3. カイラル・クォーク・ソリトン模型

3.1. Fundamentals of Chiral Quark Soliton Model

♣ milestone in the history of CQSM

[1988] D. Diakonov, V. Petrov and P. Pobylitsa

- **proposal of the model** based on

  instanton picture of QCD vacuum

[1991] M. W and H. Yoshiki

- **numerical basis** for **nonperturbative evaluation** of nucleon observables including **Dirac sea quarks**
- **a possible solution to the nucleon spin puzzle**

[1993] M. W and T. Watabe

- discovery of novel $\mathbf{1/N_c}$ **correction**

  — resolution of g_A-problem—

[1996,1997] D. Diakonov et al.

- application to **PDF** of the nucleon

basic lagrangian

[1988] D.Diakonov, V.Petrov and P.Pobylitsa

- physical consequence of the **instanton liquid picture** of QCD vacuum
- quarks acquires **dynamical mass** due to the **spontaneous breaking of chiral symmetry**
- this mass is momenntum dependentwith $M(k=0) \simeq 350 MeV$
- **Goldstone pion** also appears in resultant effective action

$$\Downarrow$$

$$
\begin{aligned}
Z &= \int \mathcal{D}\pi^a \int \mathcal{D}\psi\, \mathcal{D}\psi^\dagger \; \exp \int d^4x \left\{ \psi_f^\dagger(x)\, i\, \partial\!\!\!/\, \psi^f(x) \right. \\
&+ \left. i\, \frac{d^4k\, d^4l}{(2\pi)^8}\, e^{i(k-l)\cdot x} \sqrt{M(k)M(l)}\; \psi^\dagger(k)\, e^{i\gamma_5 \boldsymbol{\pi}(x)\cdot\boldsymbol{\tau}/f_\pi}\, \psi(l) \right\}
\end{aligned}
$$

constant mass approximation

$$
\mathcal{L}_{CQM} = \bar{\psi}\,(\, i\, \partial\!\!\!/ - M\, U^{\gamma_5}(x)\,)\,\psi \quad \text{with} \quad U^{\gamma_5}(x) = e^{\,i\,\gamma_5\, \boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\,/\,f_\pi}
$$

no kinetic term for $\boldsymbol{\pi}(x)$

NJL model との関係

$$\mathcal{L}_{NJL} \;=\; \bar{\psi}\, i\, \partial\!\!\!/\, \psi \;+\; \frac{1}{2} G\, [\, (\bar{\psi}\psi)^2 \;+\; (\bar{\psi}\, i\, \gamma_5\, \boldsymbol{\tau}\psi)^2]$$

真空汎関数

$$\begin{aligned} Z_{NJL} \;&=\; \int \mathcal{D}\psi\, \mathcal{D}\psi^\dagger\; e^{i \int d^4x\, \mathcal{L}_{NJL}} \\ &=\; \int \mathcal{D}\psi\, \mathcal{D}\psi^\dagger\, \mathcal{D}\sigma\, \mathcal{D}\boldsymbol{\pi}'\; e^{i \int d^4x\, \mathcal{L}'_{NJL}} \end{aligned}$$

$$\mathcal{L}'_{NJL} \;=\; \bar{\psi}\, [\, i\, \partial\!\!\!/ \;-\; g\, (\sigma + i\, \gamma_5\, \boldsymbol{\tau} \cdot \boldsymbol{\pi}')\,]\, \psi - \frac{g^2}{2\, G}\, (\sigma^2 + \boldsymbol{\pi}'^2)$$

$$\left\{ \begin{array}{c} \sigma \\ \\ \boldsymbol{\pi}' \end{array} \right\} \quad : \quad \text{auxiliary (”collective”) meson fields}$$

nonliner constraint (by hand)

$$\sigma^2 \;+\; \boldsymbol{\pi}^2 \;=\; f_\pi^2$$

reparametrization

$$g\, (\sigma + i\, \gamma_5\, \boldsymbol{\tau} \cdot \boldsymbol{\pi}') \;=\; e^{i\, \gamma_5\, \boldsymbol{\pi} \cdot \boldsymbol{\tau} / f_\pi} \;\equiv\; M\, U^{\gamma_5} \qquad (M = g f_\pi)$$

