{-1}s^{-1} = 10^{-36} \) cm\(^{-2}\) s\(^{-1}\)

### 3.2 RHIC and PHENIX detector

#### 3.2.1 RHIC

The RHIC at Brookhaven National Laboratory (BNL) can accelerate various beams from proton to gold. The proton beams are polarized longitudinally or transversely, and are used to study proton spin structure, especially polarized gluon distribution function, \( \Delta g \). Table 3.1 shows the main specifications of RHIC for the proton beam. Currently four experiments, PHENIX, STAR, BRAHMS, and PHOBOS, are running.
3.2.2 PHENIX detector

The PHENIX detector is classified into three parts; the Central Arms, Muon Arms, and Inner Detectors. Figure 3.3 shows a schematic drawing.

The Central Arms have tracking systems and Electro-Magnetic Calorimeters (EMCal). The tracking systems consist of the Pad Chambers (PC), the Drift Chambers (DC), the Ring-Imaging CHerenkov (RICH) detectors, etc. The PC and DC measure the track information of charged particles, and the RICH identifies the type of charged particles. The EMCal measures the position and energy of photons and electrons. Figure 3.4 shows the cross section of the PHENIX Central Arms.

The Muon Arms have the Muon Trackers (MuTr) and the Muon Identifiers (MuID). The MuTr consists of three multi-plane drift chambers, and the MuID consists of alternating layers of steel absorbers and Iarocci-type streamer tubes.

The Inner Detectors are the Beam-Beam Counters (BBC), the Zero-Degree Calorimeters (ZDC), and the Multiplicity-Vertex Detectors (MVD). The BBCs measure the number of charged particles in forward and backward region to determine the collision time, the collision $z$-vertex, and the relative luminosity.

Table 3.2 summarizes the coverage of the main PHENIX detector subsystems.

Hereinafter we briefly describe the specifications of the EMCal, DC, and PC in the Central Arms [17][18][19].

The EMCal system is located at a 5 m distance from the beam pipe to measure the position and energy of mainly photons and electrons. The system consists of four sectors in each of the East and West Arms, and each sector has a size of $2 \times 4$ m$^2$. The system has two types of towers; one is the Pb-Scintillator calorimeter (PbSc) and is used in all four sectors in the West Arms and two ones in the East Arms, and the other is the Pb-Glass calorimeter (PbGl) and is used in two sectors in the East Arms. A tower of the PbSc has a size of $5.35 \times 5.35 \times 37.5$ cm$^3$ (see Fig. 3.5), and 72×36 towers compose one sector. A tower of the PbGl has a size of $4 \times 4 \times 40$ cm$^3$, and 96×48 towers compose one sector. The energy resolution of each the PbSc and PbGl is given by

$$\frac{\sigma(E)}{E_{\text{PbSc}}} (%) = \frac{8.1}{\sqrt{E(\text{GeV})}} \oplus 2.1$$ (3.1)

$$\frac{\sigma(E)}{E_{\text{PbGl}}} (%) = \frac{5.9}{\sqrt{E(\text{GeV})}} \oplus 0.8$$ (3.2)
Figure 3.2: Schematic layout of the RHIC accelerator complex (only relevant devices for polarized $pp$ collisions). (from [9])
Figure 3.3: The schematic drawing of PHENIX detector.

Figure 3.4: The cross section of PHENIX Central Arms.
Table 3.2: Coverage of the main PHENIX Detector Subsystems. $\Delta \eta$ is the pseudorapidity coverage, and $\Delta \phi$ is the azimuthal angle coverage.

<table>
<thead>
<tr>
<th>Element</th>
<th>$\Delta \eta$</th>
<th>$\Delta \phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnet: central (CM)</td>
<td>$\pm 0.35$</td>
<td>360°</td>
</tr>
<tr>
<td>muon (MMS)</td>
<td>$-1.1 \sim -2.2$</td>
<td>360°</td>
</tr>
<tr>
<td>muon (MMN)</td>
<td>$1.1 \sim 2.4$</td>
<td>360°</td>
</tr>
<tr>
<td>Silicon (MVD)</td>
<td>$\pm 2.6$</td>
<td>360°</td>
</tr>
<tr>
<td>Beam-beam (BBC)</td>
<td>$\pm (3.1 \sim 3.9)$</td>
<td>360°</td>
</tr>
<tr>
<td>NTC</td>
<td>$\pm (1 \sim 2)$</td>
<td>320°</td>
</tr>
<tr>
<td>ZDC</td>
<td>$\pm 2$ mrad</td>
<td>360°</td>
</tr>
<tr>
<td>Drift chambers (DC)</td>
<td>$\pm 0.35$</td>
<td>$90^\circ \times 2$</td>
</tr>
<tr>
<td>Pad chambers (PC)</td>
<td>$\pm 0.35$</td>
<td>$90^\circ \times 2$</td>
</tr>
<tr>
<td>TEC</td>
<td>$\pm 0.35$</td>
<td>$90^\circ \times 2$</td>
</tr>
<tr>
<td>RICH</td>
<td>$\pm 0.35$</td>
<td>$90^\circ \times 2$</td>
</tr>
<tr>
<td>ToF</td>
<td>$\pm 0.35$</td>
<td>$45^\circ \times 2$</td>
</tr>
<tr>
<td>T0</td>
<td>$\pm 0.35$</td>
<td>$45^\circ \times 2$</td>
</tr>
<tr>
<td>PbSc EMCal (East)</td>
<td>$\pm 0.35$</td>
<td>45°</td>
</tr>
<tr>
<td>PbSc EMCal (West)</td>
<td>$\pm 0.35$</td>
<td>90°</td>
</tr>
<tr>
<td>PbGl EMCal (East)</td>
<td>$\pm 0.35$</td>
<td>45°</td>
</tr>
<tr>
<td>$\mu$ tracker (South)</td>
<td>$-1.15 \sim -2.25$</td>
<td>360°</td>
</tr>
<tr>
<td>$\mu$ tracker (North)</td>
<td>$1.15 \sim 2.44$</td>
<td>360°</td>
</tr>
<tr>
<td>$\mu$ identifier (South)</td>
<td>$-1.15 \sim -2.25$</td>
<td>360°</td>
</tr>
<tr>
<td>$\mu$ identifier (North)</td>
<td>$1.15 \sim 2.44$</td>
<td>360°</td>
</tr>
</tbody>
</table>
The DC system is located in the region from 2 to 2.4 m from the beam pipe to measure the position and momentum of charged particles with the help of PC. The system consists of one frame in each of the East and West Arms, and each frame has a cylindrical shape and a size of 2.5 m × 90° in z-φ direction (see Fig. 3.6). Each frame is divided into 20 sectors covering 4.5° in φ direction. Each sector has six types of wire modules stacked radially, and each module contains four sense planes forming cells with a 2~2.5 cm drift space in the φ direction (see Fig. 3.7). Two of the six type wires in one sector, which are called X1 and X2, run in parallel to the beam to perform precise track measurements in r-φ, and the other four type wires, which are called U1, V1, U2 and V2, are used to measure the z coordinate of the track. The requirements for the φ → e⁺e⁻ mass measurement are:

- single wire resolution better than 150 μm in r-φ,
- single two track separation better than 1.5 mm,
- single wire efficiency better than 99%, and
- spatial resolution in the z direction better than 2 mm.

