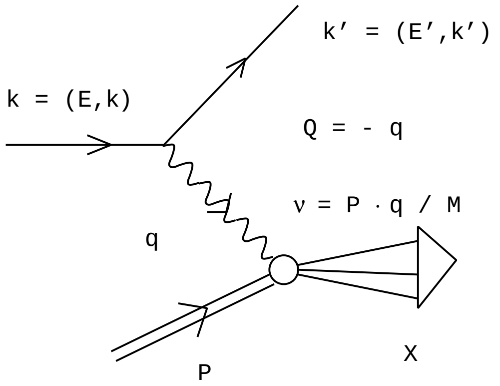
4.1. DIS and Parton Distribution Functions

inclusive cross section



$$\frac{d^2\sigma}{dE'\,d\Omega'} = \frac{\alpha^2}{M\,Q^4}\,\frac{E'}{E}\,l_{\mu\nu}\,W^{\mu\nu}$$

$l_{\mu\nu}$: lepton tensor
$W_{\mu\nu}$: hadron tensor

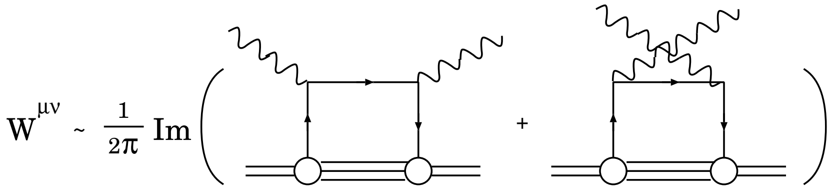
hadron tensor (unpolarized case)

$$\begin{aligned} W^{\mu\nu} &= \frac{1}{4\,\pi}\int d^4\xi\; e^{iq\cdot\xi}\;\langle P|[J^\mu(\xi), J^\nu(0)]|P\rangle \\ &= W_1(Q^2,\nu)\left(-\,g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \\ &+\; W_2(Q^2,\nu)\,\frac{1}{M^2}\left(P^\mu - \frac{(P\cdot q)q^\mu}{q^2}\right)\left(P^\nu - \frac{(P\cdot q)q^\nu}{q^2}\right) \end{aligned}$$

Bjorken limit

$$Q^2 \to \infty,\;\; \nu \to \infty,\quad x_B = \frac{Q^2}{2\,M\,\nu}\;\; \text{fixed} \qquad (0 < x_B < 1)$$

leading diagram



in Bjorken limit

$$W^{\mu\nu} = -\frac{1}{2\pi} \text{Im} \int_{-\infty}^{\infty} dx \; \text{Tr} \left[\gamma^{\mu} \frac{x \not{P} + \not{q}}{(xP+q)^2 + i\varepsilon} \gamma^{\nu} M_{\alpha\beta}(x) \right]$$

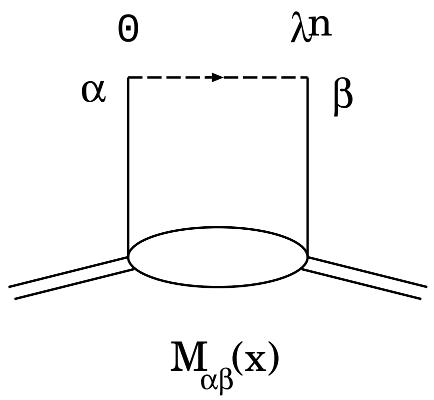
where

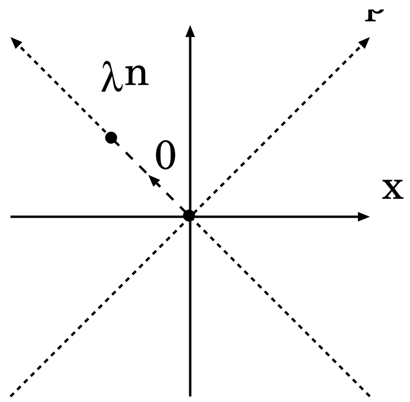
$$M_{\alpha\beta}(x) = \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle P | \bar{\psi}_{\beta}(0) \psi_{\alpha}(\lambda n) | P \rangle$$

2 light-like vectors $(n^2 = 0, p^2 = 0, n \cdot p = 0)$

$$n^{\mu} = \frac{1}{\sqrt{2}\Lambda}(1,0,0,-1), \qquad p^{\mu} = \frac{\Lambda}{\sqrt{2}}(1,0,0,1)$$

- $M_{\alpha\beta}(x)$ contains 2 quark fields separated along the light-cone





struck quark propagates along the light-cone

soft part of DIS

$$\Gamma(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \Gamma \psi(\lambda n) | PS \rangle$$

9 independent quark distribution functions

$$\int \frac{d\lambda}{2\pi}\ e^{i\lambda x}\ < PS|\,\bar{\psi}(0)\,\gamma_\mu \psi(\lambda n)\,|PS >$$

$$= 2\ \{\underbrace{f_1(x)}_{2}\,p_\mu\ +\ M^2\ \underbrace{f_4(x)}_{4}\ n_\mu\}$$

$$\int \frac{d\lambda}{2\pi}\ e^{i\lambda x}\ < PS\,|\,\bar{\psi}(0)\,\psi(\lambda n)\,|\,PS > =\ 2\,M\ \underbrace{e(x)}_{3}$$

$$\int \frac{d\lambda}{2\pi}\ e^{i\lambda x}\ < PS\,|\,\bar{\psi}(0)\,\gamma_\mu\,\gamma_5\,\psi(\lambda n)\,|\,PS >$$

$$=\ 2\ \{\underbrace{g_1(x)}_{2}\,p_\mu\,S\cdot n + \underbrace{g_T(x)}_{3}\ S_{\perp\mu} + M^2\ \underbrace{g_3(x)}_{4}\ n_\mu\,S\cdot p\}$$

$$\int \frac{d\lambda}{2\pi}\ e^{i\lambda x}\ < PS\,|\,\bar{\psi}(0)\,i\,\sigma_{\mu\nu}\,\gamma_5\,\psi(\lambda n)\,|\,PS >$$

$$\begin{aligned} = \qquad & 2\ \{\underbrace{h_1(x)}_{2}\,(\,S_{\perp\mu}\,p_\nu - S_{\perp\nu}\,p_\mu)\,/\,M \\ & +\ \underbrace{h_L(x)}_{3}\ M\,(\,p_\mu\,n_\nu - p_\nu\,n_\mu)\,S\cdot n \\ & +\ \underbrace{h_3(x)}_{4}\ M\,(\,S_{\perp\mu}\,n_\nu - S_{\perp\nu}\,n_\mu)\,\} \end{aligned}$$

frequently used notation

$$f_1(x) \equiv q(x), \quad g_1(x) \equiv \Delta q(x), \quad h_1(x) \equiv \delta q(x)$$