ゆえに

$$\mathcal{L}'_{NJL} \;\sim\; \bar{\psi}\, (\, i\, \partial\!\!\!/ - M U^{\gamma_5})\, \psi$$

有効中間子模型（Skyrme 模型）との関係

—— 4 次までの derivative 展開 ——

$$\mathcal{L}_{eff}^{(n)} \sim C(\Lambda)\, \mathrm{tr}_f \,(\partial_\mu U\, \partial^\mu U^\dagger) + \frac{N_c}{32\,\pi^2}\, \mathrm{tr}_f \left\{ \frac{1}{12}\,[L_\mu, L_\nu]^2 - \frac{1}{3}\,(\partial_\mu L^\mu)^2 + \frac{1}{6}\,(L_\mu L^\mu)^2 \right\} + \cdots$$

ここで

$$C(\Lambda) = -\,i\,N_c \int^\Lambda \frac{d^4k}{(2\pi)^4}\,\frac{M^2}{(k^2 + i\epsilon - M^2)^2}$$

$$L_\mu = \partial_\mu U\, U^\dagger$$

ゆえに

$$\mathcal{L}_{eff}^{(n)} \sim \frac{f_\pi^2}{4}\, \mathrm{tr}_f \,(\partial_\mu U\, \partial^\mu U^\dagger) + \frac{N_c}{32\pi^2}\, \mathrm{tr}_f \left\{ \frac{1}{12}\,[L_\mu, L_\nu]^2 - \frac{1}{3}\,(\partial_\mu L^\mu)^2 + \frac{1}{6}\,(L_\mu L^\mu)^2 \right\} + \cdots$$

— **destablizing term** —

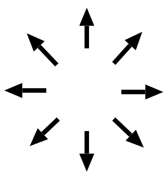
Cf.)

$$\mathcal{L}_{Skyrme} = \frac{f_\pi^2}{4}\, \mathrm{tr}_f \,(\partial_\mu U\, \partial^\mu U^\dagger) + \frac{1}{32\, e^2}\, \mathrm{tr}_f [L_\mu, L_\nu]^2$$

$$N_c = 3, \quad e = 2\pi$$

Soliton construction without derivative expansion

1st step : start with static $\boldsymbol{\pi}(\boldsymbol{x})$ of **hedgehog shape**



$$\boldsymbol{\pi}(\boldsymbol{x}) = \hat{\boldsymbol{r}}\, F(r)$$

M.F. for quarks

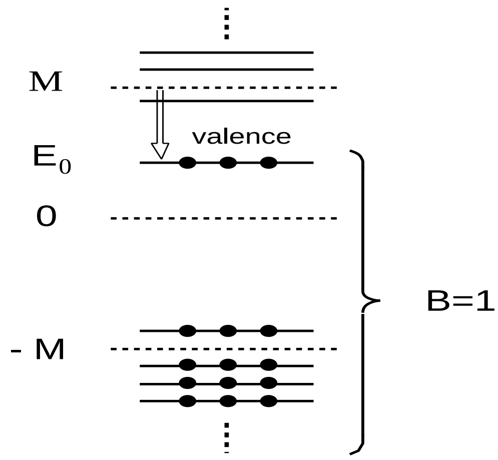
$$\left(\begin{array}{l} F(0) - F(\infty) = n\,\pi \\ n \;:\; \text{winding number} \end{array} \right)$$

Dirac eq.

$$H\,|\,m\,\rangle \;=\; E_m\,|\,m\,\rangle$$

$$H = \frac{\boldsymbol{\alpha}\cdot\nabla}{i} + M\beta\,(\cos F(r) + i\,\gamma_5\,\boldsymbol{\tau}\cdot\hat{\mathbf{r}}\sin F(r))$$

breaks "rotational" invariance



Energy of $|\,Q_H >$

$$E_{static} \;=\; N_c\,E_0 \;+\; E_{v.p.}$$

$$E_{v.p.} \sim N_c \left(\sum_{m<0} E_m \;-\; \sum_{k<0} \epsilon_k \right) \Longrightarrow$$ **regularize** (**physical cutoff**)