The PC system is composed of multiwire proportional chambers and form three separate layers, which are called PC1, PC2 and PC3, of the Central.
Arms tracking system. The PC1 is located behind the DC and is used for determining the momentum vector together with the DC by providing the $z$ coordinate. The PC1 consists of a single plane of anode and field wires lying in a gas volume between two cathode planes (see Fig. 3.8). One cathode is segmented into pixels and the other is solid copper, and signals from the pixels are routed outside the gas volume. The position resolution along the wire is 1.6 mm and the position resolution across the wire is 2.3 mm. The two-track resolution along the wire is 2.4 cm and the two-track resolution across the wire is 2.9 cm.

The momentum resolution determined with the DC and PC1 is given by

$$\frac{\sigma(p)}{p} = 8.1 \cdot p(\text{GeV}/c) + 0.9(\%) \quad \text{at} \quad p \gtrsim 0.5 \text{ GeV}/c \quad (3.3)$$

### 3.3 Experimental condition

#### 3.3.1 Data taking

The PHENIX experiment has various trigger configurations to efficiently select many interesting rare events. The PHENIX Level-1 trigger is a hardware trigger and consists of two separate subsystems: the Local Level-1 (LL1) trigger and the Global Level-1 (GL1) trigger. The input data from detector
Figure 3.7: The layout of DC wire positions within one sector and inside the anode plane (left). A schematic diagram, top view, of the stereo wire orientation (right). (from [18])

Figure 3.8: The PC vertical cut through a chamber. (from [18])
systems are processed individually by the LL1 trigger systems to produce a data set of trigger bit for each RHIC beam crossing. The GL1 trigger system receives the data from the LL1 triggers and provide a trigger decision.

The LL1 triggers used in the jet measurement are the BBCLL1 and EMCal/RICH Trigger (ERT). The BBCLL1 trigger is fired when at least one charged particle hits either of the BBC. Usually the BBCLL1 trigger is used as a minimum bias trigger. The ERT is fired by the EMCal and RICH, and has various configurations concerning the threshold energy or the number of EMCal modules in which the total detected energy is calculated. Table 3.3 shows main ERT configurations concerning the photon detection. Figure 3.9 shows the turn-on curve of ERT_Gamma3 trigger. The smooth curve around $E \sim 1.4$ GeV is caused by variations of the EMCal gain and of the trigger threshold. The efficiency is almost flat and close to unity above $E \sim 2$ GeV.

The measurement of the double helicity asymmetry needs the information of the beam polarizations and the relative luminosity. The beam polarizations are measured with proton-Carbon Coulomb-Nuclear Interference (pC CNI) polarimeter at RHIC. The relative luminosity is measured with BBC as a scaler sum of the number of forward particles. Values of them are obtained fill by fill, where fill means each beam injection and we assume that the values are stable in each fill. One fill stays typically for 10 hours. Figure 3.10 and 3.11 show the beam polarizations and the relative luminosity, respectively, vs. the fill number.

### 3.3.2 Run and event selection

The total integrated luminosity of longitudinally polarized proton-proton collision data in the PHENIX RUN 2003 is $\sim 0.27$pb$^{-1}$. The whole data are segmented into runs. Several runs should be excluded due to its unsuitable conditions to this analysis; runs with bad trigger efficiency, bad beam polarization, too few statistics, and photon converter condition. The integrated luminosity of data we actually used was $\sim 0.22$pb$^{-1}$.

We used events which fired the BBCLL1 and ERT_Gamma3 trigger, that
Figure 3.9: The ERT_Gamma3 trigger efficiency

Figure 3.10: Beam polarization of two proton beam rings (blue and yellow) vs. fill
3.3.3 Particle selection

We used photons detected as a electro-magnetic cluster by the EMCal and charged particles detected as a track by the DC and PC1.

When a photon is detected with the EMCal, its energy is shared between 2×2 or 3×3 towers. Some towers of the EMCal were hot or dead in the run period, therefore such warn towers and towers around them were masked, which are 12.1% of all. Figure 3.12 and 3.13 show the status of all modules of the EMCal. The fraction of the DC live area is 94% for the West Arm and 99% for the East Arm.

To select true photon signals from all EMCal clusters we applied a $p_T$ cut, a charged track veto, and an electro-magnetic shower shape cut. The $p_T$ cut reduces low energy noise and was set to $> 0.4$ GeV/c. The charged track veto reduces charged particle contaminations and was performed by checking whether each EMCal cluster has a matching charged track within 3σ of their
Figure 3.12: Condition map of each tower in East Arm. (white: good, gray: edge of sector or round of warn tower, black: warn tower)
Figure 3.13: Condition map of each tower in West Arm. (white: good, gray: edge of sector or round of warn tower, black: warn tower)
position resolutions. The shower shape cut reduces hadron contaminations and uses “photon probability parameter” which is calculated for all clusters from each electro-magnetic shower shape. We required a photon probability parameter of $> 0.01$ for all clusters, which means that the efficiency of a photon cluster is 99% and the rejection factor of hadron cluster is $\sim 2$.

To select true charged particle signals from all tracks we applied a $p_T$ cut and a track quality cut. The $p_T$ cut was set to $0.4 < p_T < 4.0 \text{ GeV}/c$ to suppress an effect of curving tracks with the magnetic field by the lower limit and to eliminate fake high-$p_T$ tracks with the upper limit. The track quality cut reduces unreliable tracks and uses “quality parameter” which is calculated for all tracks. We required a standard quality parameter of “$= 31$ OR $> 61$” for all tracks, which means that “X1 used, X2 used, UV found and unique, PC1 found” OR “X1 or X2 used, UV found and unique, PC1 found and unique”.

3.4 Simulation conditions

3.4.1 PYTHIA event generator

We used the PYTHIA version 6.220 with the same conditions as the PHENIX experiment, proton-proton collisions in their center-of-mass frame at $\sqrt{s} = 200$ GeV. With the MSEL set to 1, the PYTHIA generated QCD high-$p_T$ process. With the MSTP(51) set to 4046 and MSTP(52) set to 2, the external CTEQ5L PDF was used. With the CKIN(3) set to 1.5, the PYTHIA generated the events which had a $\hat{p}_T$ of $> 1.5 \text{ GeV}/c$, here $\hat{p}_T$ is transverse momenta of scattered partons in center-of-mass frame of two scattered partons. The modified MSEL and CKIN(3) reduces the time for event generation and does not affect any physics results because low-$p_T$ events are eliminated by a high-$p_T$ trigger even if generated. We call the PYTHIA with these conditions “PYTHIA default”.

To study the influence of $k_T$, which is the intrinsic transverse momentum of partons and has an order of 1 GeV/c, we changed the $k_T$ value from 1.0 GeV/c (default) to 2.0 GeV/c by setting PARP(91) to 2.0 in addition to the PYTHIA default setting. We call the PYTHIA with these conditions “PYTHIA kT”.

To consider the acceptance of the Central Arms, we selected particles whose momentum direction was geometrically in the same region as the Central Arms; $|\eta| < 0.35$ and $-11.25^\circ < \phi < 33.75^\circ$ for the East Arm, and $146.25^\circ < \phi < 180^\circ$ or $-180^\circ < \phi < -168.75^\circ$ for the West Arm. To consider the energy or momentum resolution, we smeared the generated mo-
menta of photons and charged particles with Gaussian characterized by the $\sigma$ in Eq. 3.1 and 3.3. The difference between the PbSc and PbGl was ignored. An influence of the masked or dead areas in the EMCal, DC, and PC1 was not taken into account.

### 3.4.2 Underlying event and Multi-Parton Interaction

When a hard parton-parton scattering event takes place in proton-proton collisions, the event contains particles which originate from the two outgoing partons and particles which come from the breakup of the proton, i.e. beam-beam remnants. The former particles consist of the two outgoing jets and initial- and final-state radiation. The latter particles consist of everything except the former ones, and are called ‘underlying event’. The two kinds of particles cannot be completely distinguished with topological selection of events, therefore reconstructed jets are contaminated with the underlying event.