Ramark on the antiquark distributions

$$q(x) \;=\; \int_{-\infty}^{\infty} dz_0 \; e^{\,i\,x M_N\,z_0} \; \langle\, N \,|\, \bar{\psi}(0)\,(1+\gamma^0\,\gamma^3)\,\psi(z)\,|\, N \,\rangle$$

$$\bar{q}(x) \;=\; \int_{-\infty}^{\infty} dz_0 \; e^{\,i\,x M_N\,z_0} \; \langle\, N \,|\, \bar{\psi}^c(0)\,(1+\gamma^0\,\gamma^3)\,\psi^c(z)\,|\, N \,\rangle$$

where

$$\psi^c \;=\; C\;\bar{\psi}^T, \qquad C \;:\; \textbf{charge conjugation matrix}$$

one can prove

$$\boxed{\bar{q}(x) \;=\; -\,q(-x), \qquad (0 < x < 1)}$$

for longitudinally polarized distribution

$$\Delta q(x) \;=\; \int_{-\infty}^{\infty} dz_0 \; e^{\,i\,x M_N\,z_0} \; \langle\, N \,|\, \bar{\psi}(0)\,(1+\gamma^0\,\gamma^3)\,\boldsymbol{\gamma_5}\,\psi(z)\,|\, N \,\rangle$$

because of

$$C^{-1}\,(1+\gamma^0\,\gamma^3)\,\boldsymbol{\gamma_5}\,C \;=\; -\,C^{-1}\,(1+\gamma^0\,\gamma^3)\,C$$

it follows that

$$\boxed{\Delta\bar{q}(x) \;=\; +\,\Delta q(-x), \qquad (0 < x < 1)}$$

4.2. CQSM and Parton Distribution Functions (PDF)

quark distribution functions

$$q(x) \quad = \quad \frac{1}{4\pi} \int_{-\infty}^{\infty} dz^0 \; e^{\,i\,x\,M_N\,z^0}$$

$$\times \; \underbrace{\langle N(\boldsymbol{P}) \,|\, \psi^\dagger(0)\, O\, \psi(z) \,|\, N(\boldsymbol{P}) \rangle}\; |_{z^3 = -z^0,\, z_\perp = 0}$$

nucleon matrix element of **bilocal operator**

$\Downarrow$

taking full account of this **nonlocality** !

Novel N_c dependencies of twist-2 distributions

$$\begin{cases} u(x) + d(x) & \sim \quad N_c \, [\, O(\Omega^0) \; + \quad 0 \quad] \\ u(x) - d(x) & \sim \quad N_c \, [\quad \mathbf{0} \quad + \; O(\Omega^1) \,] \\ \Delta u(x) + \Delta d(x) & \sim \quad N_c \, [\quad \mathbf{0} \quad + \; O(\Omega^1) \,] \\ \Delta u(x) - \Delta d(x) & \sim \quad N_c \, [\, O(\Omega^0) \; + \; \underbrace{O(\Omega^1)} \,] \end{cases}$$

$$\boldsymbol{\Omega \;\propto\; 1\,/\,N_c}$$

basis of analysis

$$\langle N(\boldsymbol{P}) \,|\, \psi^\dagger(0)\, O\, \psi(z) \,|\, N(\boldsymbol{P}) \rangle$$

$$\sim \int d^3x\, d^3y\; e^{-i\boldsymbol{P}\cdot\boldsymbol{x}}\, e^{i\boldsymbol{P}\cdot\boldsymbol{y}} \int \mathcal{D}\pi \int \mathcal{D}\psi\, \mathcal{D}\psi^\dagger$$
$$\times\; J_N\left(\frac{T}{2}, \boldsymbol{x}\right) \cdot \psi^\dagger(0)\, O_a\, \psi(z) \cdot J_N^\dagger\left(-\frac{T}{2}, \boldsymbol{y}\right)\; e^{i\int d^4x\, \mathcal{L}_{CQSM}}$$

where

$$J_N(x) \;=\; \frac{1}{N_c!}\, \epsilon^{\alpha_1\cdots\alpha_{N_c}}\, \Gamma^{\{f_1\cdots f_N\}}_{JJ_3,TT_3}\; \psi_{\alpha_1 f_1}(x) \cdots \psi_{\alpha_{N_c} f_{N_c}}(x)$$

composite operator carrying the quantum numbers JJ_3, TT_3 of the nucleon

rotational zero mode

$$U^{\gamma_5}(\boldsymbol{x}, t) \;=\; A(t)\, U_0^{\gamma_5}(\boldsymbol{x})\, A^\dagger(t) \quad : \quad A(t) \in SU(2)$$

variable transform : $\psi(x) \;\rightarrow\; \psi_A(x) \;=\; A(t)\, \psi(t)$

$$e^{i\int d^4x\, \bar{\psi}\,(\,i\partial\!\!\!/ - MU^{\gamma_5}\,)\,\psi} \;=\; e^{i\int d^4x\, \psi_A^\dagger\,(\,i\partial_t - H - \Omega\,)\,\psi_A}$$
$$\psi^\dagger(0)\, O_a\, \psi(z) \;=\; \psi_A^\dagger(0)\; \underbrace{A^\dagger(0)\, O_a\, A(z_0)}\; \psi_A(z)$$

$$\Downarrow$$

perturbative expansion in $\boldsymbol{\Omega}$

2 new features in PDF calculation

$$\psi_A^\dagger(0)\,\underbrace{A^\dagger(0)\,O_a\,A(z_0)}\,\psi_A(z)$$

1. Ω can operate **between** 0 and z_0 !
2. **nonlocality** (in time) of $A^\dagger(0)\,O_a\,A(z_0)$

novel nonlocality correction

$$\begin{aligned} A^\dagger(0)O_aA(z_0) &= A^\dagger(0)\,O_a\,A(0) + z_0\,A^\dagger(0)\,O_a\,\dot{A}(0) + \cdots \\ &= A^\dagger(z_0)\,O_a\,A(z_0) - z_0\,\dot{A}^\dagger(z_0)\,O_a\,A(z_0) + \cdots \end{aligned}$$

in quantization : ($\Omega_a \to \hat{J}_a/I$)

$$\begin{aligned} A^\dagger(0)\,O_a\,A(z_0) &\to A^\dagger\,O_a\,A + \frac{1}{2}\,z_0\,(\,A^\dagger\,O_a\,A\,A^\dagger\,\dot{A} - \dot{A}^\dagger\,A\,A^\dagger\,O_a\,A\,) \\ &= \tilde{O}_a + i\,z_0\,\frac{1}{2}\{\Omega,\tilde{O}_a\} \end{aligned}$$

$$\Downarrow$$

2nd term : $O(\Omega^1)$ correction from **nonlocality**

schematically

$$q(x) = \frac{1}{4\pi} \int_{-\infty}^{\infty} dz_0 \; e^{ix\, M_N\, z_0}$$
$$\times \int \mathcal{D}A \quad \Psi^{(J)*}_{M_J M_T}[A] \; \langle O(0, z_0) \rangle_A \; \Psi^{(J)}_{M_J M_T}[A]$$