Hartree condition

$$\frac{\delta}{\delta F(r)}\, E_{static}[F(r)] \;=\; 0.$$

運動方程式（Hartree 方程式）の導出

$$
\begin{aligned}
E_{static}[F(r)] &= E_{val}[F(r)] + E_{v.p.}[F(r)] \\
&= N_c \sum_{m \le 0} E_m[F(r)] - \text{vacuum subtraction}
\end{aligned}
$$

ここで

$$
H\,\psi_m(\boldsymbol{r}) = E_m\,\psi_m(\boldsymbol{r})
$$

$$
H = \frac{\boldsymbol{\alpha}\cdot\nabla}{i} + \beta M\left(\cos F(r) + i\,\gamma_5\,\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\sin F(r)\right)
$$

$F(r)$ について変分

$$
\delta E_{static}[F(r)]
$$

$$
\begin{aligned}
&= N_c \sum_{m\le 0} \delta E_m[F(r)] \\
&= N_c \sum_{m\le 0} \delta \int \psi_m^\dagger(\boldsymbol{r})\,H\,\psi_m(\boldsymbol{r})\,d^3r \\
&= N_c \sum_{m\le 0} \int \{\,\delta\psi_m^\dagger(\boldsymbol{r})\,H\,\psi_m(\boldsymbol{r}) + \psi_m^\dagger(\boldsymbol{r})\,H\,\delta\psi_m(\boldsymbol{r}) \\
&\qquad\qquad + \psi_m^\dagger(\boldsymbol{r})\,\delta H\,\psi_m(\boldsymbol{r})\}\,d^3r \\
&= N_c \sum_{m\le 0} \Big\{ E_m \int \left(\delta\psi_m^\dagger(\boldsymbol{r})\,\psi_m(\boldsymbol{r}) + \psi_m^\dagger(\boldsymbol{r})\,\delta\psi_m(\boldsymbol{r})\right) d^3r \\
&+ \delta F(r)\left[\int \psi_m^\dagger(\boldsymbol{r})\,\beta M\left(-\sin F(r) + i\,\gamma_5\,\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\cos F(r)\right)\psi_m(\boldsymbol{r})\,d^3r\right]\Big\}
\end{aligned}
$$

$\int \psi_m^\dagger(\boldsymbol{r})\psi_m(\boldsymbol{r})d^3r = 1$ より第1項は0ゆえ

$$N_c \sum_{m\le 0} \int \bar{\psi}_m(\boldsymbol{r})\left(-\sin F(r) + i\,\gamma_5\,\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\cos F(r)\right)\psi_m(\boldsymbol{r}) \;=\; 0$$

$\delta E_{static}[F(r)] = 0 \quad \text{for} \quad \forall\delta F(r)$ より

$$S(r)\sin F(r) = P(r)\cos F(r)$$

ここで

$$S(r) \;=\; \sum_{m\le 0} \bar{\psi}_m(\boldsymbol{r})\,\psi_m(\boldsymbol{r}) \qquad : \text{ scalar quark density}$$

$$P(r) \;=\; \sum_{m\le 0} \bar{\psi}_m(\boldsymbol{r})\,i\,\gamma_5\,\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\,\psi_m(\boldsymbol{r}) \quad : \text{ pseudoscalar density}$$

$$\Downarrow$$

need **vacuum subtraction** and **regularization**

自己無撞着問題

soliton profile		**Dirac Hamiltonian**
$F(r)$	$\Longrightarrow$	$H[F(r)]$
$\Uparrow$		$\Downarrow$
運動方程式（停留条件）		**Dirac 方程式**
$S(r)\sin F(r) = P(r)\cos F(r)$	$\Longleftarrow$	$H\,\vert m\rangle \;=\; E_m\,\vert m\rangle$

Numerical method for handing infinite Dirac sea

sigle particle Dirac equation

$$H|m\rangle = E_m|m\rangle$$

$$H = \frac{\boldsymbol{\alpha}\cdot\nabla}{i} + M\beta\left(\cos F(r) + i\gamma^5\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\sin F(r)\right)$$