The PYTHIA reproduces the underlying event with Multi-Parton Interaction (MPI) mechanism. The MPI includes many parton-parton scatterings in one event. Figure 3.14 is a schematic drawing of MPI. First the hardest parton-parton scattering is generated with its initial- and final-state radiation, next any multiple interactions, and finally beam remnants are attached to the initiator partons of the hardest scattering to form Lund strings.

By default PYTHIA employs the MPI for reproducing the underlying event, but it does not reproduce the data with satisfactory precision. Therefore we modified MPI parameters as shown in Tab. 3.5. The values in Tab. 3.5 have been tuned with CDF data [20]. We call the PYTHIA with these conditions “PYTHIA MPI”.

<table>
<thead>
<tr>
<th>Table 3.4: PYTHIA parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>parameter</td>
</tr>
<tr>
<td>MSEL</td>
</tr>
<tr>
<td>MSTP(51)</td>
</tr>
<tr>
<td>MSTP(52)</td>
</tr>
<tr>
<td>CKIN(3)</td>
</tr>
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</table>
Figure 3.14: Schematic drawing of multi-parton interaction.

Table 3.5: PYTHIA MPI parameters. (see appendix)

<table>
<thead>
<tr>
<th>parameter</th>
<th>default</th>
<th>used</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSTP(81)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>MSTP(82)</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>PARP(81)</td>
<td>1.9</td>
<td>1.9</td>
</tr>
<tr>
<td>PARP(82)</td>
<td>1.9</td>
<td>2.0</td>
</tr>
<tr>
<td>PARP(83)</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>PARP(84)</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>PARP(85)</td>
<td>0.33</td>
<td>0.9</td>
</tr>
<tr>
<td>PARP(86)</td>
<td>0.66</td>
<td>0.95</td>
</tr>
<tr>
<td>PARP(89)</td>
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<td>1800</td>
</tr>
<tr>
<td>PARP(90)</td>
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<td>0.25</td>
</tr>
<tr>
<td>PARP(67)</td>
<td>1.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>
Chapter 4

Results and discussions

4.1 Total transverse momentum

We evaluated a vectorial sum of transverse momenta of all detected particles in one of the Central Arms which detected a trigger photon:

\[ p_{\text{sum}}^T \equiv \left| \sum_i p_{T,i} \right| \]  

(4.1)

This \( p_{\text{sum}}^T \) is correlated with the transverse momentum of a jet, which reflects the transverse momentum of a scattered parton, \( p_{T,\text{parton}} \). In the summation, we used all particles in one arm, not assigning any threshold to the distance \( R \) defined in Eq. 2.21. It is because the Central Arm acceptance makes the same effect as the threshold distance due to their finite size.

The ratio of the \( p_{\text{sum}}^T \) to \( p_{T,\text{parton}} \),

\[ \alpha \equiv \frac{p_{\text{sum}}^T}{p_{T,\text{parton}}} \]  

(4.2)

is one when all particles in a jet are detected. But in practice the \( R \) is less than one due to the fact that only photons and charged particles are detected, this means that neutrons, neutrinos, etc. are not detected. Furthermore, some photons or charged particles go outside the detector acceptance. \( R \) can be even greater than one due to the underlying event. By measuring \( p_{\text{sum}}^T \) and evaluating \( \alpha \) with simulation, we can obtain \( p_{T,\text{parton}} \).

Figure 4.1 show the \( p_{\text{sum}}^T \) distributions. The red, black, and green lines correspond to the real data, the PYTHIA default output, and the PYTHIA MPI output, respectively. These PYTHIA outputs were normalized with

\[ \frac{L_{\text{exp}}}{L_{\text{sim}}} \cdot R_{\text{trig}} \cdot \frac{1 - R_{\text{mask}}}{1} \]  

(4.3)
Here, $L_{\exp} \approx 0.23 \text{pb}^{-1}$ and $L_{\text{sim}}$ are the integrated luminosity of real data and simulation, respectively, $R_{\text{trig}}$ is the efficiency of BBCLL1 and ERT-gamma3 trigger (78.5 %), and $R_{\text{mask}}$ is the ratio of the masked EM-Cal towers to all (12.1 %).

The PYTHIA MPI output agrees with the real data above $\sim 6 \text{ GeV/c}$ and is $\lesssim 30\%$ larger at low-$p_T^{\text{sum}}$ range. And the PYTHIA default output agrees with the real data above $\sim 10 \text{ GeV/c}$ and is $\lesssim 3$ times smaller at low-$p_T^{\text{sum}}$ range.

The difference in the $p_T^{\text{sum}}$ distribution between the PYTHIA default and PYTHIA MPI should be caused by only the number of $2 \rightarrow 2$ scatterings per event. Possible phenomena caused by the several $2 \rightarrow 2$ scatterings are 1) that some events have a larger $p_T^{\text{sum}}$ than the true $p_T$ of a jet due to an overlap between jets and 2) that even if the photon from the hardest scattering goes outside the acceptance, some events satisfy the trigger condition ($p_T > 2 \text{ GeV/c}$) with the photon from non-hardest scatterings. The first one shifts the $p_T^{\text{sum}}$ distribution to higher side. The second one increases the yield in the $p_T^{\text{sum}}$ distribution.

The real data support the PYTHIA MPI output, that is the model with several $2 \rightarrow 2$ scatterings per event. The effect of several $2 \rightarrow 2$ scatterings becomes small at high-$p_T^{\text{sum}}$ range, and both the PYTHIA default and PYTHIA MPI agree with the real data.

### 4.2 Jet structure

#### 4.2.1 Transverse momentum density

We measured $p_T$ density as a function of azimuthal angles between a trigger photon and other detected particles. The azimuthal angle for each particle, $\Delta \phi_i$, is defined as

$$\Delta \phi_i \equiv |\phi_i - \phi_{\text{trig}}|$$ (4.4)

and the $p_T$ density, $D_{p_T}$, can be obtained as

$$D_{p_T}(\Delta \phi) = \frac{1}{N_{\text{evt}} \cdot W_{\text{bin}}} \sum_{\Delta \phi_i > \Delta \phi} p_T i$$ (4.5)

Here $N_{\text{evt}}$ is the number of events and $W_{\text{bin}} (= 0.1 \text{ rad})$ is a width of $\Delta \phi$ bin. We name the region at $\Delta \phi \lesssim 0.7$ ‘toward’ region and the region at $\Delta \phi \gtrsim 0.7$ ‘transverse’ region (see Fig. 4.2). Since particles from a jet are concentrated along the jet direction, the $D_{p_T}$ at the transverse region are sensitive to the underlying event.
Figure 4.1: $p_T^{\text{sum}}$ distribution (red: real data, black: PYTHIA default output, green: PYTHIA MPI output).

Figure 4.2: Toward and transverse region in $\phi$ space with respect to the direction of a triggered photon.
To avoid the effect of the PHENIX Central Arm acceptance in the calculation of $D_{p_T}$, we limited the $\phi$ direction of trigger photons within $20^\circ$ from the edge of the PHENIX Central Arms, and didn’t employ photons and charged particles which were in the $\phi$ area between the trigger photon and the near edge. With this method the $D_{p_T}(\Delta \phi)$ distribution is not affected by the finite acceptance of the PHENIX Central Arms up to $70^\circ$ ($\sim 1.2$ rad).