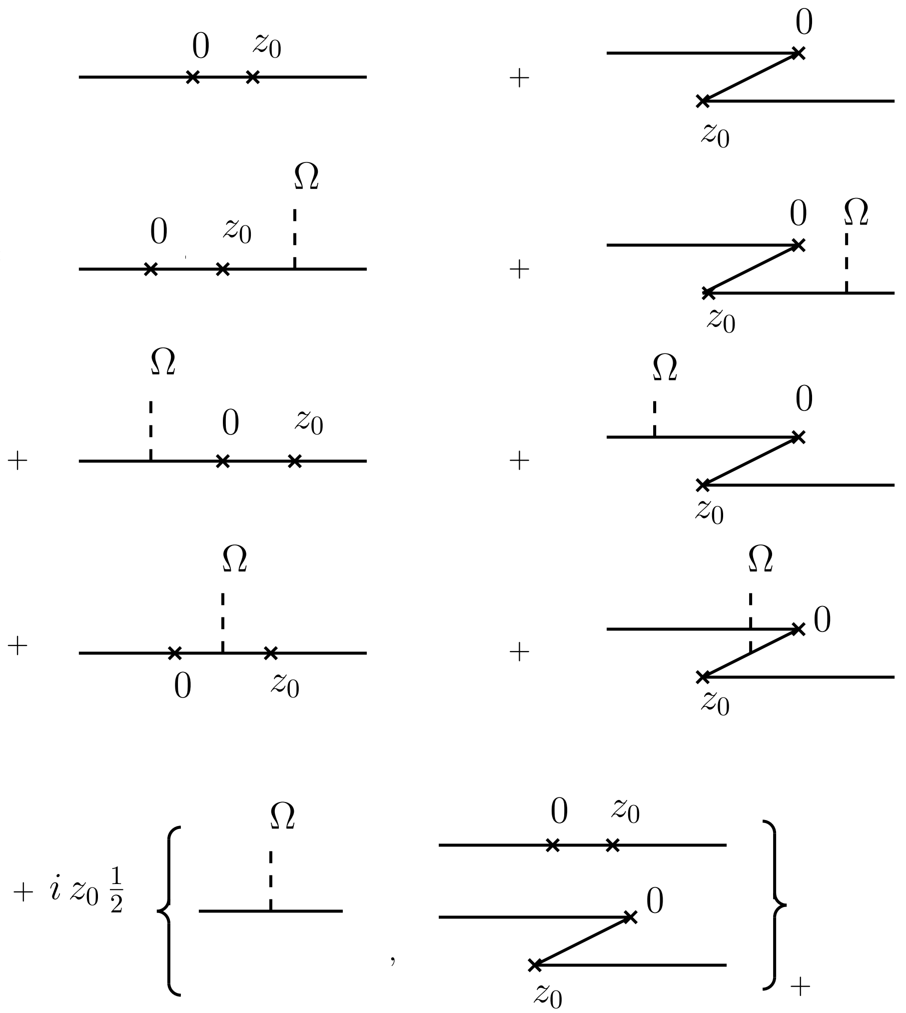
where

$$\langle O(0, z_0) \rangle_A = O(\Omega^0) + O(\Omega^1) + \cdots$$

with

$$O(\Omega^0) =$$

$$O(\Omega^1) =$$



$O(\Omega^1)$ correction resulting from expansion of $A^\dagger(0) O_a A(z_0)$ in z_0

nonlocality correction in time

sample form of $O(\Omega^0)$ contribution

$$\Delta u(x) - \Delta d(x) \;=\; \langle D_{33} \rangle_{p\uparrow} \cdot M_N \; N_c \sum_{n \leq 0} \langle\, n \,|\, (1 + \gamma^0 \gamma^3) \, \gamma_5 \, \delta_n \,|\, n \,\rangle$$

with

$$\delta_n \;\equiv\; \delta(x M_N - E_n - \hat{p}_3), \qquad (-1 < x < 1)$$

sample form of $O(\Omega^1)$ contribution

$$\Delta u(x) + \Delta d(x) \;=\; [\, \Delta u(x) + \Delta d(x) \,]^{(1)}_{\{A,B\}} + \underbrace{[\, \Delta u(x) + \Delta d(x) \,]^{(1)}_{C}}_{\text{nonlocality correction}}$$

with

$$[\, \Delta u(x) + \Delta d(x) \,]^{(1)}_{\{A,B\}} \;=\; \langle 2\, J_3 \rangle_{p\uparrow} \cdot M_N \, \frac{N_c}{2I} \times \sum_{m = all, n \leq 0} \frac{1}{E_m - E_n} \, \langle\, n \,|\, \tau_3 \,|\, m \,\rangle \, \langle\, m \,|\, \tau_3 \, (1 + \gamma^0 \gamma^3) \, \gamma_5 \, \delta_n \,|\, n \,\rangle$$

$$[\, \Delta u(x) + \Delta d(x) \,]^{(1)}_{C} \;=\; \langle 2\, J_3 \rangle_{p\uparrow} \cdot \frac{N_c}{4I} \times \frac{d}{dx} \sum_{n \leq 0} \langle\, n \,|\, \tau_3 \, (1 + \gamma^0 \gamma^3) \, \gamma_5 \, \delta_n \,|\, n \,\rangle$$

from

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} dz_0 \; i \, z_0 \;\; e^{\, i \, (x M_N - E_n - \hat{p}_3) \, z_0} \;=\; \frac{1}{M_N} \, \frac{d}{dx} \, \delta(x M_N - E_n - \hat{p}_3)$$

♣ **naive expression** for $\Delta u(x) + \Delta d(x)$
obtained with ignorance of nonlocality effects

$$\Delta u(x) + \Delta d(x) \;=\; \langle 2J_3 \rangle_{p\uparrow} \cdot M_N \frac{N_c}{2\,I}$$
$$\times \sum_{m>0,n\leq 0} \frac{1}{E_m - E_n} \;\langle n|\,\tau_3\,|m\rangle\langle m|\,\tau_3\,(1+\gamma^0\,\gamma^3)\,\gamma_5\;\delta_n\,|n\rangle$$