境界条件

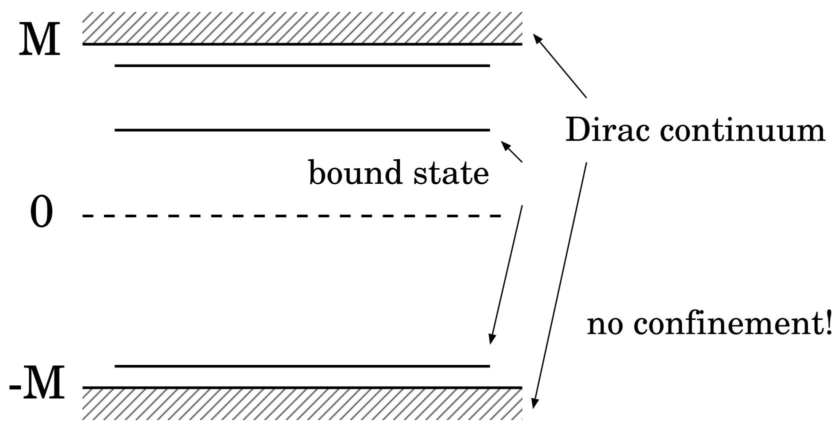
$$F(0) = n\pi, \quad F(\infty) = 0 \qquad (n=1)$$

$$\left.\begin{array}{l}[H,\boldsymbol{J}] \neq 0 \\ [H,\boldsymbol{\tau}] \neq 0\end{array}\right\} \Rightarrow \boldsymbol{K} = \boldsymbol{J} + \frac{1}{2}\boldsymbol{\tau} \quad (\text{grand spin}) \Rightarrow [H,\boldsymbol{K}] = 0$$

eigenstates

$$|m\rangle = |K^{parity}M_K\rangle \qquad (K^p = 0^{\pm}, 1^{\pm}, 2^{\pm}, \cdots)$$

general level structure



平面波基底

$$H_0\,|k\rangle \;=\; \epsilon_k\,|k\rangle, \qquad H_0 \;=\; \frac{\boldsymbol{\alpha}\cdot\nabla}{i} + M\beta \quad : \quad \text{自由粒子解}$$

角運動量基底

$$\begin{aligned}
\boldsymbol{K} &= \boldsymbol{J} + \frac{1}{2}\boldsymbol{\tau} \;=\; \boldsymbol{L} + \frac{1}{2}\boldsymbol{\sigma} + \frac{1}{2}\boldsymbol{\tau} \quad : \quad \text{coupling scheme} \\
|0\rangle &= |(L=K)\,J=K+\frac{1}{2}\,;\,KM_K\rangle \qquad (K\geq 0) \\
|1\rangle &= |(L=K)\,J=K-\frac{1}{2}\,;\,KM_K\rangle \qquad (K\geq 1) \\
|2\rangle &= |(L=K+1)\,J=K+\frac{1}{2}\,;\,KM_K\rangle \qquad (K\geq 0) \\
|3\rangle &= |(L=K-1)\,J=K-\frac{1}{2}\,;\,KM_K\rangle \qquad (K\geq 1)
\end{aligned}$$

平面波解の具体形

	natural parity $(0^+,1^-,2^+,\cdots)$	unnatural parity $(0^-,1^+,2^-,\cdots)$
$E>0$	$\varphi_a^{(n)} = N\begin{pmatrix} i\,j_K(kr)\,\|0\rangle \\ \frac{k}{\|\epsilon_k\|+M}\,j_{K+1}(kr)\,\|2\rangle \end{pmatrix}$	$\varphi_a^{(u)} = N\begin{pmatrix} i\,j_{K+1}(kr)\|0\rangle \\ -\frac{k}{\|\epsilon_k\|+M}\,j_K(kr)\,\|2\rangle \end{pmatrix}$
	$\varphi_b^{(n)} = N\begin{pmatrix} i\,j_K(kr)\|1\rangle \\ -\frac{k}{\|\epsilon_k\|+M}\,j_{K-1}(kr)\,\|3\rangle \end{pmatrix}$	$\varphi_b^{(u)} = N\begin{pmatrix} i\,j_{K-1}(kr)\,\|3\rangle \\ \frac{k}{\|\epsilon_k\|+M}\,j_K(kr)\,\|1\rangle \end{pmatrix}$
$E<0$	$\chi_a^{(n)} = N\begin{pmatrix} i\,\frac{k}{\|\epsilon_k\|+M}\,j_K(kr)\,\|0\rangle \\ -\,j_{K+1}(kr)\,\|1\rangle \end{pmatrix}$	$\chi_a^{(u)} = N\begin{pmatrix} i\,\frac{k}{\|\epsilon_k\|+M}\,j_{K+1}(kr)\,\|2\rangle \\ j_K(kr)\|0\rangle \end{pmatrix}$
	$\chi_b^{(n)} = N\begin{pmatrix} i\,\frac{k}{\|\epsilon_k\|+M}\,j_K(kr)\,\|1\rangle \\ j_{K-1}(kr)\|3\rangle \end{pmatrix}$	$\chi_b^{(u)} = N\begin{pmatrix} i\,\frac{k}{\|\epsilon_k\|+M}\,j_{K-1}(kr)\,\|2\rangle \\ -\,j_K(kr)\|1\rangle \end{pmatrix}$