Figures 4.3–4.10 show the $D_{p_T}(\Delta \phi)$ distributions for each $p_T^{\text{sum}}$ range. The red, black, blue, and green lines correspond to the real data, the PYTHIA default output, the PYTHIA $k_T$ output, and the PYTHIA MPI output, respectively. These results can be separately discussed at the ‘toward’ region and the ‘transverse’ region.

In the ‘toward’ region, the PYTHIA MPI output agrees well with the real data. It shows that the shape of jets produced by the PYTHIA is consistent with the real data. The PYTHIA default output has larger $D_{p_T}(\Delta \phi)$ particularly at low-$p_T^{\text{sum}}$ range, although the PYTHIA parameters which is related to jet production are identical between the PYTHIA default and the PYTHIA MPI. It can be explained by the fact that the $p_T^{\text{sum}}$ of each event is fixed here ($4 < p_T^{\text{sum}} < 5$ GeV/$c$ etc.) and thus the total $p_T$ at the ‘toward’ region should become large to compensate a shortage of $p_T$ at the ‘transverse’ region.

In the ‘transverse’ region, the PYTHIA default output is generally smaller than the real data. It shows that the PYTHIA default does not have sufficient underlying events. The PYTHIA MPI output has an enhancement in comparison with the PYTHIA default, but is still smaller than the real data. Therefore we can conclude that the MPI tune improves the reproductivity of the underlying event but some insufficiency remains.

The PYTHIA $k_T$ output does not differ from the PYTHIA default output. It means that the $k_T$ tune alone affects little the $p_T$ density distribution.

### 4.2.2 Thrust distribution

Thrust $T$ represents the topology of particles in one event, and is defined as

$$T \equiv \max_u \left( \sum_i \frac{|\vec{p}_i \cdot \vec{u}|}{\sum_j |\vec{p}_j|} \right)$$

(4.6)

Here, $\vec{u}$ is a unit vector which is called the thrust axis and is directed to maximize $T$, and $\vec{p}_i$ is a momentum of each particle in one arm. $T$ is equal to one when all $\vec{p}_i$ are collinear, and $T$ decreases as the jet cone size decreases
Figure 4.3: $p_T$ density at $4 < p_T^{\text{sum}} < 5$ GeV/$c$ from data and four different simulations

Figure 4.4: $p_T$ density at $5 < p_T^{\text{sum}} < 6$ GeV/$c$ from data and four different simulations

Figure 4.5: $p_T$ density at $6 < p_T^{\text{sum}} < 7$ GeV/$c$ from data and four different simulations
Figure 4.6: $p_T$ density at $7 < p_T^{\text{sum}} < 8$ GeV/$c$ from data and four different simulations

Figure 4.7: $p_T$ density at $8 < p_T^{\text{sum}} < 9$ GeV/$c$ from data and four different simulations

Figure 4.8: $p_T$ density at $9 < p_T^{\text{sum}} < 10$ GeV/$c$ from data and four different simulations
Figure 4.9: $p_T$ density at $10 < p_T^{\text{sum}} < 11$ GeV/c from data and four different simulations

Figure 4.10: $p_T$ density at $11 < p_T^{\text{sum}} < 12$ GeV/c from data and four different simulations
Figure 4.11: Thrust value in the case of collinear ($T = 1.0$), isotropic ($T = 0.5$), and isotropic in one Central Arm ($T = 0.88$).

(see Fig. 4.11). The above equation is approximately equal to

$$ T = \frac{\sum_i |p_i \cdot \hat{p}|}{\sum_i |p_i|} \quad (4.7) $$

$$ \hat{p} = \frac{\sum_i p_i}{\sum_i |p_i|} \quad (4.8) $$

Here, $\hat{p}$ is the unit vector of vectorial sum of $p_i$. This equation can be calculated without iterations to determine the thrust axis.

In the PHENIX Central Arm acceptance, $\Delta \eta = 0.7$ and $\Delta \phi = 90^\circ$, the mean value of the thrust of high-multiplicity isotropic events is equal to 0.88 (see Fig. 4.11). The thrust distribution of low-multiplicity isotropic events can be calculated under the following simple conditions. First, the number of particles per event is fixed since it is currently impossible to evaluate its mean value and deviation as a function of $p_T^{\text{sum}}$, which depends upon experimental setup, event selection, particle selection, etc. Second, the particle production cross section is assumed to be proportional to $\exp(-6 p_T [\text{GeV}/c])$ and is independent of $\eta$ and $\phi$. Third, the similar cuts to the experiment are applied numerically; the geometrical acceptance ($|\eta| < 0.35$, $\Delta \phi = 90^\circ$) and the momentum limit ($p_T > 0.4 \text{ GeV}/c$). Figure 4.12 shows the thrust distribution of isotropic events with a fixed number of particles ($N = 2 \sim 7$). Particularly the thrust distribution of $N = 2$ events is steep. Thus we applied a cut of $N \geq 3$. Figure 4.13 shows the mean value and RMS of the number of particles per event in each $p_T^{\text{sum}}$ range. Figure 4.14 shows the fraction of events that remained after this cut as a function of $p_T^{\text{sum}}$. The $N \geq 3$ cut does not exclude much events except at low-$p_T^{\text{sum}}$.

Figures 4.15-4.22 show the thrust distribution in each $p_T^{\text{sum}}$ range. The red, black, green, and purple lines correspond to the real data, the PYTHIA
Figure 4.12: Thrust distribution for isotropic events with $N = 2 \sim 7$ in the PHENIX Central Arm acceptance. As the number of particles increases ($2 \rightarrow 7$), the mean value of thrust becomes smaller (black $\rightarrow$ purple).
Figure 4.13: The mean value and RMS of the number of particles per event.

Figure 4.14: The fraction of events which remained after $N \geq 3$ cut as a function of $p_T^{\text{sum}}$. 
default output, the PYTHIA MPI output, and the isotropic output, respectively. These PYTHIA outputs were normalized with Eq. 4.3. We superposed the isotropic output which had the number of particles nearest to the mean number in each $p_T^{\text{sum}}$ range. The number of particles for each $p_T^{\text{sum}}$ range is 3 (for $4 < p_T^{\text{sum}} < 5$ GeV/c), 4 (for $5 - 6$), 4 (for $6 - 7$), 5 (for $7 - 8$), 5 (for $8 - 9$), 6 (for $9 - 10$), 6 (for $10 - 11$), and 6 (for $11 - 12$) as shown in the figures. Their vertical values are arbitrary.

The PYTHIA MPI output shows general agreement with the real data. But the absolute yields between them at low-$p_T^{\text{sum}}$ range ($< 7$ GeV/c) are different within a factor of $< 1.5$. And the real data have a slightly broader tail at $T \sim 0.9$. The former is identical to the difference in $p_T^{\text{sum}}$ distribution (Fig. 4.1). The latter can be caused by the insufficiencies of PYTHIA MPI tune which appeared in the ‘transverse’ region ($\Delta \phi \gtrsim 0.7$) of $p_T$ density distributions (Fig. 4.3 etc.). Although we need detailed studies in these differences, the general agreement indicates that the real data can be described with the jet production (high-$p_T$ jet) process without the elastic or diffractive ones.

In all the real data, PYTHIA default output, and PYTHIA MPI output, the thrust distribution becomes sharper as $p_T^{\text{sum}}$ increases. It is due to the jet characteristic that its transverse momentum ($j_T$) is independent of its longitudinal momentum ($p_T^{\text{jet}}$) and is almost constant.