— from **occupied** to **nonoccupied** —

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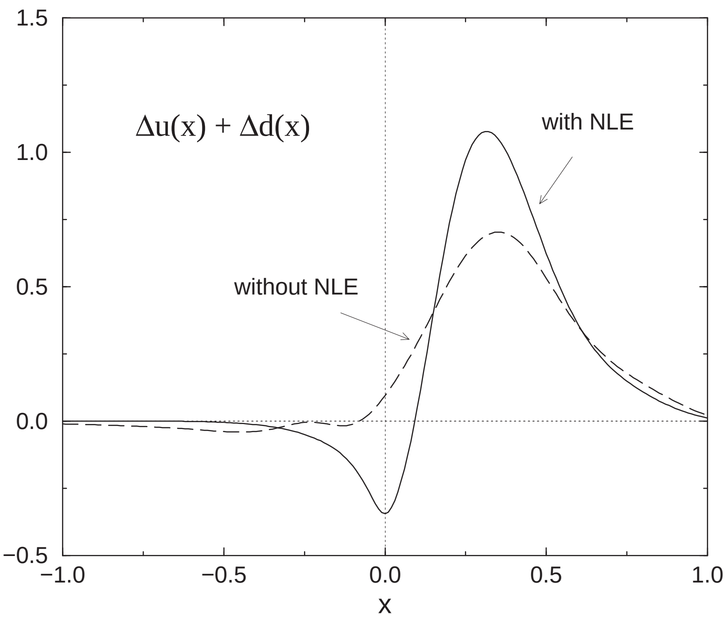
importance of **nonlocality effects**



$$\Delta u(-x) \;+\; \Delta d(-x) \;=\; [\,\Delta\bar{u}(x) \;+\; \Delta\bar{d}(x)\,] \qquad (\,0 < x < 1\,)$$

Comparison with High Energy Data

- **only 1 parameter** of the CQSM (dynamical quark mass M) is fixed from the analyses of **nucleon LE observables**

$$M = 375\,\mathrm{MeV} \quad (\text{ this gives } M_{PV} \simeq 562\,\mathrm{MeV})$$

$$\Downarrow$$

parameter free predictions for PDF

- use predictions of CQSM as **initial-scale distributions**

$$u(x),\ \bar{u}(x),\ d(x),\ \bar{d}(x),\ \ \Delta u(x),\ \Delta\bar{u}(x),\ \Delta d(x),\ \Delta\bar{d}(x)$$

$$s(x) = \bar{s}(x) = 0,\ g(x) = 0,\ \Delta s(x) = \Delta\bar{s}(x) = 0,\ \Delta g(x) = 0$$

- **scale dependence of PDF** : (setting $N_c = 3$)

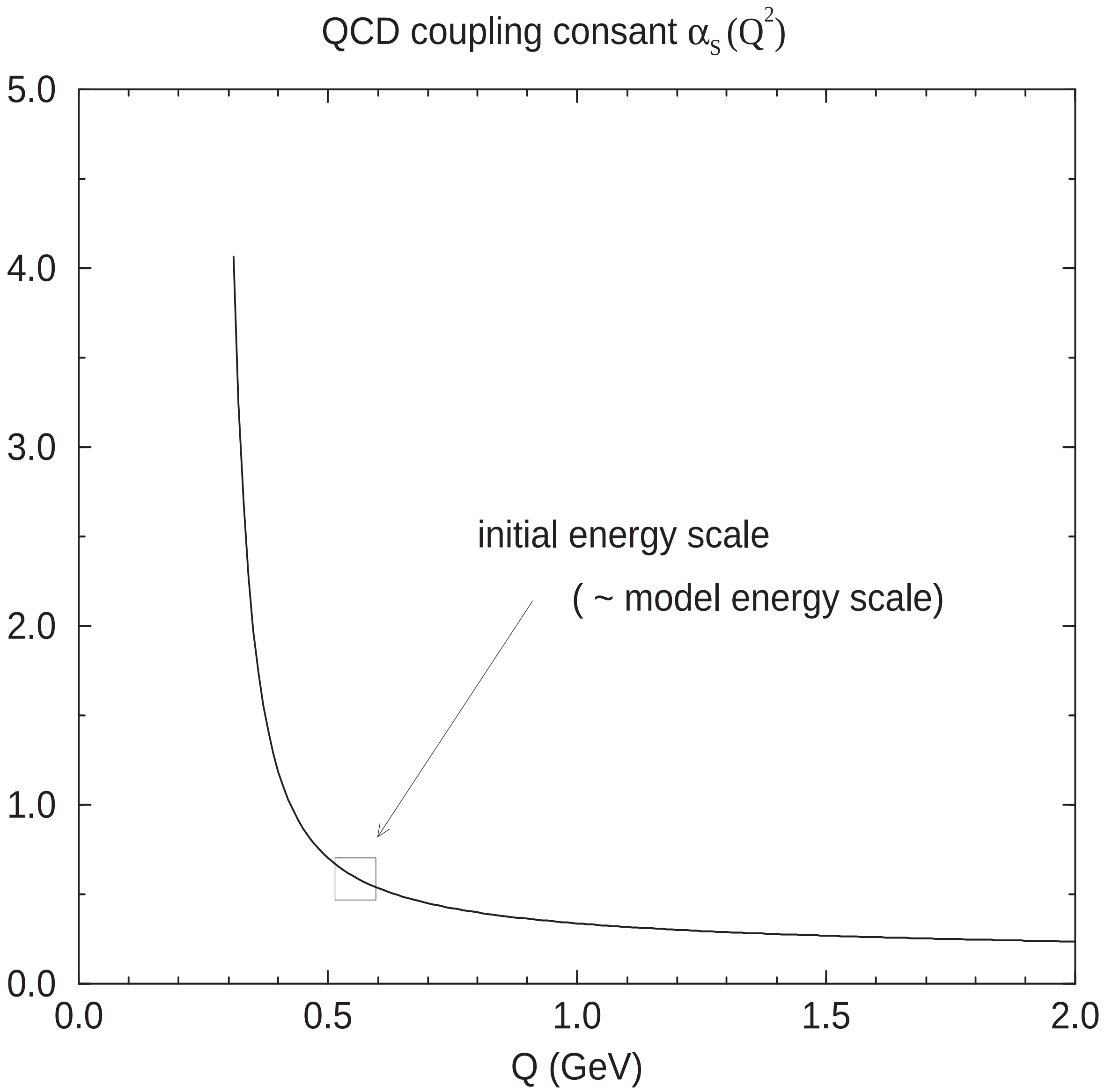
Fortran Program of DGLAP evolution eqs. at **NLO**