isospin degeneracy ！

Kahana-Ripka の離散化平面波基底

(I) **離散化** :

$$j_K(k_i^K D) \;=\; 0 \qquad (\, D \;\gg\; \text{soliton size}\,)$$

$$\int_0^D dr\, r^2 \; j_K(k_i r)\, j_{K\pm1}(k_j r)$$
$$= \int_0^D dr\, r^2 \; j_{K\pm1}(k_i r)\, j_{K\pm1}(k_J r) \;=\; \delta_{ij}\, \frac{D^3}{2}\, [\, j_{K\pm1}(k_i D)]^2$$

$$\text{for} \;\; k_i, k_j \;\; \text{such that} \;\; j_K(k_i D) \;=\; j_K(k_j D) \;=\; 0$$

$$\Downarrow$$

平面波基底の**直交性**

(II) **有限化** : all $k_i^K \;\leq\; k_{max}$

- Numerical check of the following limit

$$\left\{ \begin{array}{c} D \\ k_{max} \end{array} \right\} \;\rightarrow\; \infty$$

2nd step : **quantization** of **collective rotational motion**

- energy degeneracy under isospin (spatial) rotation

$$E_{static}[R\,U_0^{\gamma_5}R^\dagger] \;=\; E_{static}[U_0^{\gamma_5}] \quad : \quad R \in SU(2)$$

$$\Downarrow$$

- spontaneous rotation of hedgehog mean-field (“**zero mode**”)

$$U^{\gamma_5}(\boldsymbol{x},t) \;=\; A(t)\,U_0^{\gamma_5}(\boldsymbol{x})\,A^\dagger(t) \quad : \quad A(t) \in SU(3)$$

then

$$\begin{aligned} S_{eff}[U] &= -i\,N_c\,\mathrm{Sp}\log\left[\,i\,\partial\!\!\!/ - M\,A(t)\,U_0^{\gamma_5}(\boldsymbol{x})\,A^\dagger(t)\,\right] \\ &= -i\,N_c\,\mathrm{Sp}\log\;A(t)\,\gamma^0\,(\,i\,\partial_t - H - \Omega\,)\,A^\dagger(t) \\ &= -i\,N_c\,\mathrm{Sp}\log\,(\,i\,\partial_t - H - \Omega) \end{aligned}$$

with

$$\Omega \;\equiv\; -\,i\,A^\dagger(t)\,\dot{A}(t) \;\equiv\; \frac{1}{2}\,\Omega_a\,\tau_a$$

: **collective angular velocity**

we then get

$$\begin{aligned} S_{eff}[U] &= S_{eff}[U_0] \;+\; \{\,S_{eff}[U] \;-\; S_{eff}[U_0]\,\} \\ &= S_{eff}[U_0] \;-\; i\,N_c\;\mathrm{Sp}\;\log\left(1 - \frac{1}{i\partial_t - H}\,\Omega\right) \end{aligned}$$