In comparison with the isotropic output, all the real data, PYTHIA default output, and PYTHIA MPI output have larger thrust value and steeper distribution except for $4 < p_T^{\text{sum}} < 5$ GeV/c. Therefore, even if the isotropic events exist under the real data, the contamination is negligible because the possible maximum contamination is limited with the yields at low-thrust region ($T \sim 0.9$) and thus the isotropic events should be small. A quantitative estimation can be performed under the assumption that the real data are composed of the linear sum of the PYTHIA MPI output and the isotropic output.

It is expected that an event with large-thrust value has a small contamination of the underlying event. Therefore, by applying a large-thrust cut, we may obtain good $\alpha = p_T^{\text{sum}} / p_T^{\text{parton}}$ defined in Eq. 4.2.

### 4.3 Ratio of $p_T$ sum of detected particles to $p_T$ of original parton

We evaluated the $\alpha$ in Eq. 4.2 with the PYTHIA simulator. This $p_T^{\text{parton}}$ can be obtained from an event list from the PYTHIA. By iteratively tracing a
Figure 4.15: Thrust distribution at $4 < p_T^{\text{min}} < 5 \text{ GeV/c}$

Figure 4.16: Thrust distribution at $5 < p_T^{\text{min}} < 6 \text{ GeV/c}$
Figure 4.17: Thrust distribution at $6 < p_T^{miss} < 7$ GeV/$c$

Figure 4.18: Thrust distribution at $7 < p_T^{miss} < 8$ GeV/$c$
Figure 4.19: Thrust distribution at $8 < \sum p_T < 9$ GeV/c

Figure 4.20: Thrust distribution at $9 < \sum p_T < 10$ GeV/c
Figure 4.21: Thrust distribution at $10 < p_T^{\text{sum}} < 11 \text{ GeV/c}$

Figure 4.22: Thrust distribution at $11 < p_T^{\text{sum}} < 12 \text{ GeV/c}$
parent particle or string from a trigger photon, we can select the original parton from two scattered partons. But, there are events in which the string traced from a trigger photon does not include neither scattered parton. The ratio of these events is 5% at \( p_T^{\text{sum}} \sim 5 \text{ GeV}/c \) and 2% at \( p_T^{\text{sum}} \gtrsim 10 \text{ GeV}/c \). Currently we discarded them in the \( R \) evaluation. We should evaluate the ratio with other event simulators in future to precisely check the systematic error of this method.

Figures 4.23–4.30 show \( \alpha \) distribution in each \( p_T^{\text{sum}} \) range. The black and green lines in each left plot correspond to the PYTHIA default output and the PYTHIA MPI output, respectively. At the higher side of peaks, especially in the low-\( p_T^{\text{sum}} \) range, there is a broad tail which should be caused by the underlying event. With the outputs of the PYTHIA MPI, we evaluated the mean value and the uncertainty of the ratio by fitting the top part of each peak with a Gaussian and calculating RMS in lower and upper side separately. The results are shown in the right plots of Fig. 4.23–4.30 and listed in Tab. 4.1. The ratios are around 0.85 in the measured \( p_T^{\text{sum}} \) range.

## 4.4 Double helicity asymmetry

The \( A_{LL} \) are evaluated fill by fill in each \( p_T^{\text{sum}} \) range and the results are fitted with a constant, since the beam polarization and the relative luminosity were measured fill by fill to decrease systematic errors. The statistical error of the
Figure 4.24: Ratio of $p_T^{\text{sum}}$ to $p_T^{\text{parton}}$ at $5 < p_T^{\text{sum}} < 6 \text{ GeV/c}$

Figure 4.25: Ratio of $p_T^{\text{sum}}$ to $p_T^{\text{parton}}$ at $6 < p_T^{\text{sum}} < 7 \text{ GeV/c}$

Figure 4.26: Ratio of $p_T^{\text{sum}}$ to $p_T^{\text{parton}}$ at $7 < p_T^{\text{sum}} < 8 \text{ GeV/c}$
Figure 4.27: Ratio of $p_T^{\text{sum}}$ to $p_T^{\text{parton}}$ at $8 < p_T^{\text{sum}} < 9$ GeV/c

Figure 4.28: Ratio of $p_T^{\text{sum}}$ to $p_T^{\text{parton}}$ at $9 < p_T^{\text{sum}} < 10$ GeV/c

Figure 4.29: Ratio of $p_T^{\text{sum}}$ to $p_T^{\text{parton}}$ at $10 < p_T^{\text{sum}} < 11$ GeV/c
Figure 4.30: Ratio of $p_T^{\text{sum}}$ to $p_T^{\text{parton}}$ at $11 < p_T^{\text{sum}} < 12$ GeV/c

Table 4.1: Ratio of $p_T^{\text{sum}}$ to $p_T^{\text{parton}}$

<table>
<thead>
<tr>
<th>$p_T^{\text{sum}}$ [GeV/c]</th>
<th>$p_T^{\text{sum}} / p_T^{\text{parton}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4~5</td>
<td>0.88$^{+0.63}_{-0.22}$</td>
</tr>
<tr>
<td>5~6</td>
<td>0.88$^{+0.63}_{-0.22}$</td>
</tr>
<tr>
<td>6~7</td>
<td>0.86$^{+0.63}_{-0.21}$</td>
</tr>
<tr>
<td>7~8</td>
<td>0.83$^{+0.60}_{-0.19}$</td>
</tr>
<tr>
<td>8~9</td>
<td>0.83$^{+0.33}_{-0.19}$</td>
</tr>
<tr>
<td>9~10</td>
<td>0.84$^{+0.39}_{-0.19}$</td>
</tr>
<tr>
<td>10~11</td>
<td>0.83$^{+0.34}_{-0.18}$</td>
</tr>
<tr>
<td>11~12</td>
<td>0.84$^{+0.36}_{-0.18}$</td>
</tr>
</tbody>
</table>
Table 4.2: Asymmetry vs. $p_T^{\text{sum}}$

<table>
<thead>
<tr>
<th>$p_T^{\text{sum}}$ [GeV/$c$]</th>
<th>$A_{LL}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4~5</td>
<td>-2.3 ± 2.7</td>
</tr>
<tr>
<td>5~6</td>
<td>2.4 ± 3.7</td>
</tr>
<tr>
<td>6~7</td>
<td>5.6 ± 5.1</td>
</tr>
<tr>
<td>7~8</td>
<td>-5.6 ± 7.4</td>
</tr>
<tr>
<td>8~9</td>
<td>-17 ± 10</td>
</tr>
<tr>
<td>9~10</td>
<td>-1.4 ± 14.1</td>
</tr>
<tr>
<td>10~11</td>
<td>17 ± 19</td>
</tr>
<tr>
<td>11~12</td>
<td>25 ± 25</td>
</tr>
</tbody>
</table>

$A_{LL}$ can be derived from Eq. 2.17

$$
\sigma_{A_{LL}} = \sqrt{\left(\frac{\partial A_{LL}}{\partial N_{++}}\right)^2 \cdot \sigma_{N_{++}}^2 + \left(\frac{\partial A_{LL}}{\partial N_{+-}}\right)^2 \cdot \sigma_{N_{+-}}^2}
$$

$$
= \frac{1}{P_B P_Y} \frac{2R \sqrt{N_{++}^2 \sigma_{N_{++}}^2 + N_{+++}^2 \sigma_{N_{+-}}^2}}{(N_{++}^2 + RN_{+-}^2)^2}
$$

$$
\approx \frac{1}{P_B P_Y} \frac{2R \sqrt{N_{++} N_{+-} (N_{++} + N_{+-})}}{(N_{++}^2 + RN_{+-}^2)^2} \quad (\because \sigma_N \approx \sqrt{N})
$$

(4.9)

Systematic errors are known to be smaller than the statistical error in virtue of the asymmetry measurement in which most systematic errors cancel. The estimation of systematic errors will have to be done with “beam bunch shuffling” technique. Currently only the statistical error has been estimated.