provided by Saga group

initial energy scale is fixed to be

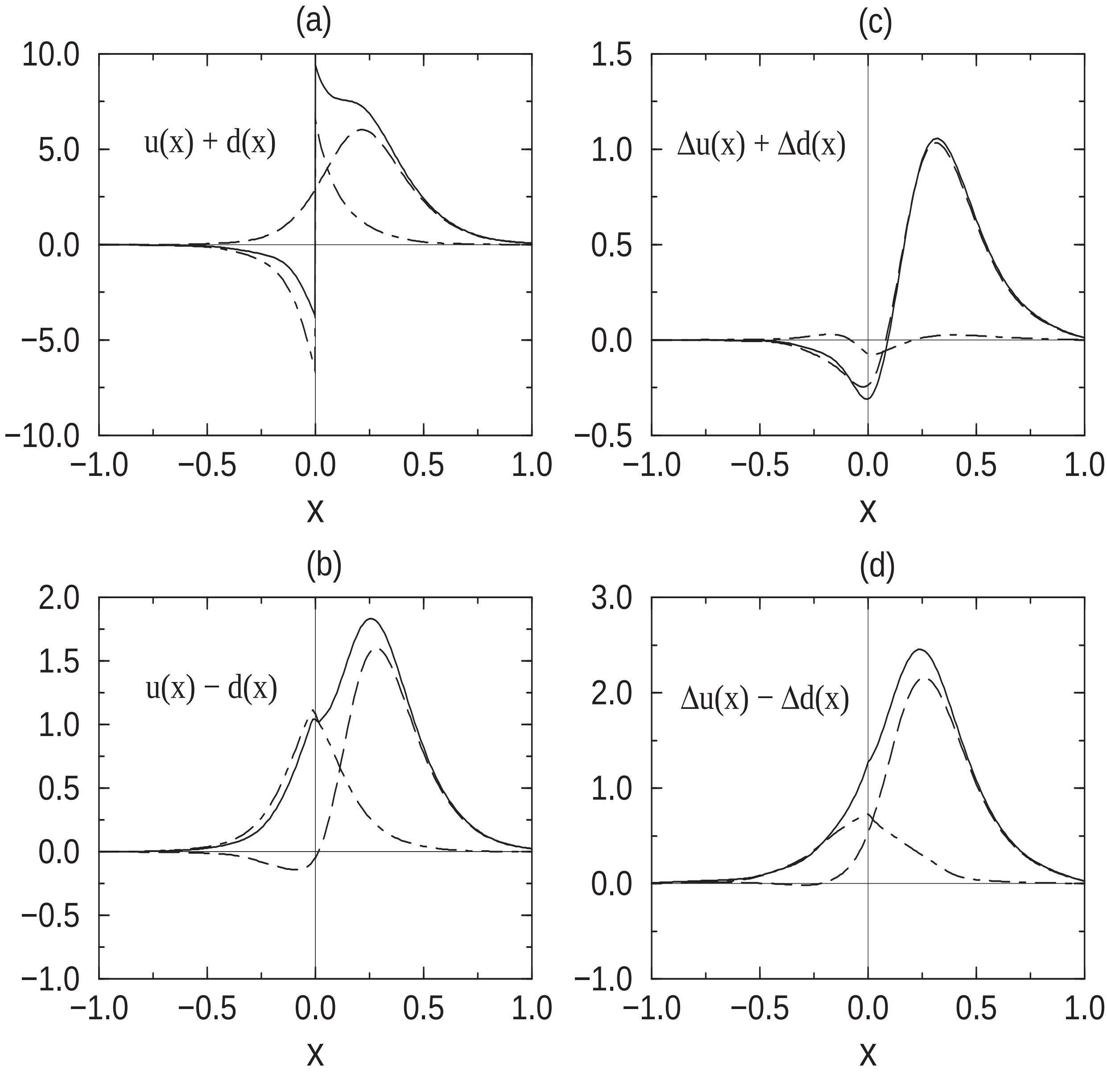
$$Q^2_{ini} = 0.30\,\mathrm{GeV}^2 \quad \simeq \quad (550\,\mathrm{MeV})^2$$

QCD running coupling constant at NLO



- **perturbative QCD** is barely applicable for taking account of $\boldsymbol{Q}^2$**-dependence** of PDF

model predictions for twist-2 PDF



$$u(-x) \ \pm \ d(-x) \quad = \ - [\, \bar{u}(x) \ \pm \bar{d}(x) \,] \qquad (0 < x < 1)$$

$$\Delta u(-x) \ \pm \ \Delta d(-x) \ = \ \Delta \bar{u}(x) \ \pm \ \Delta \bar{d}(x) \qquad (0 < x < 1)$$