perturbative expansion in Ω

$$
\begin{aligned}
& \mathrm{Sp}\ \log\left(1 - \frac{1}{i\partial_t - H}\,\Omega\right) \\
= & \ T \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\ \mathrm{Tr}\ \log\left(1 - \frac{1}{\omega - H}\,\Omega\right) \\
= & \ T \int \frac{d\omega}{2\pi}\ \mathrm{Tr}\left(-\frac{1}{\omega - H}\,\Omega\ -\ \frac{1}{2}\,\frac{1}{\omega - H}\,\Omega\,\frac{1}{\omega - H}\,\Omega\ +\ \cdots\right) \\
= & \ -T \int \frac{d\omega}{2\pi} \sum_n \frac{1}{\omega - E_n} \langle n\,|\,\Omega\,|\,n\rangle \qquad (\Rightarrow\ \text{vanish due to T-rev.}\,!) \\
& -T \int \frac{d\omega}{2\pi} \sum_{m,n} \frac{1}{(\omega - E_n)(\omega - E_m)} \langle n\,|\,\Omega\,|\,m\rangle\langle m\,|\,\Omega\,|\,n\rangle\ +\ \cdots
\end{aligned}
$$

then

$$
\begin{aligned}
& \mathrm{Sp}\ \log\left(1 - \frac{1}{i\partial_t - H}\,\Omega\right) \\
= & \ T \cdot \frac{1}{4}\ i \sum_{m>0,n\leq 0} \frac{\langle n\,|\,\tau_a\,|\,m\rangle\langle m\,|\,\tau_b\,|\,n\rangle}{E_m - E_n}\ \Omega_a\,\Omega_b + \cdots \\
\equiv & \ T \cdot \frac{1}{2}\ i\ \cdot\ I_{ab}\ \cdot\ \Omega_a\,\Omega_b
\end{aligned}
$$

where

$$
I_{ab}\ \equiv\ \frac{1}{2} \sum_{m>0,n\leq 0} \frac{< n\,|\,\tau_a\,|\,m >< m\,|\,\tau_b\,|\,n >}{E_m - E_n}\ =\ \delta_{ab}\ I
$$

with

$$
I\ =\ \frac{1}{2} \sum_{m>0,n\leq 0} \frac{< n\,|\,\tau_3\,|\,m >< m\,|\,\tau_3\,|\,n >}{E_m - E_n}\quad :\ \textbf{moment of inertia}
$$

effective lagrangian

$$S_{eff}[U] \equiv T \cdot L_{eff}[U]$$

with

$$L_{eff}[U] = -E_{static}[U_0] + \frac{1}{2} I \, \Omega_a^2$$

canonical quantization : (crude argument)

Ω_a ∼ **time derivative** of **collective coordinate**

canonical momentum : $\hat{J}_a \sim \dfrac{\partial L_{eff}}{\partial \Omega_a} = I \, \Omega_a$

then

$$\begin{aligned} H_{eff} &= \hat{J}_a \Omega_a - L_{eff} \\ &= E_{static}[U_0] + \frac{\hat{J}_a^2}{2I} = E_{static}[U_0] + H_{rot} \end{aligned}$$

$\hat{J}_a$: collective angular momentum operator

H_{rot} : hamiltonian of classical **symmetric top**

eigenstate of H_{rot}

$$H_{rot} \, \Psi^{(J)}_{M_J M_T}[A] = \frac{J(J+1)}{2I} \, \Psi^{(J)}_{M_J M_T}[A]$$

$$\Psi^{(J)}_{M_J M_T}[A] = \sqrt{\frac{2J+1}{8\pi^2}} \, (-1)^{T+T_3} \cdot \underbrace{D^{(J)}_{-T_3 J_3}[A]}_{\text{Wigner rotation matrix}}$$

[補遺 1] **correlation** between **iso- and spatial-rotation**

for hedgehog configuration

$$U(\boldsymbol{x}) = e^{i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)}$$

isospin rotation matrix $A \in SU(2)$

$$A\tau_j A^\dagger = \tau_a R_{aj}$$
$$R_{aj} = \frac{1}{2}\mathrm{tr}\left[\tau_a A \tau_j A^\dagger\right] \subset SO(3)$$

有名な $SU(2) \Longleftrightarrow SO(3)$ 対応

global isorotation

$$\begin{aligned} A\,U(\boldsymbol{x})\,A^\dagger &= A\,e^{i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)}\,A^\dagger \\ &= e^{i\,A\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\,A^\dagger\,F(r)} \\ &= e^{i\,\tau_a\,R_{aj}\,\hat{x}_j\,F(r)} \\ &= U(R\hat{\boldsymbol{x}}) \end{aligned}$$