Figures 4.31~4.38 show the fill-by-fill values of the $A_{LL}$ and the fit results. The reduced $\chi^2$‘s support the validity of fits and the smallness of the systematic errors. Figure 4.39 and Tab. 4.2 shows the $A_{LL}$ as a function of $p_T^{\text{sum}}$. Figure 4.40 is an expansion of Fig. 4.39 with the theoretical predictions re-calculated with $\alpha$ from $p_T$ of jet in Fig. 2.13. Also, the expected statistical error of PHENIX RUN5 data are drawn as a blue band. Within the statistical errors the result of RUN3 is not inconsistent with all theoretical predictions, but the error is not small enough to discriminate the theoretical predictions. In expected RUN5 (2005) conditions, the integrated luminosity will become 10 pb$^{-1}$ in comparison with 0.27 pb$^{-1}$ (RUN3) and the beam polarizations will become 50% in comparison with 26% (RUN3). The statistical error in RUN5 will become 24 times smaller than that in RUN3. It is expected that we can determine the $\Delta g$ in the coming RUN5.
Figure 4.31: Asymmetry for each fill at $4 < p_T^{\text{sum}} < 5 \text{ GeV/c}$

![Asymmetry plot for fill range 3600 to 3850 with entries, mean, RMS, and probability data]

Figure 4.32: Asymmetry for each fill at $5 < p_T^{\text{sum}} < 6 \text{ GeV/c}$

![Asymmetry plot for fill range 3600 to 3850 with entries, mean, RMS, and probability data]
Figure 4.33: Asymmetry for each fill at $6 < p_t^{\text{sum}} < 7 \text{ GeV/c}$

Figure 4.34: Asymmetry for each fill at $7 < p_t^{\text{sum}} < 8 \text{ GeV/c}$
Figure 4.35: Asymmetry for each fill at $8 < p_T^{\text{sum}} < 9$ GeV/c

Figure 4.36: Asymmetry for each fill at $9 < p_T^{\text{sum}} < 10$ GeV/c
Figure 4.37: Asymmetry for each fill at $10 < p_T^{\text{sum}} < 11 \text{ GeV/c}$

![Asymmetry plot for $10 < p_T^{\text{sum}} < 11 \text{ GeV/c}$]

<table>
<thead>
<tr>
<th>asymmetry</th>
</tr>
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<tbody>
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<td>Entries</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>RMS</td>
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<tr>
<td>$\chi^2$/ndf</td>
</tr>
<tr>
<td>Prob</td>
</tr>
<tr>
<td>$p_0$</td>
</tr>
</tbody>
</table>

Figure 4.38: Asymmetry for each fill at $11 < p_T^{\text{sum}} < 12 \text{ GeV/c}$

![Asymmetry plot for $11 < p_T^{\text{sum}} < 12 \text{ GeV/c}$]

<table>
<thead>
<tr>
<th>asymmetry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entries</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>RMS</td>
</tr>
<tr>
<td>$\chi^2$/ndf</td>
</tr>
<tr>
<td>Prob</td>
</tr>
<tr>
<td>$p_0$</td>
</tr>
</tbody>
</table>
Figure 4.39: Asymmetry from RUN3 data as a function of $p_T^{\text{sum}}$.

Figure 4.40: Asymmetry from RUN3 data as a function of $p_T^{\text{sum}}$ plotted together with theoretical curves predicted with, from top to bottom, $\Delta g = g$ input, $\Delta g = -g$ input, GRSV-std, and $\Delta g = 0$ input. The colored band represents the expected statistical error of PHENIX RUN5 data.
Chapter 5

Conclusion

The spin of the proton is expressed with the sum of spins and orbital angular momenata of quarks and gluons. The fraction of proton spin carried by the quark spins has been revealed to be only $20 \sim 30\%$. One of the goals of the PHENIX experiment is to obtain the contribution of gluon spins to the proton spin.

We measured jet production process to obtain the double helicity asymmetry, $A_{LL}$, with the PHENIX RUN3 polarized proton-proton collision data. Photons and charged particles detected with one of the PHENIX Central Arms were treated as a jet. The PYTHIA simulator with default parameters and MPI-tuned parameters were tested to estimate the ratio of detected $p_T$ to jet $p_T$.

The vectorial sum of transverse momentum of detected particles, $p_T^{\text{sum}}$, was measured. The PYTHIA MPI output agrees with the real data above $\sim 6$ GeV/c and is $\lesssim 30\%$ larger at low-$p_T^{\text{sum}}$ range. The real data support the PYTHIA MPI output, that is the model with plural $2 \rightarrow 2$ scatterings per event.

Event structure was evaluated with the transverse momentum density as a function of $\Delta \phi$. At the ‘toward’ region, the PYTHIA MPI output agrees well with the real data. It shows that the shape of jets reproduced by the PYTHIA is consistent with the real data. At the ‘transverse’ region, the PYTHIA MPI output has an enhancement in comparison with the PYTHIA default, but is still smaller than the real data. We can describe that the MPI tune improves the reproductivity of the underlying event but some insufficiency remains.

Jet structure was evaluated with thrust variable. The thrust distribution of isotropic events was calculated. In comparison with the isotropic output, all the real data, PYTHIA default output, and PYTHIA MPI output have larger thrust value and steeper thrust distribution except for a range of $4 < p_T^{\text{sum}} < 5$ GeV/c. Even if the isotropic events exist under the real data, the
contamination is negligible.

The ratio of $p_T^{\text{sum}}$ to $p_T$ of original parton was estimated with the PYTHIA MPI. The ratio was around 0.85 in the measured $p_T^{\text{sum}}$ range, $4 < p_T^{\text{sum}} < 12 \text{ GeV/c}$.

We measured the $A_{LL}$ in $4 < p_T^{\text{sum}} < 12 \text{ GeV/c}$. Within the statistical errors the result is not inconsistent with theoretical predictions, but the error is not small enough to discriminate the theoretical predictions. The expected PHENIX RUN5 statistics will enable us to determine the $\Delta g$. 
Acknowledgement

I am obliged to the following persons. I could not finish this work without their thankful supports.

Prof. Toshi-Aki Shibata provided me with the opportunity to participate in the PHENIX experiment. His advice throughout my graduate course supported me in research activities. Dr. Yuji Goto, Dr. Yasuyuki Akiba, and Dr. Atsushi Taketani paid great attention on the progress of my research and pointed out many important aspects in understanding physics results. Dr. Kensuke Okada, Mr. Takuma Horaguchi, and Mr. Yoshinori Fukao gave me advices on analysis methods as well as physics arguments. All members of Shibata lab of Tokyo Tech supported me in various aspects.
Appendix A

Underlying event and Multi-Parton Interaction

Underlying event and Multi-parton interaction are described in Chapter 3. Here some supplementary descriptions are given.

When a hard parton-parton scattering event takes place in proton-proton collisions, the event contains particles which originate from the two outgoing partons and particles which come from the breakup of the proton, i.e. beam-beam remnants. The former particles contain the two outgoing jets and initial- and final-state radiations. The latter particles contain everything except for the former ones, and are called “underlying event”. These two kinds of particles cannot be distinguished, therefore reconstructed jets are contaminated with the underlying event.

PYTHIA reproduces the underlying event with Multi-Parton Interaction (MPI) mechanism. The MPI includes many parton-parton scatterings in one event. First, the hardest parton-parton scattering is generated with its initial- and final-state radiations, next any multiple interactions, and finally beam remnants are attached to the initiator partons of the hardest scattering.