complete set of twist-2 PDF

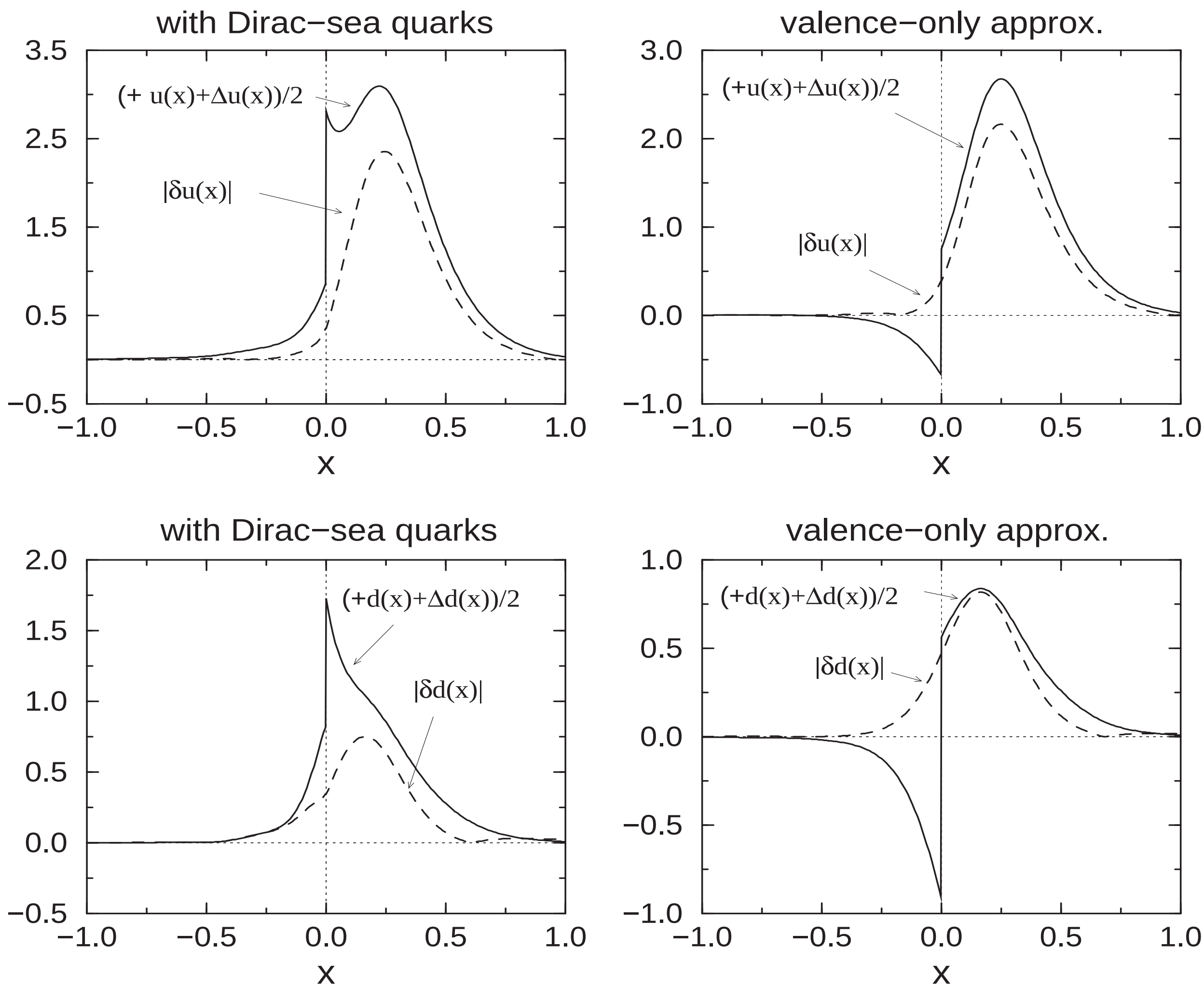
$q(x)$: unpolarized distribution

$\Delta q(x)$: longitudinally polarized distribution

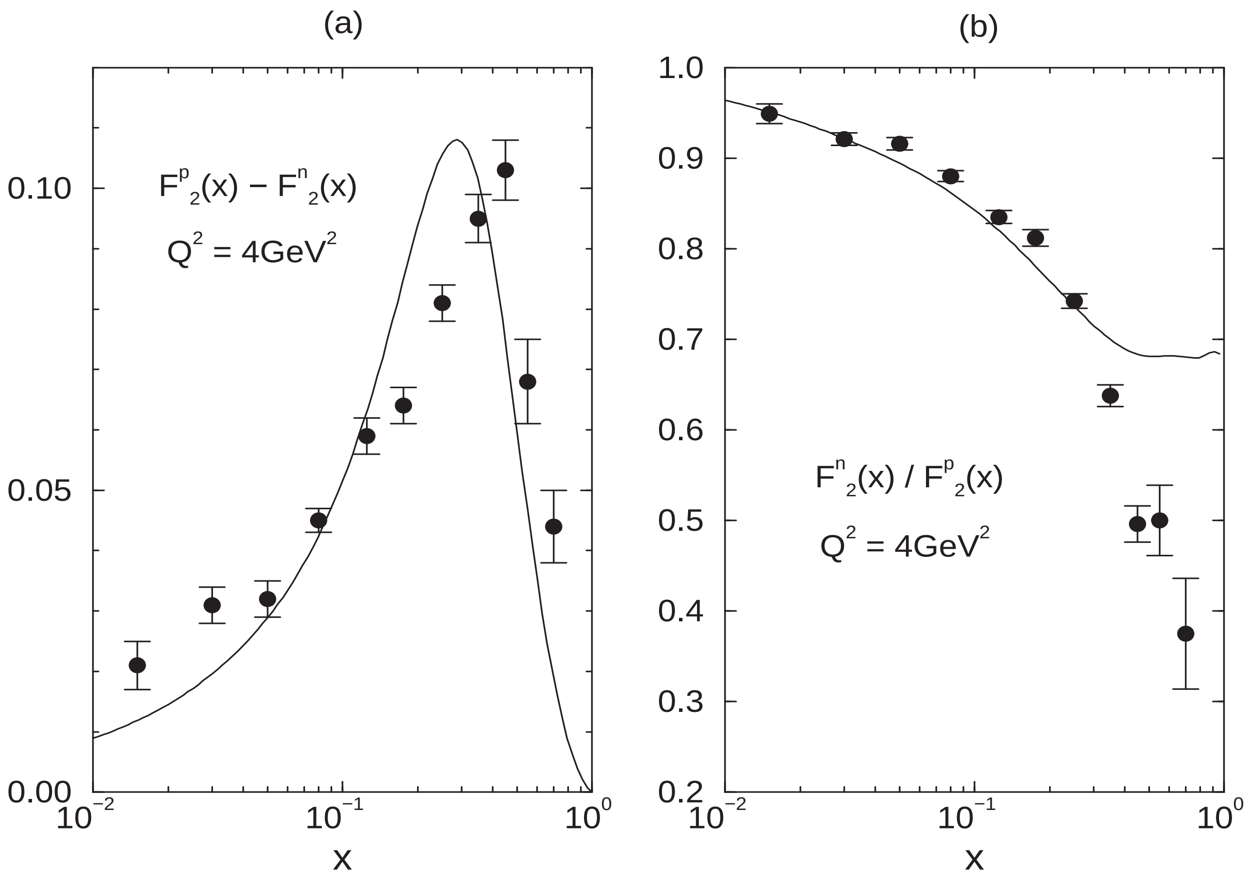
$\delta q(x)$: **transversity distribution**

Soffer inequality

$$|\pm\,\delta q(x)| \;\leq\; \frac{1}{2}\,\left(\pm\, q(x) \;+\; \Delta q(x)\right) \qquad \begin{cases} x > 0 \\ x < 0 \end{cases}$$



comparison with NMC data



Gottfried sum

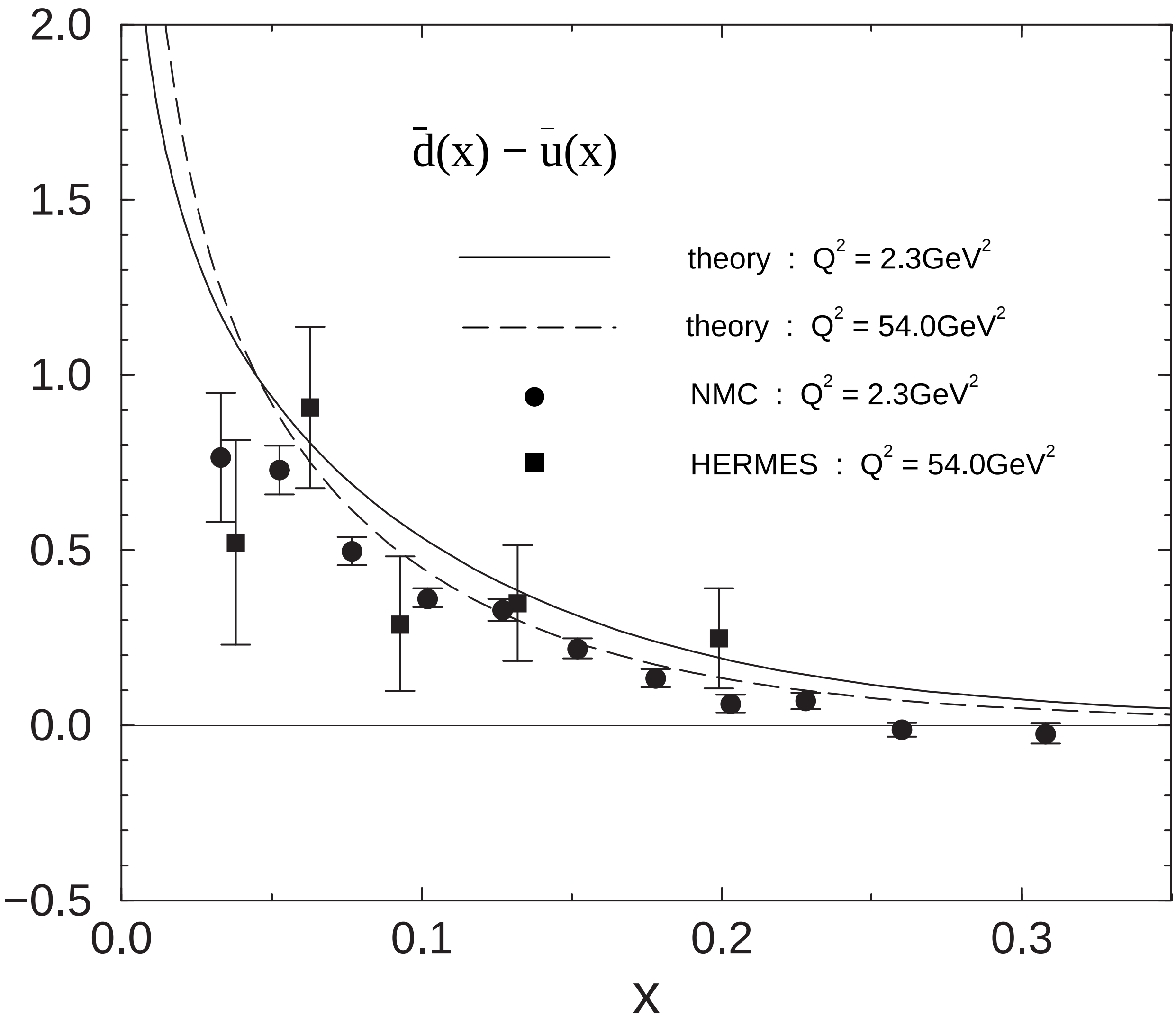
$$S_G \;=\; \int_0^1 \;\frac{F_2^p(x) - F_2^n(x)}{x}\; dx \;=\; \frac{1}{3} \;+\; \int_0^1 \;\{\, \bar{u}(x) \;-\; \bar{d}(x)\,\}$$