$R\,\hat{\boldsymbol{x}}$ = **spatial rotation**

[補遺2] relation between spin and isospin

$$\begin{cases} \Omega = i\,A^\dagger \dot{A} = \frac{1}{2}\,\Omega_a\,\tau_a \\ \omega = i\,A\,\dot{A}^\dagger = \frac{1}{2}\,\omega_a\,\tau_a \end{cases}$$

quantization rule

$$\begin{cases} \Omega_a = \mathrm{tr}\,[\,\tau_a i A^\dagger \dot{A}] & : \quad \Omega_a \to -\hat{J}_a\,/\,I \\ \omega_a = \mathrm{tr}\,[\,\tau_a i A \dot{A}^\dagger] & : \quad \omega_a \to -\hat{T}_a\,/\,I \end{cases}$$

$$\begin{aligned}
\Omega_a &= \mathrm{tr}\,[\,\tau_a\,i\,A^\dagger\,\dot{A}] = \mathrm{tr}\,[\,\tau_a \cdot i\,A^\dagger \dot{A} A^\dagger A] \\
&= -\,\mathrm{tr}\,[\,A\,\tau_a\,A^\dagger \cdot i\,A\,\dot{A}^\dagger] \\
&= -\,\frac{1}{2}\,\mathrm{tr}\,[\,i\,A\dot{A}^\dagger\,\tau_b] \cdot \mathrm{tr}\,[\,A\,\tau_a A^\dagger\,\tau_b] \\
&\qquad\qquad (\mathrm{tr}(AB) = \frac{1}{2}\,\mathrm{tr}\,(A\tau_a)\,tr(B\tau_a)) \\
&= -\,\mathrm{tr}\,[\,i\,A\,\dot{A}^\dagger\,\tau_b] \cdot \frac{1}{2}\,\mathrm{tr}\,[\,\tau_b A\,\tau_a\,A^\dagger] \\
&= -\,\omega_b\,R_{ba}
\end{aligned}$$

これより

$$\hat{J}_a = -\,\hat{T}_b\,R_{ba} \qquad (R_{ba} \subset SO(3))$$

R_{ba} は直交行列だから $\hat{J}_a^2 = \hat{T}_a^2$ が生じる。

$$\hat{J}^2 = \hat{T}^2 \quad : \quad \textbf{contraint}\ \text{on eigenstates}$$

nucleon observables : (M.E. of **quark bilinear op.**)

$$\langle \bar{\psi} O^\mu \psi \rangle = \int \mathcal{D}\psi \, \mathcal{D}\psi^\dagger \; \bar{\psi} \, O^\mu \, \psi \;\; e^{\, i \int d^4x \, \bar{\psi} (i \partial\!\!\!/ - M U^{\gamma_5}) \psi}$$

$$= \frac{1}{i} \, \frac{\delta}{\delta A_\mu} \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \; e^{i \int d^4x \, \bar{\psi} \, (\, i \partial\!\!\!/ - M U^{\gamma_5} + A_\mu O^\mu) \psi} \, |_{A_\mu \to 0}$$

$$= \frac{N_c}{i} \, \frac{\delta}{\delta A_\mu} \, \mathrm{Sp} \log (\, i \, \partial\!\!\!/ - M U^{\gamma_5} + A_\mu O^\mu) \, |_{A_\mu \to 0}$$

$$= \frac{N_c}{i} \, \mathrm{Sp} \left[\frac{1}{i \, \partial\!\!\!/ - M U^{\gamma_5}} \, O^\mu \right]$$

collective rotation

$$U^{\gamma_5}(\boldsymbol{x}, t) = A(t) \, U_0^{\gamma_5}(\boldsymbol{x}) \, A^\dagger(t)$$

$$\Downarrow$$

$$\langle \bar{\psi} O^\mu \psi \rangle = \frac{N_c}{i} \, \mathrm{Sp} \left[\frac{1}{i \partial_t - H - \Omega} \, \tilde{O}^\mu \right]$$

with

$$\tilde{O}^\mu \equiv A^\dagger(t) \, \gamma^0 \, O^\mu \, A(t)$$

perturbative expansion in Ω : ($\sim$ $1/N_c$ expansion)