A.1 Basic cross section and effective cut-off

The QCD cross section for hard $2 \rightarrow 2$ processes as a function of $p_T$ of scattered parton is written as

$$\frac{d\sigma}{dp_T^2} = \sum_{i,j,k} \int dx_1 \int dx_2 \int dt \ f_i(x_1, Q^2) \ f_j(x_2, Q^2) \ \frac{d\hat{\sigma}^{ij-k}}{dt} \ \delta\left(p_T^2 - \frac{\hat{s}}{s}\right) \ \delta$$

(A.1)
Here, $x_1$ and $x_2$ are the Bjorken’s $x$. $f_i$ and $f_j$ are the parton distribution functions. $\hat{\sigma}^{ij-kX}$ is the cross section of parton-parton scattering with flavors of $i$, $j$, and $k$. $\hat{s}$, $\hat{t}$, and $\hat{u}$ are the Mandelstam variables in parton-parton scattering. There are some limitations for an application of the formula above to small $p_T$ values. The integrals receive major contributions from the small-$x$ region, where parton distributions are poorly understood theoretically and experimentally. Also higher-order QCD corrections to the jet rates, $K$ factors, are not evident. One simple provision is to evaluate $\alpha_s$ of the hard scattering process at an optimized scale.

The total cross section of the hard parton-parton scattering above a given $p_{T\text{min}}$ is written as

$$\sigma_{\text{hard}}(p_{T\text{min}}) = \int_{p_{T\text{min}}^{2}}^{s/4} \frac{d\sigma}{dp_T^2} dp_T^2$$

(A.2)

Since the differential cross section diverges roughly like $dp_T^2/p_T^4$, $\sigma_{\text{hard}}$ is also divergent for $p_{T\text{min}} \to 0$. Now we represent the total non-diffractive (inelastic) cross section as $\sigma_{\text{nd}}(s)$. At current collider energies, $\sigma_{\text{hard}}$ becomes comparable with $\sigma_{\text{nd}}(s)$ for $p_{T\text{min}} = 1.5 \sim 2\text{GeV}$. Simply $\sigma_{\text{hard}}/\sigma_{\text{nd}}(s)$ can be the average number of parton-parton scatterings above $p_{T\text{min}}$ in an event, and this number may well be larger than unity. The average $\bar{s}$ of a scattering decreases more slowly with $p_{T\text{min}}$ than the increase of the number of interactions, so naively the total amount of scattered partonic energy becomes infinite. Therefore, one effective cut-off with non-zero $p_{T\text{min}}$ is needed.

A more credible reason for the effective cut-off is that the outgoing hadrons are color neutral objects. Therefore, when the $p_T$ of an exchanged gluon is small and the transverse wavelength correspondingly large, the gluon can no longer resolve the individual color charges, and the effective coupling decreases. This mechanism is not in contradiction with perturbative QCD calculations.

As a simple first approximation, the energy-dependence of $p_{T\text{min}}$ can be assumed to be the same as the total cross section. The default in PYTHIA is

$$p_{T\text{min}}(s) = (1.9 \text{ GeV}) \left( \frac{\sqrt{s}}{1000 \text{ GeV}} \right)^{0.16}$$

(A.3)

This dependence can be modified with PYTHIA parameters like

$$p_{T\text{min}}(s) = (\text{PARP}(82) \text{ GeV}) \left( \frac{\sqrt{s}}{\text{PARP}(89) \text{ GeV}} \right)^{\text{PARP}(90)}$$

(A.4)
A.2 Multi-parton interaction without impact parameters

We arrange all scatterings in one event in falling sequence of $x_T = 2p_T/E_{cm}$ and label $x_{T1} > x_{T2} > \cdots > x_{Ti}$ for technical convenience. As a stating point we assume that all hadron collisions are equivalent (no impact parameter dependence), and that the different parton-parton interactions take place completely independently of each other. The number of scatterings per event is then distributed according to a Poissonian with mean $\langle n \rangle = \sigma_{\text{hard}}(p_{T\text{min}})/\sigma_{\text{nd}}(s)$. For Monte Carlo generation of these interactions it is useful to define

$$f(x_T) = \frac{1}{\sigma_{\text{nd}}(s)} \frac{d\sigma}{dx_T} \quad (A.5)$$

with $d\sigma/dx_T$ obtained in analogy with Eq. (A.1). Then $f(x_T)$ is simply the probability to have a parton-parton interaction at $x_T$ under a non-diffractive hadron-hadron collision.

The probability for the hardest interaction, i.e. the one with highest $x_T = x_{T1}$, is given by

$$\frac{dP_{\text{hardest}}}{dx_{T1}} = f(x_{T1}) \cdot \exp \left\{ - \int_{x_{T1}}^{1} f(x'_T)dx'_T \right\} \quad (A.6)$$

It means the probability to have a scattering at $x_{T1}$ multiplied by the probability that there was no scattering with $x_T$ larger than $x_{T1}$. Using the same technique, the probability to have an $i$th scattering at an $x_{Ti} < x_{Ti-1} < \cdots < x_{T1} < 1$ is found to be

$$f(x_{Ti}) \cdot \frac{1}{(i-1)!} \cdot \left( \int_{x_{Ti}}^{1} f(x'_T)dx'_T \right)^{i-1} \exp \left\{ - \int_{x_{Ti}}^{1} f(x'_T)dx'_T \right\} \quad (A.7)$$

With the help of the integral

$$F(x_T) = \int_{x_T}^{1} f(x'_T)dx'_T = \frac{1}{\sigma_{\text{nd}}(s)} \int_{x_T^2/4}^{s/4} \frac{d\sigma}{d^2p_T} dp_T \quad (A.8)$$

(where we assume $F(x_T) \to \infty$ for $x_T \to 0$) and its inverse $F^{-1}$, the iterative procedure to generate a chain of scatterings 1 > $x_{T1}$ > $x_{T2}$ > $\cdots$ > $x_{Ti}$ is given by

$$x_{Ti} = F^{-1} \cdot (F(x_{Ti-1}) - \ln R_i) \quad (A.9)$$

Here the $R_i$ are random numbers evenly distributed between 0 and 1. The iterative chain is started with a fictitious $x_{T0} = 1$ and is terminated when $x_{Ti}$
is smaller than $x_{\text{Min}}$. To take into account the energy already used in harder scatterings, a conservative approach is to evaluate the parton distributions at the rescaled value $x'_T$

$$x'_T = \frac{x_i}{\sum_{j=1}^{i-1} x_j}$$

(A.10)

for $i$th scattered parton.

In a hard interaction, the number of possible string drawings is much more, and the overall situation can become quite complex when several hard scatterings are present in an event. Specifically, the string drawing now depends on the relative color arrangement, in each hadron individually, of the partons that are about to scatter. The standard string fragmentation description would have to be extended, in order to handle events where two or more valence quarks have been kicked out of an incoming hadron by separate interactions. In particular, the position of the baryon number would be unclear. As a provision against these difficulties, it is assumed in PYTHIA that all subsequent (non-hardest) interactions belong to one of the following three classes. By default, the three possibilities are assumed to be equally probable.

- Scatterings of the $gg \rightarrow gg$ type, with the two gluons in a color-singlet state, such that a double string is stretched directly between the two outgoing gluons, decoupled from the rest of the system. The possibility for this type can be modified with PARP(86)–PARP(85).

- Scatterings $gg \rightarrow gg$, but color correlations assumed to be such that each of the gluons is connected to one of the strings already present. The possibility for this type can be modified with PARP(85).