$$S_G^{th}(Q^2 = 4\,\mathrm{GeV}^2) \;\simeq\; \mathbf{0.204} \quad < \quad \frac{1}{3}$$

$$\Updownarrow$$

$$S_G^{exp}(Q^2 = 4\,\mathrm{GeV}^2) \;=\; \mathbf{0.228 \pm 0.007}$$

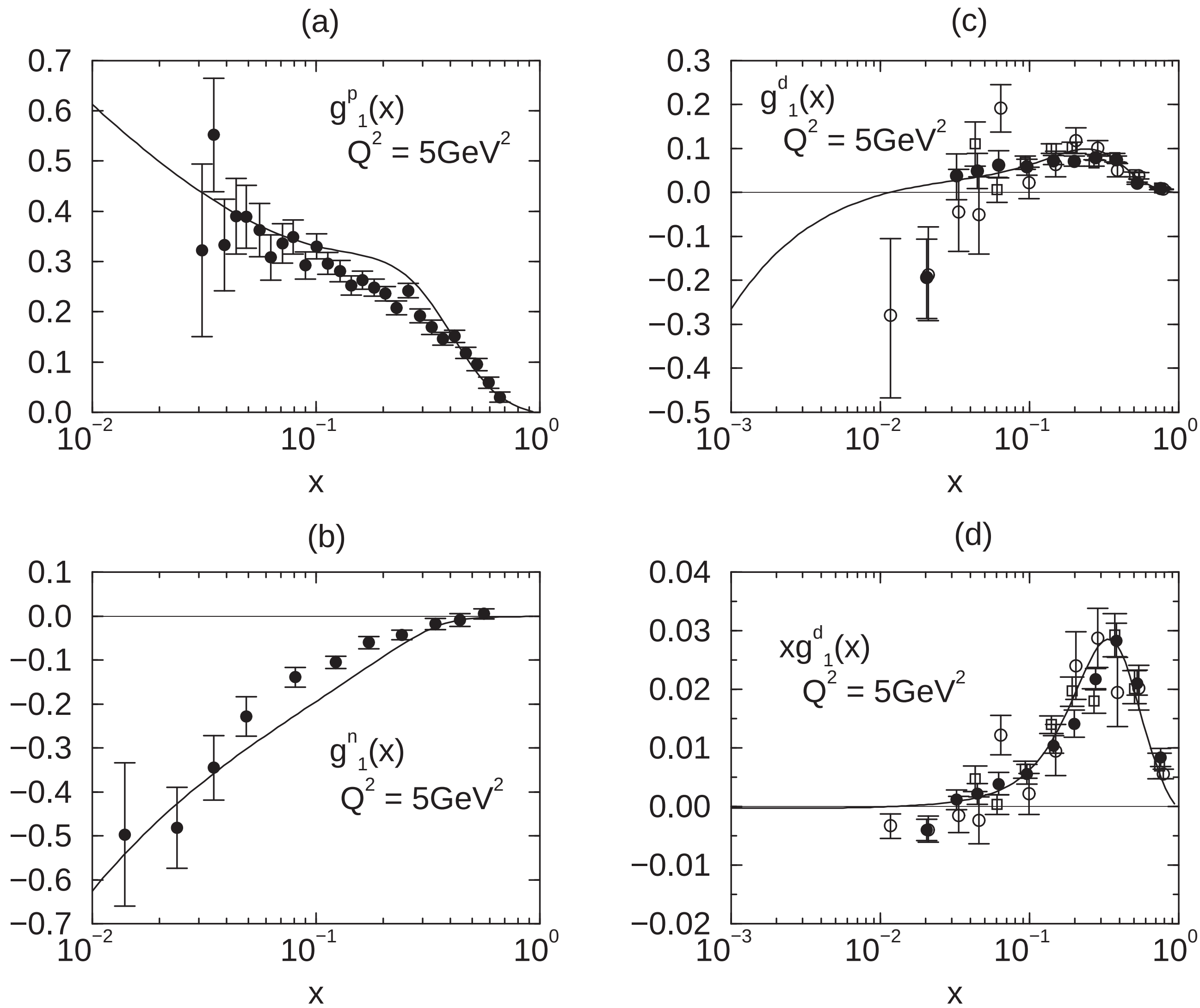
Fig.3



$\Downarrow$

$\bar{d}(x) \; > \; \bar{u}(x)$ in the proton !

comparison with EMC and SMC data



good reproduction of **neutron data**

$\Downarrow$

manifestation of **chiral symmetry** in high energy observables !