$$\frac{1}{i\partial_t - H - \Omega} = \frac{1}{i\partial_t - H} + \frac{1}{i\partial_t - H} \, \Omega \, \frac{1}{i\partial_t - H} + \cdots$$

final answer for nucleon observables

$$\langle J'M_J'M_T' \,|\, O \,|\, JM_JM_T \rangle \;=\; \int \mathcal{D}A \;\; \Psi^{(J')*}_{M_J'M_T'}[A] \; \langle O \rangle_A \; \Psi^{(J)}_{M_JM_T}[A]$$

with

$$\langle O \rangle_A \;=\; \langle O \rangle_A^{(0)} \;+\; \langle O \rangle_A^{(1)} \;+\; \cdots$$

$\boldsymbol{O\,(\Omega^0)}$: leading N_c term (Mean field prediction)

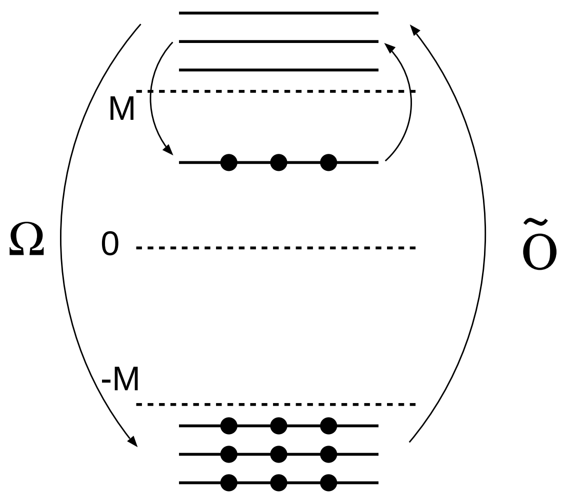
$$\langle O \rangle_A^{(0)} \;=\; N_c \sum_{n \leq 0} \langle \, n \,|\, \tilde{O} \,|\, n \, \rangle$$

diagonal sum over **occupied** states (valence + sea)

$\boldsymbol{O\,(\Omega^1)}$: $1/N_c$ correction term (Linear response)

$$\langle O \rangle_A^{(1)} \;=\; \frac{N_c}{2} \sum_{m>0, n\leq 0} \frac{1}{E_m - E_n} \left[\langle n \,|\, \tilde{O} \,|\, m \rangle \langle m \,|\, \Omega \,|\, n \rangle + (\tilde{O} \leftrightarrow \Omega) \right]$$

virtual transition from **occupied** to **nonoccupied** states



model needs regularization

$$
\begin{aligned}
S_{eff}[U] &= -\,i\,N_c\,\mathrm{Sp}\,\log\left[\,i\,\partial\!\!\!/ - MU^{\gamma_5}\right] \\
&= \frac{4N_c}{f_\pi^2}\,I_2(M)\cdot\frac{1}{2}\,(\partial_\mu \boldsymbol{\pi})^2 \;+\;\cdots
\end{aligned}
$$

where

$$
I_2(M) \;\equiv\; i\int\frac{d^4k}{(2\pi)^4}\,\frac{M^2}{(k^2-M^2)^2} \qquad : \quad \textbf{対数発散}
$$

Pauli-Villars regularization scheme

$$
\boxed{S_{eff}^{reg} \;\equiv\; S_{eff}^{M} \;-\; \left(\frac{M}{M_{PV}}\right)^2 S_{eff}^{M_{PV}}}
$$

then

$$
I_2^{reg} \;\equiv\; I_2(M) - \left(\frac{M}{M_{PV}}\right)^2 I_2(M_{PV}) \;=\; \frac{M^2}{16\pi^2}\,\log\left(\frac{M_{PV}}{M}\right)^2
$$

$$
\frac{N_c}{4\pi^2}\,M^2\,\log\left(\frac{M_{PV}}{M}\right)^2 = f_\pi^2 \quad\Longrightarrow\quad \boldsymbol{M_{PV}}
$$

other observables

$$
\langle O\rangle^{reg} \;\equiv\; \langle O\rangle^{M} \;-\; \left(\frac{M}{M_{PV}}\right)^2 \langle O\rangle^{M_{PV}}
$$