- Scatterings $gg \rightarrow q\bar{q}$, with the final pair again in a color-singlet state, such that a single string is stretched between them. The possibility for this type can be modified with 1–PARP(86).

### A.3 Multi-parton interaction with impact parameters

Up to this point, we have assumed that the initial state is the same for all hadron collisions, whereas in fact each collision also is characterized by a varying impact parameter, $b$. A small $b$ value corresponds to a large overlap between the two colliding hadrons, and hence an enhanced probability for multiple interactions.
Now we assume a spherically symmetric distribution of matter inside the hadron, $\rho(x)d^3x = \rho(r)d^3x$. Furthermore, we assume a double Gaussian distribution

$$\rho(r) \propto \frac{1 - \beta}{a_1^3} \exp\left(-\frac{r^2}{a_1^2}\right) + \frac{\beta}{a_2^3} \exp\left(-\frac{r^2}{a_2^2}\right) \quad (A.11)$$

This corresponds to a distribution with a small core region, of radius $a_2$ and containing a fraction $\beta$ of the total hadronic matter, embedded in a larger hadron of radius $a_1$. The values $\beta = 0.5$ and $a_2/a_1 = 0.2$ are default, and these can be modified with PYTHIA parameters like

$$\rho(r) \propto \frac{1 - \text{PARP}(83)}{a_1^3} \exp\left(-\frac{r^2}{a_1^2}\right) + \frac{\text{PARP}(83)}{a_2^3} \exp\left(-\frac{r^2}{a_2^2}\right) \quad (A.12)$$

The overall distance scale $a_1$ never enters in the subsequent calculations, since the non-diffractive cross section $\sigma_{nd}(s)$ is taken from literature rather than calculated from the $\rho(r)$.

For a collision with impact parameter $b$, the time-integrated overlap $O(b)$ between the matter distributions of the colliding hadrons is given by

$$O(b) \propto \int dt \int d^3x \rho(x, y, z) \rho(x + b, y, z + t)$$

$$\propto (1 - \beta)^2 \exp\left\{ -\frac{b^2}{2a_1^2} \right\} + \frac{2\beta(1 - \beta)}{a_1^2 + a_2^2} \exp\left\{ -\frac{b^2}{a_1^2 + a_2^2} \right\} + \frac{\beta^2}{2a_2^2} \exp\left\{ -\frac{b^2}{2a_2^2} \right\} \quad (A.13)$$

The larger the overlap $O(b)$ is, the more likely it is to have interactions between partons in the two colliding hadrons. In fact, there should be a linear relationship

$$\langle \tilde{n}(b) \rangle = k O(b) \quad (A.14)$$

Here $\tilde{n}$ counts the number of interactions when two hadrons pass each other with an impact parameter $b$. The constant of proportionality, $k$, is related to the parton-parton cross section and hence increases with c.m. energy.

For each given impact parameter, the number of interactions is assumed to be distributed according to a Poissonian. If the matter distribution has a tail to infinity (as the double Gaussian does), events may be obtained with arbitrarily large $b$ values. In order to obtain finite total cross sections, it is necessary to assume that each event contains at least one semi-hard
interaction. The probability that two hadrons that pass each other with an impact parameter $b$ will actually undergo a collision is then given by

$$P_{\text{int}}(b) = 1 - \exp(-\langle \tilde{n}(b) \rangle) = 1 - \exp(-kO(b))$$ \hfill (A.15)

according to Poissonian statistics. The average number of interactions per event at an impact parameter $b$ can be derived as

$$\langle n(b) \rangle = \frac{\langle \tilde{n}(b) \rangle}{P_{\text{int}}(b)} = \frac{kO(b)}{1 - \exp(-kO(b))}$$ \hfill (A.16)

The denominator comes from the removal of hadron pairs that pass without colliding, i.e. with $\tilde{n} = 0$.

The average number of parton-parton scatterings $\langle n \rangle$, described above, can be obtained by integrating $\langle n(b) \rangle$ over $b$,

$$\langle n \rangle = \frac{\int \langle n(b) \rangle P_{\text{int}}(b) d^2b}{\int P_{\text{int}}(b) d^2b} = \frac{\int kO(b) d^2b}{\int (1 - \exp(-kO(b))) d^2b} = \frac{\sigma_{\text{hard}}}{\sigma_{\text{nd}}}$$ \hfill (A.17)

For $O(b)$, $\sigma_{\text{hard}}$ and $\sigma_{\text{nd}}$ given, $k$ can always be found numerically by solving the last equality.

The absolute normalization of $O(b)$ is not important in itself, but only the relative variation with the impact parameter. It is therefore useful to introduce an enhancement factor $e(b)$, which gauges how the interaction probability for a passage with impact parameter $b$ compares with the average $\langle kO(b) \rangle$,

$$\langle \tilde{n}(b) \rangle = kO(b) = e(b) \langle kO(b) \rangle$$ \hfill (A.18)

With the knowledge of $e(b)$, the $f(x_T)$ function of the no-impact-parameter model generalizes to

$$f(x_T, b) = e(b)f(x_T)$$ \hfill (A.19)

In analogy with Eq. \( \text{A.6} \), the probability of finding the hardest scattering in an event at $x_{T1}$ is

$$\frac{dP_{\text{hardest}}}{d^2bdx_{T1}} = f(x_{T1}, b) \exp \left\{ -\int_{x_{T1}}^{1} f(x'_{T1}, b) dx'_{T1} \right\}$$

$$= e(b)f(x_{T1}) \exp \left\{ -e(b) \int_{x_{T1}}^{1} f(x'_{T1}) dx'_{T1} \right\}$$ \hfill (A.20)

With this $dP_{\text{hardest}}/d^2bdx_{T1}$, the $b$-dependent iterative procedure (Eq. \( \text{A.9} \)) can be obtained.
A.4 Configurable parameters

We list and briefly explain the PYTHIA parameters that were used in the Multi-Parton Interaction tune.

- MSTP(81) (default 1): master switch for multiple interactions.
  - 0: off
  - 1: on

- MSTP(82) (default 1): structure of multiple interactions.
  - 0: simple two-string model without any hard interactions.
  - 1: multiple interactions assuming the same probability in all events.
  - 2: multiple interactions assuming a varying impact parameter and a hadronic matter overlap consistent with a Gaussian matter distribution.
  - 3: multiple interactions assuming a varying impact parameter and a hadronic matter overlap consistent with a double Gaussian matter distribution.

- PARP(82) (default 1.9 GeV): effective minimum transverse momentum $p_{T_{\text{min}}}$ (in truer sense, regularization scale $p_{T_{0}}$) of the transverse momentum spectrum for multiple interactions, used in Eq. A.4

- PARP(83), PARP(84) (default 0.5, 0.2): parameters of an assumed double Gaussian matter distribution used in Eq. A.12

- PARP(85) (default 0.33): probability that an additional interaction in the multiple interaction formalism gives two gluons, with color connections to nearest neighbors in momentum space.

- PARP(86) (default 0.66): probability that an additional interaction in the multiple interaction formalism gives two gluons, either as described in PARP(85) or as a closed gluon loop.

- PARP(89) (default 1000 GeV): reference energy scale used in Eq. A.12 at which PARP(82) gives the $p_{T_{\text{min}}}$ values directly.

- PARP(90) (default 0.16): power of the energy-rescaling term of the $p_{T_{\text{min}}}$ parameters, used in Eq. A.12

- PARP(67) (default 1.0): the $Q^{2}$ scale of the hard scattering is multiplied by PARP(67) to define the maximum parton virtuality allowed in space-like showers.
Bibliography