核子スピンの謎とは ？

EMC 実験 (1988) とその後の追実験により

- 核子のスピンの中クォークの固有スピンから来る部分は約２０％のみである

$$\Downarrow$$

残りの約８０％を担うものは何か？

— **核子スピンのパズル** —

核子スピン和則 （$\Leftarrow$ Lorentz 不変性）

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta g + L_g$$

$$\Downarrow$$

$$\text{答え} = L_q \ \text{または}\ \Delta g \ \text{または}\ L_g$$

[**留意点**]

- 核子のスピン・コンテントは観測のエネルギー・スケールに依存する量？

$$\Downarrow$$

スケールに言及せず議論しても意味がない

CQSMの予言する核子スピン・コンテンツ

M. W. and H. Yoshiki (1991)

[※] 模型の成り立つ**低エネルギー・スケール**でのコンテンツと理解する。

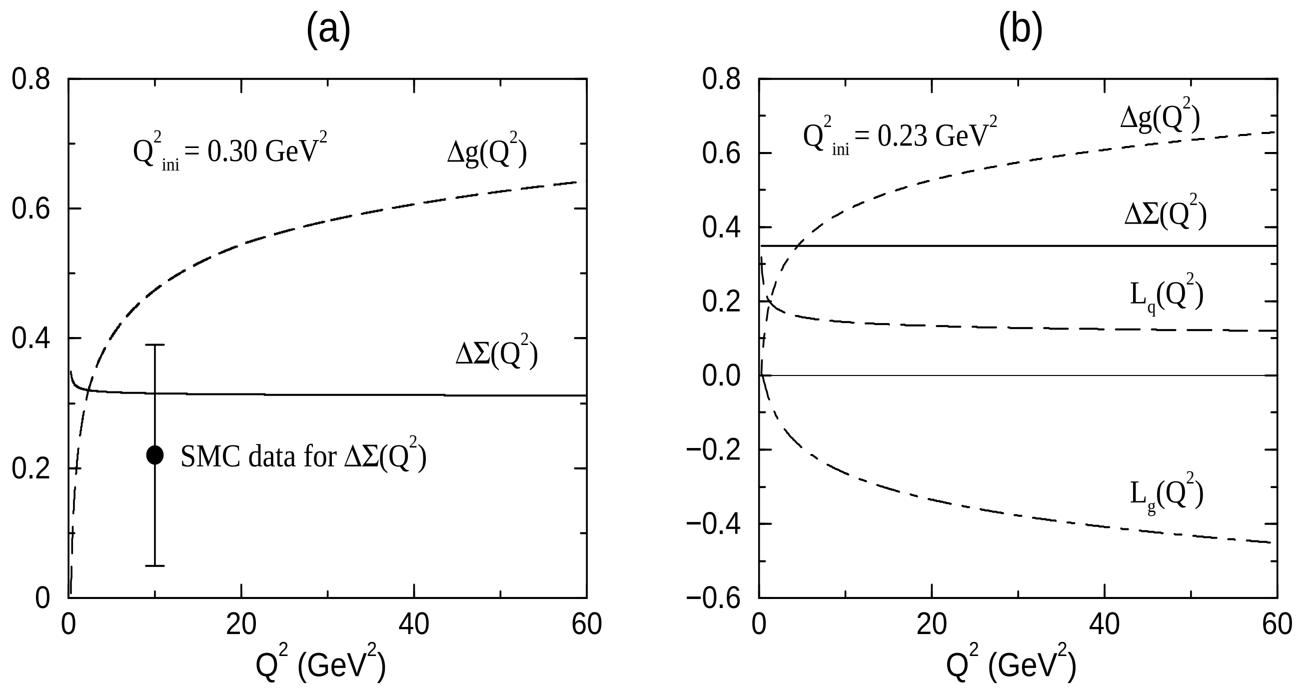
contents	quark	antiquark	total
$\Delta\Sigma$	0.40	- 0.05	**0.35**
$2\,L_q$	0.46	**0.19**	**0.65**
$\Delta\Sigma\,+\,2\,L_q$	0.86	0.15	1.00

- 核子スピンの約３５％のみが**クォークの固有スピン**に由来
- 残りの約６５％はクォーク及び反クォークの**軌道角運動量**が運ぶ。

⇕

回転するハリネズミ型ソリトンとしての核子描像

核子のスピン・コンテントのスケール依存性



- 低エネルギー・スケールでは小さくても Δg はエネルギーと共に急激に増大する

- 我々がナイーブに核子のスピン・コンテントというときには非摂動的 QCD の領域、すなわち低エネルギー・スケールでのそれを想定している

- カイラル・クォーク・ソリトン模型は低エネルギー・スケールで

$$\Delta\Sigma \simeq 0.35, \quad 2L_q \simeq 0.65, \quad \Delta g = \text{小さい？}$$

と主張している

検証可能か？ $\Longrightarrow$ GPD と Ji の角運動量和則！

axial charge と tensor charge の特徴的差異について

R.L. Jaffe and X. Ji, N. P. B375 (1992) 527

$$g_A^{(0,3)} = \int_0^1 dx \, \{ [\Delta u(x) + \Delta \bar{u}(x)] \pm [\Delta d(x) + \Delta \bar{d}(x)] \}$$
$$g_T^{(0,3)} = \int_0^1 dx \, \{ [\, \delta u(x) \, - \, \delta \bar{u}(x)] \pm [\, \delta d(x) \, - \, \delta \bar{d}(x)] \}$$

NRQM

$$g_A^{(3)} = g_T^{(3)} = \frac{5}{3}$$
$$g_A^{(0)} = g_T^{(0)} = 1$$

MIT bag model (with lower component)

$$g_A^{(3)} = \frac{5}{3} \cdot \int (f^2 - \frac{1}{3} g^2), \qquad g_T^{(3)} = \frac{5}{3} \cdot \int (f^2 + \frac{1}{3} g^2)$$
$$g_A^{(0)} = 1 \cdot \int (f^2 - \frac{1}{3} g^2), \qquad g_T^{(0)} = 1 \cdot \int (f^2 + \frac{1}{3} g^2)$$

where

$$\psi_{g.s.} = \begin{pmatrix} f(r) \, \chi_s \\ g(r) \, i \, \boldsymbol{\sigma} \cdot \hat{\boldsymbol{r}} \, \chi_s \end{pmatrix}$$

important observation

$$g_A^{(0)} / g_A^{(3)} = g_T^{(0)} / g_T^{(3)} = 3/5$$

$$\Downarrow$$

limit of **valence quark model** without chiral symmetry ?

axial versus tensor charges

	CQSM	MIT-bag	Lattice QCD*)	Experiment
$g_A^{(3)}$	**1.40**	1.06	0.99	**1.254** $\pm$ 0.006 (Q^2-indep.)
$g_A^{(0)}$	**0.35**	0.64	0.18	**0.31** $\pm$ 0.07 ($Q^2 = 10\ \text{GeV}^2$)
$g_T^{(3)}$	**1.22**	1.34	1.07	–
$g_T^{(0)}$	**0.67**	0.80	0.56	–
$g_A^{(0)}/g_A^{(3)}$	**0.25**	0.60	0.18	**0.24**
$g_T^{(0)}/g_T^{(3)}$	**0.55**	0.60	0.52	–

*) Y.Kuramashi, at Quark Lepton Nuclear Physics, (RCNP), 1997

$\Downarrow$

- qualitative difference between **transversity distributions** and **longitudinally polarized distributions**

$$g_A^{(0)}/g_A^{(3)} = g_T^{(0)}/g_T^{(3)} \quad \text{(NRQM, MIT bag model)}$$

$$g_A^{(0)}/g_A^{(3)} << g_T^{(0)}/g_T^{(3)} \quad \text{(\textbf{CQSM}, Lattice)}$$